MINIMAL g^*p -SEPARATION AXIOMS IN TOPOLOGICAL SPACES

A. Vadivel^{1 §}, A. Swaminathan²

^{1,2}Department of Mathematics (FEAT)

Annamalai University

Annamalainagar, Tamil Nadu, 608 002, INDIA

Abstract: In this paper, some new separation axioms called minimal g p - T_0 , minimal g p - c -regular, minimal g p - o -regular and minimal g p -completely regular spaces are introduced and investigated. Some of their basic properties in topological spaces are studied. Also their relationship among themselves as well as with other known separation axioms have been studied.

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Key Words: minimal g p - T_0 , minimal g p - c -regular space, minimal g p - o -regular space and minimal g p -completely regular space

1. Introduction

In the years 2001 and 2003, F. Nakaoka and N. Oda [2, 3, 4] introduced and studied minimal open (resp. minimal closed) sets and maximal open (resp. maximal closed) sets, which are subclasses of open (resp. closed) sets. The complements of minimal open sets and maximal open sets are called maximal closed sets and minimal closed sets respectively. Also S. S. Benchalli, Basavaraj M. Ittanagi, R. S. Wali and others, introduced and studied minimal open sets and maps in topological spaces [5, 6]. Recently, A. Vadivel and A. Swaminathan [7, 8] have introduced and studied minimal g p -open sets and maps in topological spaces.

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[§]Correspondence author

2. Preliminaries

Definition 2.1.. (see [2]) A proper nonempty open subset U of a topological space X is said to be a minimal open set if any open set which is contained in U is ϕ or U .

Definition 2.2.. (see [3]) A proper nonempty open subset U of a topological space X is said to be maximal open set if any open set which contains U is X or U .

Definition 2.3.. (see [4]) A proper nonempty closed subset F of a topological space X is said to be a minimal closed set if any closed set which is contained in F is ϕ or F .

Definition 2.4.. (see [4]) A proper nonempty closed subset F of a topological space X is said to be maximal closed set if any closed set which contains F is X or F .

Definition 2.5.. (see [5]) Let X and Y be the topological spaces. A map $f : X \rightarrow Y$ is called:

(1) minimal continuous (briefly min-continuous) if $f^{-1}(M)$ is an open set in X for every minimal open set M in Y .

(2) minimal irresolute (briefly min-irresolute) if $f^{-1}(M)$ is minimal open set in X for every minimal open set M in Y .

(3) minimal open (briefly min-open) if $f(M)$ is an open set in Y for every minimal open set M in X .

(4) strongly minimal open (briefly s-min open) if $f(M)$ is minimal open set in Y for every minimal open set M in X .

(5) minimal closed (briefly min-closed) if $f(F)$ is a closed set in Y for every minimal closed set F in X .

(6) strongly minimal closed (briefly s-min closed) if $f(F)$ is minimal closed set in Y for every minimal closed set F in X .

Definition 2.6.. (see [9]) A subset A of a space (X, τ) is said to be g p -closed if $pCl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in (X, τ) . The complement of a g p -closed set is called g p -open.

Definition 2.7.. (see [7]) A proper nonempty g p -open subset U of a topological space X is said to be a minimal g p -open set if any g p -open set which is contained in U is ϕ or U .

Definition 2.8.. (see [7]) A proper nonempty g p -open subset U of a

topological space X is said to be maximal g p -open set if any g p -open set which contains U is X or U .

Definition 2.9.. (see [7]) A proper nonempty g p -closed subset F of a topological space X is said to be a minimal g p -closed set if any g p -closed set which is contained in F is ϕ or F .

Definition 2.10.. (see [7]) A proper nonempty g p -closed subset F of a topological space X is said to be maximal g p -closed set if any g p -closed set which contains F is X or F .

Definition 2.11.. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be g p -continuous [9] if $f^{-1}(V)$ is g p -open set of (X, τ) for every open set V of (Y, σ) .

Definition 2.12.. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be g p -open map [9] if $f(V)$ is g p -open set of (Y, σ) for every open set V of (X, τ) .

Definition 2.13.. (see [8]) Let X and Y be the topological spaces. A map $f : X \rightarrow Y$ is called:

- (1) minimal g p -continuous (briefly min- g p -continuous) if $f^{-1}(M)$ is a g p -open set in X for every minimal g p -open set M in Y .
- (2) minimal g p -irresolute (briefly min- g p -irresolute) if $f^{-1}(M)$ is minimal g p -open set in X for every minimal g p -open set M in Y .
- (3) minimal g p -open (briefly min- g p -open) if $f(M)$ is a g p -open set in Y for every minimal g p -open set M in X .
- (4) strongly minimal g p -open (briefly s-min- g p -open) if $f(M)$ is minimal g p -open set in Y for every minimal g p -open set M in X .
- (5) minimal g p -closed (briefly min- g p -closed) if $f(F)$ is a g p -closed set in Y for every minimal g p -closed set F in X .
- (6) strongly minimal g p -closed (briefly s-min- g p -closed) if $f(F)$ is minimal g p -closed set in Y for every minimal g p -closed set F in X .

Definition 2.14.. A space (X, τ) is said to be g pre-regular [1] (briefly g p -regular) if for every g p -closed set F and a point $x \notin F$, there exist disjoint pre-open sets U and V such that $F \subseteq U$ and $x \in V$.

Definition 2.15.. A space (X, τ) is said to be g pre-normal [1] (briefly g p -normal) if for any pair of disjoint g p -closed sets A and B , there exist disjoint pre-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

3. Minimal g^*p-T_0 -Spaces

Definition 3.1.. (see) A topological space X is said to be minimal g $p-T_0$ (briefly, min- g $p-T_0$) space if for any two distinct points x, y in X , there exists a minimal g p -open set containing one of them but not the other.

Example 3.1. Let $X = \{a, b, c\}$ be with $\tau = \{\phi, X, \{a\}, \{b, c\}\}$. Then (X, τ) is a min- g $p-T_0$ space. Let x, y be two distinct points in X . Then $\{x\}$ is a minimal g p -open set containing x but not y . Therefore (X, τ) is a min- g $p-T_0$ space.

Theorem 3.1.. Every min- g $p-T_0$ space is a g $p-T_0$ space.

Proof. Let X be a min- g $p-T_0$ space. To prove X is a g $p-T_0$ space. Let x, y be two distinct points in X . Since X is min- g $p-T_0$ space, there exists a minimal g p -open set M in X such that $x \in M, y \notin M$. Since every minimal g p -open set is a g p -open set, M is a g p -open set in X . Thus, for any two distinct points x, y in X , there exists a g p -open set M in X such that $x \in M, y \notin M$. Hence X is a g $p-T_0$ space. $\square$

Remark 3.1.. The converse of the Theorem 3.1. need not be true.

Example 3.2. Let $X = \{a, b, c\}$ be with $\tau = \{\phi, X, \{c\}, \{b, c\}, \{a, c\}\}$. Then (X, τ) is a g $p-T_0$ space but it is not min- g $p-T_0$ space, since $a, b \in X$ with $a \neq b$, there is no minimal g p -open set containing a but not b or containing b but not a .

Lemma 3.1.. Let X be a topological space and let Y be an open subspace of X . If A is minimal g p -open set in Y then there exists a minimal g p -open set U in X such that $Y \cap U = A$.

Proof. Let Y be an open subspace of a topological space X . Suppose A is a minimal g p -open set in Y , which implies A is a g p -open set in Y . Then there exists g p -open set U in X such that $Y \cap U = A$. To prove U is a minimal g p -open set in X . Suppose U is not a minimal g p -open set in X . Then there exists a g p -open set V in X such that $\phi \neq V$ is proper subset of U . Consider $Y \neq V$. Now we have $V \subset U$ which implies $Y \cap V \subset Y \cap U$ which implies $Y \cap V \subset A$. Since V is nonempty proper subset of U , $Y \cap V$ is nonempty proper subset of $Y \cap U = A$. Also $Y \cap V$ is a g p -open set in Y . Therefore A cannot be a minimal g p -open set in Y , which is contradiction to the hypothesis. Therefore U must be a minimal g p -open set in X . $\square$

Theorem 3.2.. If $f : X \rightarrow Y$ is a bijection, strongly minimal g p -open map and X is min- g $p-T_0$ space then Y is also min- g $p-T_0$ space.

Proof. Let $f : X \rightarrow Y$ be a bijection, strongly minimal g p -open map and X be a min- g p - T_0 space. To prove Y is min- g p - T_0 space. Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since f is bijection, there exist $x_1, x_2 \in X$ with $x_1 \neq x_2$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since X is min- g p - T_0 space, there exists a minimal g p -open set M in X such that $x_1 \in M, x_2 \notin M$. Since f is strongly minimal g p -open map, $f(M)$ is a minimal g p -open set in Y . Now we have $x_1 \in M \Rightarrow f(x_1) \in f(M) \Rightarrow y_1 \in f(M)$ and $x_2 \notin M \Rightarrow f(x_2) \notin f(M) \Rightarrow y_2 \notin f(M)$. Thus, for any two distinct points $y_1, y_2 \in Y$, there exists a minimal g p -open set $f(M)$ in Y such that $y_1 \in f(M), y_2 \notin f(M)$. Hence Y is min- g p - T_0 space. $\square$

Theorem 3.3.. *If $f : X \rightarrow Y$ is a bijection, minimal g p -open map and X is min- g p - T_0 space then Y is a g p - T_0 space.*

Proof. Similar to that of Theorem 3.2. $\square$

Theorem 3.4.. *If $f : X \rightarrow Y$ is a bijection, minimal g p -irresolute map and Y is min- g p - T_0 space then X is also min- g p - T_0 space.*

Proof. Let $f : X \rightarrow Y$ be a bijection, minimal g p -irresolute map and Y be a min- g p - T_0 space. To prove X is min- g p - T_0 space. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Since f is bijection, there exist $y_1, y_2 \in Y$ with $y_1 \neq y_2$ such that $f(x_1) = y_1$ and $f(x_2) = y_2 \Rightarrow x_1 = f^{-1}(y_1)$ and $x_2 = f^{-1}(y_2)$. Since Y is min- g p - T_0 space, there exists a minimal g p -open set M in Y such that $y_1 \in M, y_2 \notin M$. Since f is minimal g p -irresolute, $f^{-1}(M)$ is a minimal g p -open set in X . Now we have $y_1 \in M \Rightarrow f^{-1}(y_1) \in f^{-1}(M) \Rightarrow x_1 \in f^{-1}(M)$ and $y_2 \notin M \Rightarrow f^{-1}(y_2) \notin f^{-1}(M) \Rightarrow x_2 \notin f^{-1}(M)$. Thus, for any two distinct points x_1, x_2 in X , there exists a minimal g p -open set $f^{-1}(M)$ in X such that $x_1 \in f^{-1}(M), x_2 \notin f^{-1}(M)$. Hence X is a min- g p - T_0 space. $\square$

Theorem 3.5.. *If $f : X \rightarrow Y$ is a bijection, minimal g p -continuous map and Y is min- g p - T_0 space then X is a g p - T_0 space.*

Proof. Similar to that of Theorem 3.4. $\square$

4. Minimal g^*p -c-Regular Spaces and Minimal g^*p -Completely Regular Spaces

Definition 4.1.. A topological space X is said to be:

(1) minimal g p -c-regular (briefly min g p -c-regular) if for every point $x \in X$ and each minimal g p -closed set F in X such that $x \notin F$, there exist preopen sets U, V in X such that $x \in U, F \subset V$ and $U \cap V = \phi$.

(2) minimal g p -o-regular (briefly min g p -o-regular) if for every point $x \in X$ and each g p -closed set F in X such that $x \notin F$, there exist minimal preopen sets M, N in X such that $x \in M$, $F \subset N$ and $M \cap N = \phi$.

(3) minimal g p -completely regular (briefly min g p -completely regular) if for every point $x \in X$ and each minimal g p -closed set F in X such that $x \notin F$, there exists a g p -continuous map $f : X \rightarrow [0, 1]$ such that $f(x) = 0$ and $f(F) = \{1\}$.

Theorem 4.1.. *Every regular space is min- g p -c-regular space.*

Proof. Let X be a regular space. To prove X is min g p -c-regular space. Let $x \in X$ and F be any minimal g p -closed set in X such that $x \notin F$. Since every minimal g p -closed set is a g p -closed set, F is a g p -closed set in X such that $x \notin F$. Since X is regular space, there exist preopen sets U, V in X such that $x \in U$, $F \subset V$ and $U \cap V = \phi$. Thus, for every point $x \in X$ and each minimal g p -closed set F in X such that $x \notin F$, there exist preopen sets U, V in X such that $x \in U$, $F \subset V$ and $U \cap V = \phi$. Hence X is a min g p -c-regular space. $\square$

Theorem 4.2.. *Every completely regular space is min- g p -completely regular space.*

Proof. Let X be a completely regular space. To prove X is min- g p -completely regular space. Let $x \in X$ and F be any minimal g p -closed set in X such that $x \notin F$. Since every minimal g p -closed set is a g p -closed set, F is a g p -closed set in X such that $x \notin F$. Since X is completely regular space, there exists a g p -continuous map $f : X \rightarrow [0, 1]$ such that $f(x) = 0$ and $f(F) = \{1\}$. Thus, for every point $x \in X$ and each minimal g p -closed set F in X such that $x \notin F$, there exists a g p -continuous map $f : X \rightarrow [0, 1]$ such that $f(x) = 0$ and $f(F) = \{1\}$. Hence X is a min- g p -completely regular space. $\square$

Theorem 4.3.. *Every min- g p -completely regular space is min- g p -c-regular space.*

Proof. Let X be a min- g p -completely regular space. To prove X is min- g p -c-regular space. Let $x \in X$ and F be any minimal g p -closed set in X such that $x \notin F$. Since X is min- g p -completely regular space, there exists a g p -continuous map $f : X \rightarrow [0, 1]$ such that $f(x) = 0$ and $f(F) = \{1\}$. Also it is easy to see that the space $[0, 1]$ with relativesed usual topology is Hausdorff space. Hence there exist g p -open subsets G and H of $[0, 1]$ such that $0 \in G$, $1 \in H$ and $G \cap H = \phi$. Since f is g p -continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are g p -open subsets in X such that

$$f^{-1}(G) \cap f^{-1}(H) = f^{-1}(G \cap H) = f^{-1}(\phi) = \phi$$

Further

$$f(x) = 0 \in G \Rightarrow f(x) \in G \Rightarrow x \in f^{-1}(G)$$

and

$$f(F) = \{1\} \subset H \Rightarrow f(F) \subset H \Rightarrow F \subset f^{-1}(H)$$

Thus, for every point $x \in X$ and each minimal g p -closed set F in X such that $x \notin F$, there exist g p -open sets $f^{-1}(G)$, $f^{-1}(H)$ in X such that $x \in f^{-1}(G)$, $F \subset f^{-1}(H)$ and $f^{-1}(G) \cap f^{-1}(H) = \phi$. Hence X is a min- g p -c-regular space. $\square$

Theorem 4.4.. *Let X be a topological space. If X is a min- g p -c-regular space then for every point $x \in X$ and each maximal g p -open set U containing x , there exists a preopen set V in X such that $x \in V \subset pcl(V) \subset U$.*

Proof. Let $x \in X$ and U be any maximal g p -open set in X such that $x \in U$. Then $X - U$ is a minimal g p -closed set in X such that $x \notin X - U$. Since X is min- g p -c-regular space, there exist preopen sets V , W in X such that $x \in V$, $X - U \subset W$ and $V \cap W = \phi$. Now we have

$$\begin{aligned} V \cap W = \phi &\Rightarrow V \subset X - W \Rightarrow pcl(V) \subset pcl(X - W) = X - W \\ &\Rightarrow pcl(V) \subset X - W \longrightarrow (1) \end{aligned}$$

since $X - W$ is a preclosed set in X . Also we have

$$X - U \subset W \Rightarrow X - W \subset U \longrightarrow (2)$$

From (1) and (2), we have $pcl(V) \subset U$. Therefore $x \in V \subset pcl(V) \subset U$. Hence the proof. $\square$

Theorem 4.5.. *Let X be a topological space. If X is a min- g p -o-regular space then for every point $x \in X$ and each preopen set M containing x , there exists a minimal g p -open set N in X such that $x \in N \subset pcl(N) \subset M$.*

Proof. Let $x \in X$ and M be any preopen set in X such that $x \in M$. Then $X - M$ is a g p -closed set in X such that $x \notin X - M$. Since X is min- g p -o-regular space, there exist minimal preopen sets N , W in X such that $x \in N$, $X - M \subset W$ and $N \cap W = \phi$. Now we have

$$\begin{aligned} N \cap W = \phi &\Rightarrow N \subset X - W \Rightarrow pcl(N) \subset pcl(X - W) = X - W \\ &\Rightarrow pcl(N) \subset X - W \longrightarrow (3), \end{aligned}$$

since $X - W$ is a g p -closed set in X . Also we have

$$X - M \subset W \Rightarrow X - W \subset M. \longrightarrow (4)$$

From (3) and (4), we have $pcl(N) \subset M$. Therefore $x \in N \subset pcl(N) \subset M$. Hence the proof. $\square$

Theorem 4.6.. *A topological space X is min- g p -completely regular if and only if for every point $x \in X$ and each maximal g p -open set G containing x , there exists a g p -continuous map $f : X \rightarrow [0, 1]$ such that $f(x) = 0$ and $f(y) = 1$ for all $y \in X - G$.*

Proof. Let X be a min- g p -completely regular space and G be any maximal g p -open set containing $x \in X$. Then $X - G$ is a minimal g p -closed set in X such that $x \notin X - G$. Since X is min- g p -completely regular, there exists a g p -continuous map $f : X \rightarrow [0, 1]$ such that $f(x) = 0$ and $f(y) = 1$ for all $y \in X - G$.

Conversely, let $x \in X$ and F be any minimal g p -closed set in X such that $x \notin F$. Therefore $X - F$ is a maximal g p -open set containing x . By hypothesis, there exists g p -continuous map $f : X \rightarrow [0, 1]$ such that $f(x) = 0$ and $f(y) = 1$ for all $y \in X - (X - F) = F$. Hence the space X is a min- g p -completely regular space. $\square$

Theorem 4.7.. *Every min- g p -c-regular, g p -normal space is min- g p -completely regular space.*

Proof. Let X be a min- g p -c-regular and g p -normal space. To prove X is min- g p -completely regular space. Let $x \in X$ and F be any minimal g p -closed set in X such that $x \notin F$. Now $x \in X - F$, where $X - F$ is a maximal g p -open set in X . Since X is min- g p -c-regular space, by characterization of min- g p -c-regular space, there exists a preopen set U in X such that $x \in U \subset pcl(U) \subset X - F$ which implies $x \in pcl(U)$ and $pcl(U) \cap F = \phi$. Now $pcl(U)$ and F are disjoint preclosed sets in X , since every minimal g p -closed set is a closed. Again since X is g p -normal, by Urysohn's lemma, there exists a g p -continuous map $f : X \rightarrow [0, 1]$ such that $f(pcl(U)) = \{0\}$ and $f(F) = 1$. That is $f(x) = 0$ for all $x \in pcl(U)$ and $f(x) = 1$ for all $x \in F$ (Since $x \in pcl(U)$ we have $f(x) = 0$). Thus, for every point $x \in X$ and each minimal g p -closed set F in X such that $x \notin F$, there exists a g p -continuous map $f : X \rightarrow [0, 1]$ such that $f(x) = 0$ and $f(x) = 1$ for all $x \in F$. Hence the space X is a min- g p -completely regular space. $\square$

Theorem 4.8.. *If $f : X \rightarrow Y$ is a bijection, minimal g p -irresolute, g p -open map and X is min- g p -c-regular space then Y is also min- g p -c-regular space.*

Proof. Let $f : X \rightarrow Y$ be a bijection, minimal g p -irresolute, g p -open map and X be a min- g p -c-regular space. To prove Y is min- g p -c-regular space. Let $y \in Y$ and F be any minimal g p -closed set in Y such that $y \notin F$. Since f is bijection, there exists a point $x \in X$ such that $f(x) = y \Rightarrow x = f^{-1}(y)$. Again since f is minimal g p -irresolute, $f^{-1}(F)$ is a minimal g p -closed set in X . Also

$$y \notin F \Rightarrow f^{-1}(y) \notin f^{-1}(F) \Rightarrow x \notin f^{-1}(F).$$

Since X is min- g p -c-regular space, there exist preopen sets U, V in X such that $x \in U$, $f^{-1}(F) \subset V$ and $U \cap V = \phi$. Since f is a g p -open map, $f(U)$ and $f(V)$ are preopen sets in Y . Now we have

$$x \in U \Rightarrow f(x) \in f(U) \Rightarrow y \in f(U), f^{-1}(F) \subset V \Rightarrow f(f^{-1}(F)) \subset f(V) \Rightarrow F \subset f(V)$$

and

$$U \cap V = \phi \Rightarrow f(U \cap V) = f(\phi) \Rightarrow f(U) \cap f(V) = f(\phi) = \phi,$$

since f is a bijection. Thus, for every point $y \in Y$ and each minimal g p -closed set F in Y such that $y \notin F$, there exist preopen sets $f(U), f(V)$ in Y such that $y \in f(U)$, $F \subset f(V)$ and $f(U) \cap f(V) = \phi$. Hence Y is a min g p -c-regular space. $\square$

Theorem 4.9.. *If $f : X \rightarrow Y$ is a bijection, g p -continuous, strongly minimal g p -closed map and Y is min- g p -c-regular space then X is also min- g p -c-regular space.*

Proof. Let $f : X \rightarrow Y$ be a bijection, g p -continuous, strongly minimal g p -closed map and Y be a min- g p -c-regular. To prove X is min- g p -c-regular space. Let $x \in X$ and F be any minimal g p -closed set in X such that $x \notin F$. Since f is bijection, there exists a point $y \in Y$ such that $f(x) = y \Rightarrow x = f^{-1}(y)$. Also since f is strongly minimal g p -closed map, $f(F)$ is a minimal g p -closed set in Y such that

$$x \notin F \Rightarrow f(x) \notin f(F) \Rightarrow y \notin f(F).$$

Since Y is min- g p -c-regular space, there exist preopen sets U, V in Y such that $y \in U$, $f(F) \subset V$ and $U \cap V = \phi$. Since f is g p -continuous, $f^{-1}(U), f^{-1}(V)$ are preopen sets in X . Now we have

$$y \in U \Rightarrow f^{-1}(y) \in f^{-1}(U) \Rightarrow x \in f^{-1}(U), f(F) \subset V \Rightarrow f^{-1}(f(F)) \subset f^{-1}(V) \Rightarrow F \subset f^{-1}(V)$$

and

$$f^{-1}(U \cap V) = f^{-1}(\phi) \Rightarrow f^{-1}(U) \cap f^{-1}(V) = \phi,$$

since f is a bijection. Thus, for every point $x \in X$ and each minimal g p -closed set F in X such that $x \notin F$, there exist preopen sets $f^{-1}(U)$, $f^{-1}(V)$ in X such that $x \in f^{-1}(U)$, $F \subset f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Hence X is a min g p - c -regular space. $\square$

Theorem 4.10.. *If $f : X \rightarrow Y$ is a bijection, g p -continuous, strongly minimal g p -open map and X is min- g p - o -regular space then Y is also min- g p - o -regular space.*

Proof. Let $f : X \rightarrow Y$ be a bijection, g p -continuous, strongly minimal g p -open map and X be a min- g p - o -regular space. To prove Y is min- g p - o -regular space. Let $y \in Y$ and F be any g p -closed set in Y such that $y \notin F$. Since f is bijection, there exists a point $x \in X$ such that $f(x) = y \Rightarrow x = f^{-1}(y)$. Again since f is g p -continuous, $f^{-1}(F)$ is a g p -closed set in X . Also,

$$y \notin F \Rightarrow f^{-1}(y) \notin f^{-1}(F) \Rightarrow x \notin f^{-1}(F).$$

Since X is min- g p - o -regular space, there exist minimal preopen sets M , N in X such that $x \in M$, $f^{-1}(F) \subset N$ and $M \cap N = \phi$. Since f is strongly minimal g p -open map, $f(M)$, $f(N)$ are minimal preopen sets in Y . Now we have

$$x \in M \Rightarrow f(x) \in f(M) \Rightarrow y \in f(M), \quad f^{-1}(F) \subset N \Rightarrow f(f^{-1}(F)) \subset f(N) \Rightarrow F \subset f(N)$$

and

$$M \cap N = \phi \Rightarrow f(M \cap N) = f(\phi) \Rightarrow f(M) \cap f(N) = f(\phi) = \phi,$$

since f is a bijection. Thus, for every point $y \in Y$ and each g p -closed set F in X such that $y \notin F$, there exist minimal preopen sets $f(M)$, $f(N)$ in Y such that $y \in f(M)$, $F \subset f(N)$ and $f(M) \cap f(N) = \phi$. Hence Y is a min g p - o -regular space. $\square$

Theorem 4.11.. *If $f : X \rightarrow Y$ is a bijection, g p -continuous, minimal g p -open map and X is min- g p - o -regular space then Y is a g p -regular space.*

Proof. Similar to that of Theorem 4.10. $\square$

Theorem 4.12.. *If $f : X \rightarrow Y$ is a bijection, minimal g p -irresolute, g p -closed map and Y is min- g p - o -regular space then X is also min- g p - o -regular.*

Proof. Let $f : X \rightarrow Y$ be a bijection, minimal g p -irresolute, g p -closed map and Y be a min- g p -o-regular space. To prove X is min- g p -o-regular space. Let $x \in X$ and F be any g p -closed set in X such that $x \notin F$. Since f is bijection, there exists a point $y \in Y$ such that $f(x) = y \Rightarrow x = f^{-1}(y)$. Also since f is g p -closed map, $f(F)$ is a g p -closed set in Y such that

$$x \notin F \Rightarrow f(x) \notin f(F) \Rightarrow y \notin f(F).$$

Since Y is min- g p -o-regular space, there exist minimal preopen sets M, N in Y such that $y \in M$, $f(F) \subset N$ and $M \cap N = \phi$. Since f is minimal g p -irresolute, $f^{-1}(M), f^{-1}(N)$ are minimal preopen sets in X . Now we have

$$y \in M \Rightarrow f^{-1}(y) \in f^{-1}(M) \Rightarrow x \in f^{-1}(M), f(F) \subset N \Rightarrow f^{-1}(f(F)) \subset f^{-1}(N) \Rightarrow F \subset f^{-1}(N)$$

and

$$f^{-1}(M \cap N) = f^{-1}(\phi) \Rightarrow f^{-1}(M) \cap f^{-1}(N) = \phi,$$

since f is a bijection. Thus, for every point $x \in X$ and each g p -closed set F in X such that $x \notin F$, there exist minimal preopen sets $f^{-1}(M), f^{-1}(N)$ in X such that $x \in f^{-1}(M), F \subset f^{-1}(N)$ and $f^{-1}(M) \cap f^{-1}(N) = \phi$. Hence X is a min g p -o-regular space. $\square$

Theorem 4.13.. *If $f : X \rightarrow Y$ is a bijection, minimal g p -continuous, g p -closed map and Y is min- g p -o-regular space then X is a g p -regular space.*

Proof. Similar to that of Theorem 4.12.. $\square$

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