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PRIME GAMMA NEAR-RINGS WITH DERIVATIONS

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Abstract: Let N be a prime Γnear-ring with the center $Z(N)$. The objective of this paper is to study derivations on N. We prove two results:

(a) Let N be 2-torsion free and let D_1 and D_2 be derivations on N such that D_1D_2 is also a derivation. Then $D_1 = 0$ or $D_2 = 0$ if and only if $[D_1(x), D_2(y)]_\alpha = 0$ for all x, $y \in N$, $\alpha \in \Gamma$;

(b) Let n be an integer greater than 1, N be $n!$-torsion free, and D be a derivation with $D^n(N) = \{0\}$. Then $D(Z(N)) = \{0\}$.

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1. Introduction

The notion of derivations in near-rings has been introduced by Bell and Mason [2]. They obtained some basic properties of derivations in near-rings. Then Asci [1] investigated commutativity conditions for a Γ-near-ring with derivations. Cho and Jun [7] studied some characterizations of Γ-near-rings and some regularity conditions. In classical ring theory, Posner [14], Herstein [11], Bergen

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[5], Bell and Daif [4] studied derivations in prime and semiprime rings and obtained commutativity results of prime and semiprime rings with derivations. In near ring theory, Bell and Mason [3], and also Cho [8] worked on derivations in prime and semiprime near-rings.

In this paper, we deal with the prime Γ-near-rings with derivations. Here we extend the results of Wang [20] on prime near-rings of to Γ-near-rings.

2. Preliminaries

A Γ-near-ring is a triple $(N,\ +,\ \Gamma)$, where:

(i) $(N, +)$ is a group (not necessarily abelian),

(ii) Γ is a non-empty set of binary operations on N such that for each $\alpha \in \Gamma$,

$(N, +, \alpha)$ is a left near-ring.

(iii) $x\alpha(y\beta z) = (x\alpha y)\beta z$,for all $x, y, z \in N$and $\alpha,\ \beta \in \Gamma$.

Examples of Γ-near-rings and motivations to study has been given in [16, 17].

Throughout this paper, N will denote a zero-symmetric left Γ-near-ring with multiplication center Z(N). A Γ-near-ring N is called a prime Γ-near-ring if N has the property that for x, y∈ N, $x\Gamma N\Gamma y = \{\ 0\}$ implies $x = 0$ or $y = 0$. A Γ-near-ring N is called a semiprime Γ-near-ring if N has the property that for $x \in N$, $x\Gamma N\Gamma x = \{\ 0\}$ implies $x = 0$. A derivation D on N is an additive endomorphism of N with the property that for all $x,\ y \in N$ and $\alpha \in \Gamma$, $D(x\alpha y) = x\alpha D(y) + D(x)\alpha y$. An additive endomorphism D of N is called a derivation on N if $D(x\alpha y) = x\alpha D(y) + D(x)\alpha y$ for all $x,\ y \in N$, $\alpha \in \Gamma$ A Γ-near-ring N is called commutative if $(N,\ +)$ is abelian, and 2-torsion free if $2x = 0$ implies $x = 0$.

3. Derivations in Prime Γ-Near-Rings

We begin with the following lemmas on derivations on prime Γ-near-rings N.

Lemma 3.1. *Let D be an additive endomorphism of N. Then:*

$$D(x\alpha y) = x\alpha D(y) + D(x)\alpha y$$

for all $x, y \in N$and $\alpha \in \Gamma$, if and only if

$$D(x\alpha y) = D(x)\alpha y + x\alpha D(y)$$

for all $x,\ y \in N$ *and* $\alpha \in \Gamma$.

Proof. We assume that $D(x\alpha y) = x\alpha D(y) + D(x)\alpha y$ for all $x, y \in N$ and $\alpha \in \Gamma$.

Since

$$x\alpha(y + y) = x\alpha y + x\alpha y$$

and

$$D(x\alpha(y+y)) = x\alpha D(y+y)+D(x)\alpha(y+y) = x\alpha D(y)+x\alpha D(y)+D(x)\alpha y+D(x)\alpha y$$

and

$$D(x\alpha y + x\alpha y) = D(x\alpha y) + D(x\alpha y) = x\alpha D(y) + D(x)\alpha y + x\alpha D(y) + D(x)\alpha y$$

we get

$$x\alpha D(y) + D(x)\alpha y = D(x)\alpha y + x\alpha D(y),$$

so

$$D(x\alpha y) = D(x)\alpha y + x\alpha D(y),$$

for all $x, y \in N \alpha \in \Gamma$.

The converse is proved in a similar way.

Note that due to Lemma 3.1, D is a derivation if and only if

$$D(x\alpha y) = D(x)\alpha y + x\alpha D(y), \text{ for all } x, y \in N \text{and } \alpha \in \Gamma.$$

We make use the following lemma from [15], Lemma 3.5.

Lemma 3.2. *Suppose that* N *is a prime* Γ*-near-ring.*

(i) any nonzero element of the center of N *is not zero divisor.*

(ii) If there exist a nonzero element of $Z(N)$ *such that* $x + x \in Z(N)$*, then* $(N, +)$ *is commutative.*

(iii) Let dbe a nonzero derivation on N*. If one of the* $x\Gamma d(N) = \{\ 0\}$ *and* $d(N)\Gamma x = \{\ 0\}$ *holds then* $x = 0$*.*

Lemma 3.3. *Let* D *be a derivation on* N*. Then* N *satisfies the following partial distributive laws*

(i) $(x\alpha D(y) + D(x)\alpha y)\beta z = x\alpha D(y)\beta z + D(x)\alpha y\beta z$ *for all* $x,\ y,\ z \in N$ *and* $\alpha,\ \beta \in \Gamma$

(ii) $(D(x)\alpha y + x\alpha D(y))\beta z = D(x)\alpha y\beta z + x\alpha D(y)\beta z$ *for all* $x,\ y,\ z \in N$ *and* $\alpha,\ \beta \in \Gamma$

Proof. (i) Consider $D((x\alpha y)\beta z) = D(x\alpha(y\beta z))$ for all x, y, $z \in N$ and α, $\beta \in \Gamma$. Then by using Lemma 3.1, we obtain the required result.

(ii) Consider $D((x\alpha y)\beta z) = D(x\alpha(y\beta z)$. Then we make use Lemma 3.1 to obtain,

$$D((x\alpha y)\beta z) = D(x\alpha y)\beta z + x\alpha y\beta D(z) = (D(x)\alpha y + x\alpha D(y))\beta z + x\alpha y\beta D(z)$$

and

$$\begin{aligned} &D(x\alpha(y\beta z)) = D(x)\alpha y\beta z + x\alpha D(y\beta z) \\ &= D(x)\alpha y\beta z+x\alpha(D(y)\beta z+y\beta D(z)) = D(x)\alpha y\beta z+x\alpha D(y)\beta z+x\alpha y\beta D(z), \end{aligned}$$

for all x, y, $z \in N$ and α, $\beta \in \Gamma$.

Comparing the above two relations we get the required result:

$$(D(x)\alpha y + x\alpha D(y))\beta z = D(x)\alpha y\beta z + x\alpha D(y)\beta z$$

for all x, y, $z \in N$ and α, $\beta \in \Gamma$.

Now we prove our main results.

Theorem 3.4. *Let N be a 2-torsion-free prime Γnear-ring, and let D_1 and D_2 be derivations on N such that D_1D_2 is also a derivation. Then the following two conditions are equivalent:*

1. *either $D_1 = 0$ or $D_2 = 0$;*
2. *(ii) $[D_1(x), D_2(y)]_\alpha = 0$ for all $xy \in N$, $\alpha \in \Gamma$*

Proof. We need prove only the part (ii) $\Rightarrow$ (i) since (i) $\Rightarrow$ (ii) is obvious. Consider

$$D_1D_2(x\alpha y) = x\alpha D_1D_2(y) + D_1D_2(x)\alpha y$$

for all x, y, $z \in N$ and $\alpha \in \Gamma$.

On the other hand, both D_1 and D_2 are derivations, therefore

$$\begin{aligned} D_1D_2(x\alpha y) =& D_1(D_2(x\alpha y)) \\ =& D_1(x\alpha D_2(y) + D_2(x)\alpha y) \\ =& D_1(x\alpha D_2(y)) + D_1(D_2(x)\alpha y) \\ =& x\alpha D_1D_2(y) + D_1(x)\alpha D_2(y) + D_2(x)\alpha D_1(y) + D_1D_2(x)\alpha y, \end{aligned}$$

for all x, $y \in N$ and $\alpha \in \Gamma$.

The above two relations for $D_1D_2(x\alpha y)$ give

$$D_1(x)\alpha D_2(y) + D_2(x)\alpha D_1(y) = 0 \text{ for all } x,\, y \in N \text{ and } \alpha \in \Gamma. \tag{1}$$

Replacing x by $x\beta D_2(z)$, $z \in N$, $\beta \in \Gamma$ in (1), by using Lemma 31 and Lemma 3.3 we get

$$\begin{aligned} 0 =& D_1(x\beta D_2(z))\alpha D_2(y) + D_2(x\beta D_2(z))\alpha D_1(y) \\ =& (D_1(x)\beta D_2(z) + x\beta D_1D_2(z))\alpha D_2(y) + (x\beta D_2^2(z) + D_2(x)\beta D_2(z))\alpha D_1(y) \\ =& D_1(x)\beta D_2(z)\alpha D_2(y) + x\beta D_1D_2(z)\alpha D_2(y) + x\beta D_2^2(z)\alpha D_1(y) \\ &+ D_2(x)\beta D_2(z)\alpha D_1(y) \\ =& D_1(x)\beta D_2(z)\alpha D_2(y) + x\beta(D_1D_2(z)\alpha D_2(y) + D_2^2(z)\alpha D_1(y)) \\ &+ D_2(x)\beta D_2(z)\alpha D_1(y), \end{aligned}$$

for all x, y, $z \in N$ and α, $\beta \in \Gamma$.

Then by using the equation (1) we obtain

$$x\beta(D_1D_2(z)\alpha D_2(y) + D_2^2(z)\alpha D_1(y)) = 0.$$

If we replace x by $D_2(z)$ in (1) then we get

$$D_1D_2(z)\alpha D_2(y) + D_2^2(z)\alpha D_1(y) = 0.$$

Therefore

$$D_1(x)\beta D_2(z)\alpha D_2(y) + D_2(x)\beta D_2(z)\alpha D_1(y) = 0 \quad \text{for all } x,\, y,\, z \in N \text{ and } \alpha,\, \beta \in \Gamma. \tag{2}$$

Replacing x and y by z in (1), respectively, we obtain

$$D_2(z)\alpha D_1(y) = -D_1(z)\alpha D_2(y) \text{ for all } y,\, z \in N \text{ and } \alpha \in \Gamma,$$

and

$$D_1(x)\alpha D_2(z) = -D_2(x)\alpha D_1(z) \text{ for all } x,\, z \in N \text{ and } \alpha \in \Gamma.$$

Since N is a zero-symmetric left Γnear-ring, then due to (2) we obtain

$$\begin{aligned} 0 =& (-D_2(x)\beta D_1(z))\alpha D_2(y) + D_2(x)\beta(-D_1(z)\alpha D_2(y)) \\ =& D_2(x)\beta(-D_1(z))\alpha D_2(y) + D_2(x)\beta(-D_1(z)\alpha D_2(y)) \\ =& D_2(x)\beta[(-D_1(z))\alpha D_2(y) - D_1(z)\alpha D_2(y)] \end{aligned}$$

for all $x,\, y,\, z \in N$ and $\alpha,\, \beta \in \Gamma$.

If $D_2 \neq 0$ then thanks to Lemma 3.2 we have

$$(-D_1(z))\alpha D_2(y) - D_1(z)\alpha D_2(y) = 0.$$

That is

$$D_1(z)\alpha D_2(y) = (-D_1(z))\alpha D_2(y) \text{ for all } y,\, z \in N \text{ and } \alpha \in \Gamma. \tag{3}$$

The condition (ii) provides

$$(-D_1(z))\alpha D_2(y) = D_1(-z)\alpha D_2(y) = D_2(y)\alpha D_1(-z)$$

$$= D_2(y)\alpha(-D_1(z)) = -D_2(y)\alpha D_1(z) = -D_1(z)\alpha D_2(y).$$

Therefore

$$(-D_1(z))\alpha D_2(y) = -D_1(z)\alpha D_2(y) \text{ for all } y,\, z \in N \text{ and } \alpha \in \Gamma. \tag{4}$$

From (3) and (4) we obtain $2D_1(z)\alpha D_2(y) = 0$ for all $y,\, z \in N$ and $\alpha \in \Gamma$ Since N is 2-torsion-free, this gives $D_1(z)\alpha D_2(y) = 0$ for all $y,\, z \in N$ and $\alpha \in \Gamma$ Therefore $D_1(z)\alpha D_2(N) = \{\, 0\}$. But $D_2 \neq 0$, so $D_1(z) = 0$ for all $z \in N$, that is $D_1 = 0$.

Note that Lemma 3.5. from [15] can be derived now as the following corollary from the theorem.

Corollary 3.5 *Let N be a 2-torsion free prime Γnear-ring, and let D be a derivation on N such that $D^2 = 0$. Then $D = 0$.*

Proof. It is clear that $D^2 = 0$ is a derivation on N, and we have

$$\begin{aligned} 0 = D^2(x\alpha y) &= D(x\alpha D(y) + D(x)\alpha y) = D(x\alpha D(y)) + D(D(x)\alpha y) \\ &= x\alpha D^2(y) + D(x)\alpha D(y) + D(x)\alpha D(y) + D^2(x)\alpha y = 2D(x)\alpha D(y) \end{aligned}$$

for all $x,\, y \in N$ and $\alpha \in \Gamma$. Since N is 2-torsion free we obtain $D(x)\alpha D(y) = 0$ for all $x,\, y \in N$ and $\alpha \in \Gamma$ Similarly we get $D(y)\alpha D(x) = 0$ for all $x,\, y \in N$ and $\alpha \in \Gamma$ Therefore $[D(x),\, D(y)]_\alpha = 0$ for all $x,\, y \in N$ and $\alpha \in \Gamma$. Hence by Theorem 3.4, we get $D = 0$.

Another consequence of Theorem 3.4 is the following

Corollary 3.6. *Let N be a Γnear-ring and D_1 and D_2 be derivations on N such that D_1D_2 is a derivation. Then D_2D_1 is also a derivation.*

Proof. Obviously D_2D_1 is an additive endomorphism of N. By Theorem 3.4 we have

$$\begin{aligned} D_2D_1(x\alpha y) =& D_2(D_1(x)\alpha y + x\alpha D_1(y)) = D_2(D_1(x)\alpha y) + D_2(x\alpha D_1(y)) \\ =& D_2D_1(x)\alpha y + (D_1(x)\alpha D_2(y) + D_2(x)\alpha D_1(y)) + x\alpha D_2D_1(y) \\ =& D_2D_1(x)\alpha y + x\alpha D_2D_1(y) \end{aligned}$$

for all $xy \in N$ and $\alpha \in \Gamma$.

This completes the proof.

The following is an extension of Wang [20] on Leibniz's rule for derivations of rings to Γnear-rings.

Theorem 3.7. *Let N be a $n!$-torsion free Γ-near-ring Let n be an integer $n \geq 2$ and D be a derivation on N. Then*

$$\begin{aligned} D^n(x\alpha y) = D^n(x)\alpha y + \binom{n}{1} D^{n-1}(x)\alpha D(y) + \cdots + \binom{n}{i} D^{ni}(x)\alpha D^i(y) \\ + \cdots + \binom{n}{n-1} D(x)\alpha D^{n-1}(y) + x\alpha D^n(y), \end{aligned}$$

for all x, $y \in N$ and $\alpha \in \Gamma$.

Proof. By Theorem 3.4 it can easily seen that

$$D(x)\alpha y + nx\alpha D(y) = nx\alpha D(y) + D(x)\alpha y$$

for all x, $y \in N$, $\alpha \in \Gamma$ and n be an integer. The same observation gives

$$nD(x)\alpha y + nx\alpha D(y) = n(D(x)\alpha y + x\alpha D(y)) \quad \text{for all } x,\ y \in N,\ \alpha \in \Gamma, \text{ and } n \text{ be an integer.} \tag{5}$$

We proceed the proof of Leibniz's rule by induction on n. Let $n = 2$. Then

$$\begin{aligned} D^2(x\alpha y) =& D(D(x)\alpha y + x\alpha D(y)) \\ =& D(D(x)\alpha y) + D(x\alpha D(y)) \\ =& D^2(x)\alpha y + D(x)\alpha D(y) + D(x)\alpha D(y) + x\alpha D^2(y) \\ =& D^2(x)\alpha y + 2D(x)\alpha D(y) + x\alpha D^2(y). \end{aligned}$$

Assume that Leibniz's rule holds for $n-1$. That is, if N is $(n-l)!$-torsion-free.

Then

$$D^{n-1}(x\alpha y) = D^{n-1}(x)\alpha y + \cdots + \binom{n-1}{i-1} D^{ni}(x)\alpha D^{i-1}(y) + \binom{n-1}{i} D^{ni-1}(x)\alpha D^{i}(y) + \cdots + x\alpha D^{n-1}(y).$$

Since $n!$-torsion-freeness implies $(n-l)!$-torsion-freeness, by (5) we have

$$D^{n}(x\alpha y) = D(D^{n-1}(x\alpha y))$$

$$= D(D^{n-1}(x)\alpha y + ... + \binom{n-1}{i-1} D^{ni}(x)\alpha D^{i-1}(y) + \binom{n-1}{i} D^{ni-1}(x)\alpha D^{i}(y) + \cdots + x\alpha D^{n-1}(y))$$

$$= D(D^{n-1}(x)\alpha y + ... + \binom{n-1}{i-1} D(D^{ni}(x)\alpha D^{i-1}(y)) + \binom{n-1}{i} D(D^{ni-1}(x)\alpha D^{i}(y)) + \cdots . + D(x\alpha D^{n-1}(y))$$

$$= D^{n}(x)\alpha y + D^{n-1}(x)\alpha D(y) + \cdots + (\binom{n-1}{i-1} D^{ni+1}(x)\alpha D^{i-1}(y) + D^{ni}(x)\alpha D^{i}(y)) + \binom{n-1}{i} (D^{ni}(x)\alpha D^{i}(y) + D^{ni-1}(x)\alpha D^{i+1}(y)) + \cdots . + D(x)\alpha D^{n-1}(y) + x\alpha D^{n}(y)$$

$$= D^{n}(x)\alpha y + \binom{n-1}{i-1} D^{ni+1}(x)\alpha D^{i-1}(y) + \binom{n-1}{i-1} D^{ni}(x)\alpha D^{i}(y) + \binom{n-1}{i} D^{ni}(x)\alpha D^{i}(y) + \binom{n-1}{i} (D^{ni-1}(x)\alpha D^{i+1}(y) + \cdots + x\alpha D^{n}(y)$$

$$= D^{n}(x)\alpha y + \cdots + \binom{n-1}{i-1} D^{ni}(x)\alpha D^{i}(y) + \binom{n-1}{i} D^{ni}(x)\alpha D^{i}(y) + \cdots + x\alpha D^{n}(y)$$

$$= D^{n}(x)\alpha y + \cdots + [\binom{n-1}{i-1} + \binom{n-1}{i}] D^{ni}(x)\alpha D^{i}(y) + \cdots\cdots + x\alpha D^{n}(y)$$

$$= D^{n}(x)\alpha y + \cdots + \binom{n}{i} D^{ni}(x)\alpha D^{i}(y) + \cdots\cdots + x\alpha D^{n}(y)$$

The proof is complete.

Lemma 3.8. *Let N be a Γnear-ring with center $Z(N)$, and let D be a derivation on N. Then $D(Z(N)) \subseteq Z(N)$.*

Proof. By Theorem 3.4, we have

$$x\alpha D(z) + z\alpha D(x) = x\alpha D(z) + D(x)\alpha z = D(x\alpha z) = D(z\alpha x)$$
$$= D(z)\alpha x + z\alpha D(x)$$

for all $z \in Z(N)$, $x \in N$ and $\alpha \in \Gamma$.

Therefore $x\alpha D(z) = D(z)\alpha x$ for all x, $z \in N$ and $\alpha \in \Gamma$. Thus $D(z) \in Z(N)$

Lemma 3.9. *Let $n \geq 2$, and let N be an n!-torsion free Γnear-ring and D be a derivation with $D^n(N) = \{\ 0\}$. Then for each $y \in N$, either $D(y) = 0$ or there exists k ($0 <k <n$) such that $D^k(y)$ is a nonzero divisor of zero.*

Proof. Since n!-torsion-freeness implies (n - l)!-torsion-freeness, we may assume that $D^{n-1}(N) \neq \{\ 0\}$ Choose x such that $D^{n-1}(x) \neq 0$. Assume that $D(y) \neq 0$. Then there exists k with $0 <k <n$ such that $D^k(y) \neq 0$ and $D^{k+1}(y) = 0$. Then due to Theorem 3.7 we obtain

$$0 = D^n(x\alpha D^{k1}(y)) = D^n(x)\alpha D^{k-1}(y) + \binom{n}{1} D^{n-1}(x)\alpha D^k(y)$$
$$+\binom{n}{2} D^{n-2}(x)\alpha D^{k+1}(y) + \cdots = \binom{n}{1} D^{n-1}(x)\alpha D^k(y) = nD^{n-1}(x)\alpha D^k(y)$$

for all $y \in N$ and $\alpha \in \Gamma$.

Since N is n!-torsion-free, then we get $D^{n-1}(x)D^k(y) = 0$ for all $y \in N$ and $\alpha \in \Gamma$. By Lemma 3.2(i) $D^k(y)$ is a nonzero divisor of zero.

We finalize the paper by the following two theorems which can be easily proven by using the previous results

Theorem 3.10. *Let n be an integer ≥ 1 and N be a prime Γnear-ring with center $Z(N)$, and let N be n!-torsion free and D be a derivation with $D^n(N) = \{0\}$. Then $D(Z(N)) = \{0\}$.*

Theorem 3.11. *Let n be a positive integer and N be an n!-torsion free Γnear-ring with no divisor of zero, then N admits no nonzero derivation D with $D^n = 0$.*

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