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STUDY ON N-IDEAL OF A BF-ALGEBRAS

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Abstract: This paper introduces the notion of N-structured ideal of a BF-algebra by using the idea of an N-function.

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1. Introduction

Y. Imai and K. Iseki in [3] introduced two new classes of abstract algebras: BCK-algebras and BCI-algebras. In 2007, Andrzej Walendziak[1] defined the notion of a BF-algebra. In 1965, Zadeh introduced the notion of fuzzy subsets. Since then many research works have been done using fuzzy subsets[6] and Intuitionistic fuzzy sets in the various classes of abstract algebraic structures. Fuzzy BF-subalgebras are one among them developed by A. Borumand Saeid and M.A. Rezvani in [2] in 2009. Recently N-Subalgebras of BF-algebras were introduced by A.R. Hadipourand A. Borumand Saeid in [4]. Also Y.B. Jun, K.H. Lee and S.Z. Song in [5] discussed about N-ideals BCK/BCI-algebras. In this paper, we investigate N-structured ideal of a BF-algebra and establish some of their basic properties.

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2. Preliminaries

For any two elements a,b in $[-1,0]$, we use the notation $a \vee b$ to represent $max(a,b)$.

Definition 2.1. (N-Fuction and N-Structure) Consider a non-empty Set S. Denote the collection of functions from S to $[-1,0]$ by $F(S,[-1,0])$. We say that a member of $F(S,[-1,0])$ is a negative valued function from S to $[-1,0]$, briefly N-function and by an N-structure(NS) on S, we mean that an ordered pair (S,η) of S and N-function η on S.

Definition 2.2. [1] A non-empty set X with a constant 0 and a single binary operation $*$, is said to be a BF-Algebra if it satisfies the following:
(i) $x * x = 0$; (ii) $x * 0 = x$ and (iii) $0 * (x * y) = y * x$ $\forall x, y \in X$.

Example 2.3. Let $X = \{0,1,2,3,4\}$ be a set with the following table:

$*$	0	1	2	3	4
0	0	4	3	2	1
1	1	0	4	3	2
2	2	1	0	4	3
3	3	2	1	0	4
4	4	3	2	1	0

Then $(X,*,0)$ is a BF-Algebra.

Definition 2.4. A partial ordering " $\leq$ " on X can be defined as $x \leq y$ if and only if $x * y = 0$.

Definition 2.5. [1] A non-empty subset S of a BF-algebra X is said to be a subalgebra if $x * y \in S$ $\forall x, y \in S$.

Definition 2.6. [1] A non-empty subset I of a BF-algebra X is said to be an ideal of X if (i) $0 \in I$;(ii) $x * y \in I$ and $y \in I$
$\Rightarrow x \in I$ $\forall x, y \in X$.

Definition 2.7. [1] An ideal I is called closed if $0 * x \in I$ $\forall x \in X$.

Definition 2.8. [4] An N-structure (X,η)on a BF-algebra X is called an N-subalgebra of X if $\eta(x * y) \leq \eta(x) \vee \eta(y)$ $\forall\ x, y \in X$.

Example 2.9. The N-structure, (X,η) on a BF-algebra X in the Examplle 2.3 defined by $\eta(x) = \begin{cases} -0.8\ ; x \neq 2 \\ -0.2\ ; x = 2 \end{cases}$ is an N-subalgebra of X.

3. N-Structured Ideal of a BF-Algebra

In this section we introduce the notion of N-structured ideal of a BF-algebra and discuss some of its properties. X represents a BF-algebra unless otherwise specified.

Definition 3.1. An N-structure (X, η) on a BF-algebra X is said to be a N-structured Ideal(N-Ideal) of X if
(i) $\eta(0) \leq \eta(x)$ and (ii) $\eta(x) \leq \eta(x * y) \vee \eta(y) \forall x, y \in X$.

Example 3.2. The N-structure, (X, η) on a BF-algebra X in the Examplle 2.3 defined by

$$\eta(x) = \begin{cases} -0.3 \; ; x = 0, 2 \\ 0 \quad ; x = 1, 3, 4 \end{cases}$$

is an N-Ideal of X.

Definition 3.3. An N-structure (X, η) on a BF-algebra X is said to be a N-structured Closed Ideal (NC-Ideal) of X if
(i) $\eta(x) \leq \eta(x * y) \vee \eta(y)$ and (ii) $\eta(0 * x) \leq \eta(x)$ $\forall x, y \in X$.

Example 3.4. Consider the BF-algebra $X = \{0, 1, 2, 3\}$ with the Cayley table given below.

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

The N-structure, (X, η) defined by $\eta(x) = \begin{cases} -0.4 & ; x = 0 \\ -0.3 & ; x = 1 \\ -0.1 & ; x = 2, 3 \end{cases}$ is an NC-Ideal of X.

Proposition 3.5. Every NC-ideal is an N-ideal.

The converse of the Proposition 3.5 is not true, in general, as seen from the following.

Example 3.6. Consider the BF-algebra $X = \{0, 1, 2, 3\}$ with the Cayley table given below.

$*$	0	1	2	3
0	0	3	0	1
1	1	0	1	3
2	2	3	0	1
3	3	1	3	0

The N-structure, (X,η) defined by $\eta(x) = \begin{cases} -0.4 & ; x = 0 \\ -0.3 & ; x = 1 \\ -0.1 & ; x = 2, 3 \end{cases}$ is N-Ideal but not an NC-Ideal of X, since $\eta(0 * 1) > \eta(1)$.

Proposition 3.7. If (X,η) is N-Ideal of X with $x \leq y$ for any $x, y \in X$ then $\eta(x) \leq \eta(y)$. (i.e) η is order-preserving.

Proof. Let $x, y \in X$ such that $x \leq y$. Then $x * y = 0$. Thus

$$\eta(x) \leq \eta(x * y) \vee \eta(y) = \eta(0) \vee \eta(y) = \eta(y).$$

Proposition 3.8. If (X,η) is N-Ideal of X with $x*y \leq z$ for any $x, y, z \in X$ then $\eta(x) \leq \eta(y) \vee \eta(z)$.

Proof. Let $x, y, z \in X$ such that $x * y \leq z$. Then $(x * y) * z = 0$.

$$\begin{aligned} Thus \quad \eta(x) \leq \eta(x * y) \vee \eta(y) &= (\eta(x * y) * z)) \vee \eta(z)) \vee \eta(y) \\ &= (\eta(0) \vee \eta(z)) \vee \eta(y) \\ &= \eta(y) \vee \eta(z) \end{aligned}$$

Using induction on n and from the above propositions we have,

Theorem 3.9. If (X,η) is N-Ideal of$X$ then for any $x, a_1, a_2, \cdots, a_n \in X$ and $(\cdots((x * a_1) * a_2) * \cdots)) * a_n = 0$ implies $\eta(x) \leq \eta(a_1) \vee \eta(a_2) \vee \cdots \vee \eta(a_n)$.

Theorem 3.10. Any N-Ideal of X is an N-subalgebra of$X$.

The converse of the theorem 3.10 is not be true in general, as seen from the following

Example 3.11. For the N-subalgebra in example 2.9. Take x = 2 and y = 4. We have $\eta(2) = -0.2$ and $\eta(2*4) \vee \eta(4) = \eta(3) \vee \eta(4) = -0.8 \vee -0.8 = -0.8$ $\Rightarrow \eta(x) > \eta(x * y) \vee \eta(y)$ and so it is not N-Ideal.

The following gives a sufficient condition for an N-subalgebra to be an N-ideal

Theorem 3.12. Let (X, η) be an N-subalgebra of X with $x * y \leq z, \forall x, y, z \in X$.
(1) If $\eta(x) \leq \eta(y) \vee \eta(z)\ \forall\ x, y, z \in X$ then (X, η) is an N-Ideal of X.
(2) If $\eta(x) \leq \eta(x * y) \vee \eta(y)\ \forall\ x, y \in X$ then (X, η) is an NC-Ideal of X.

Proof. Let (X, η) be an N-subalgebra and $x * y \leq z, \forall x, y, z \in X$.

Proof (1). For an N-subalgebra of X, we have $\eta(0) \leq \eta(x)$.
Also since $x * (x * y) \leq y$ and from the hypothesis, $\eta(x) \leq \eta(x * y) \vee \eta(y)$. Hence (X, η) is an N-Ideal of X.

Proof (2). For any $x \in X$, we have,
$\eta(0*x) \leq \eta(0) \vee \eta(x) = \eta(x*x) \vee \eta(x) \leq (\eta(x) \vee \eta(x)) \vee \eta(x) = \eta(x)$. Hence by hypothesis,$(X, \eta)$ is an NC-Ideal of X.

Theorem 3.13. If (X, η) is an NC-Ideal of X, then $K = \{x \in X\ \eta(x) = \eta(0)\}$ is an Ideal of X.

Proof. Clearly $0 \in K$ and hence $K \neq \phi$. Let $x * y$ and $y \in K$.
$\Rightarrow \eta(x * y) = \eta(y) = 0. \Rightarrow \eta(x) \leq \eta(x * y) \vee \eta(y) = 0 \vee 0 = 0.$
But $\eta(0) \leq \eta(x) \Rightarrow \eta(x) = 0$, showing that $x \in K$, completes the proof.
The following theorem shows the arbitrary union of family of NC-ideals of X is also an NC-ideal of X.

Theorem 3.14. Let $\{\eta_i\ ;\ i \in I\}$ be the family of NC-Ideals of X. Then $\bigcup_i \eta_i$ NC-Ideal of X.

Proof. Let x and $y \in X$.
Since $\{\eta_i; i \in I\}$ be the family of NC-Ideals of X, $\forall i \in I$, we have (i) $\eta_i(x) \leq \eta_i(x * y) \vee \eta_i(y)$ and (ii) $\eta_i(0 * x) \leq \eta_i(x)\ \forall x, y \in X$.
Now $\bigcup_i \eta_i(x) = Sup\{\eta_i(x)\ ;\ i \in I\} \leq Sup\{\eta_i(x * y) \vee \eta_i(y)\ ;\ i \in I\}$

$$\begin{aligned} &= Sup\{\eta_i(x * y)\ \ i \in I\} \vee Sup\{\eta_i(y)\ ;\ i \in I\} \\ &= \bigcup_i \eta_i(x * y)\ \vee\ \bigcup_i \eta_i(x * y) \end{aligned}$$

And $\bigcup_i \eta_i(0 * x) = Sup\{\eta_i(0 * x); i \in I\} \leq Sup\{\eta_i(x); i \in I\} = \bigcup_i \eta_i(x)$
Hence $\bigcup_i \eta_i$ NC-Ideal of X.

4. Conclusion

In this paper we extend the concepts of N-ideal of a BF-algebra and some of the properties have been discussed. In the paper "On BF-algebras", Andrzej

Walendziak (see [1]) proved that every BF-algebra is a BG-algebra. Hence all the results proved in this paper can easily verify for a BG-algebra and it can be extended for different ideal structures in a BF-algebra.

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