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SIMULATION THE RUNUP OF NONLINEAR SURFACE GRAVITY WAVES A STEEP COASTAL SLOPE

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Abstract: The paper considers two-dimensional numerical simulation of nonlinear gravity wave run-up to a steep coastal slope. The problem is formulated, boundary and initial conditions are described, the numerical model is based on Navier-Stokes equations. The discrete model is constructed using the method of splitting with respect to physical processes. A discrete finite-element model of this problem is developed taking into account the cell fill factor. Two-dimensional run-up model of a surface wave to a steep coastal slope, generation of reflected run back and their superposition are presented.

AMS Subject Classification: 76D33
Key Words: numerical simulation, Navier-Stokes equations, splitting method, run-up of surface gravity waves, reflection at steep coastal slope, superposition of run-up and run back waves

1. Introduction

The problem of surface gravity wave run-up to coastal slopes takes researchers' attention. In actual practice run-up is a nonlinear process accompanied by steeping, plunging, spreading-out, turbulence. Whereby, the relevant research

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questions are the wave action impacts on slopes and hydraulic engineering structures. Most existing works use variations of shallow-water equations to describe the above phenomena.

Though, to describe sufficiently the run-up of surface gravity waves onto coastal slopes one should account for viscosity, turbulence and bottom friction. The most correct here are Navier-Stokes equations that demonstrate not only nonlinear effects but turbulent processes in viscous incompressible fluid as well.

Let us discuss some studies on the run-up of surface gravity waves onto coastal slopes. Paper of Madsen et all, 2002, demonstrated calculation data of solitary wave run-up onto a vertical wall, described propagation and breaking of surface waves under shallow-water conditions.

Paper of Jensen et all, 2003, discussed observation data on run-up value reliant on breaking wave amplitude and the angle of coastal slope. In paper Kimmoun, Branger, 2007, demonstrated the results of run-up and wave breaking during its propagation along the slope coast obtained in experimental studies. Wave breaking was evaluated with provision for tapering, steepening, plunging, splashing up and air mass grabbing.

Paper of Kawasaki et all, 2010, demonstrated two-dimensional model of surface wave run-up based on multi-cycle state of the environment. Original equations were decomposed into advectional and non-advectional parts by the splitting method.

Two-dimensional numerical simulation of surface gravity wave run-up onto steep coastal slopes is proposed in this study. The problems of run back generation are described as well. The simulation is based on hydrophysical conditions of the Azov Sea. Such problems become essential in this shallow-water basin in case of storm tide. Results the numerical modelling of run up the nonlinear surface gravity waves on given beach is presented in Abbasov, 2013.

2. Statement of the Problem

Due to the problem geometry Ox axis of the coordinate system is aligned with the undisturbed fluid surface and is directed toward the coast, Oz axis is directed vertically downwards (Figure 1). Bottom line is depicted by a Power Function graph, it flows into the line of steep coastal slope. At the start time the fluid is resting. At some distance from the coast at a point $x = 0$, s disturbance is set a pressure pulse that varies harmonically. Let us analyze the run-up onto the coastal slope and further generation of the run back.

To describe two-dimensional surface gravity waves on fluid surface with

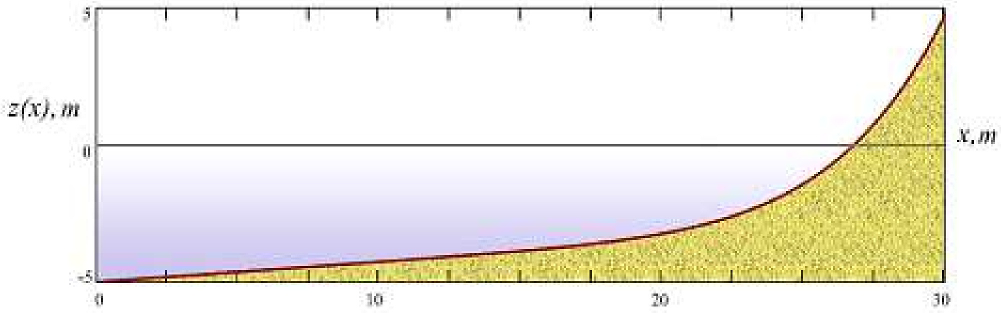

Figure 1: The coastal geometry

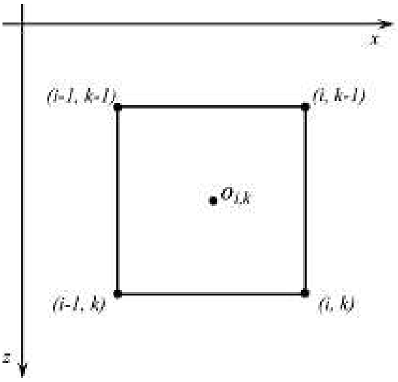

Figure 2: Cell layout and neighboring vertices

viscosity let us use two-dimensional Navier-Stokes equations, continuity equation for an incompressible fluid and hydrodynamic pressure equation (Abbasov, 2013).

Dynamic condition is observed on a free fluid surface, at the bottom of area conditions impermeable and sliding are assumed, on the left lateral border the source is located, the right lateral border is continuation of a bottom, at the initial time the fluid is resting.

3. Structure of the Discrete Model

To approximate the original equations with the variable temporary the splitting method on physical processes is applied. Discrete finite-volume model is generated by taking the coefficient of cell fill factor into account. For numerical implementation of discrete numerically simulated model an equally spaced grid is given (Holt, 1977):

$$\begin{aligned}\varpi_h = \{ & t^n = n\tau,\ x_i = ih_x,\ z_k = kh_z; \\ & i = \overline{0..N_x},\ k = \overline{0..N_z};\ N_t\tau = T,\ N_x h_x = l_x,\ N_z h_z = l_z \}\,, \end{aligned} \quad (1)$$

where τ– a time step, $h_x,\ h_z$– space steps, N_t– upper time limit, $N_x,\ N_z$– space limits.

Computational cells represent rectangles, which can be completely filled, partially filled or empty. Cell centre's and vertices are distributed to $h_x/2$ and $h_z/2$ along correspondent coordinates x, z (see Figure 2). Whereby, velocity field and pressure are rated at the vertices of the present cell; (i, k), $(i - 1, k)$, $(i, k - 1)$, $(i - 1, k - 1)$. Fill factor $o_{i,k}$ of cell (i, k) is defined by fluid head pressure inside the present cell. If the average pressure in cell vertices is more than fluid head pressure inside the cell, it is considered to be completely filled $o_{i,k} = 1$.

To approximate the original differential equations with spatial coordinates we use integro- interpolation method. Discrete equation analogs are obtained to calculate velocity vector components, pressure field, as well as discrete analogs of boundary conditions. The conservativeness of discrete model was investigated. It has been established that the integral law of conservation of momentum is observed for net analog of a finite-difference equation. Discrete equations are solved by upper relaxation method. To calculate two-dimensional velocity field and pressure field the program code "2DBayWaves" was developed.

4. Analyzing Simulation Results

As a shallow water area model, we shall use hydrophysical conditions of the Taganrog Bay of the Azov Sea (Hydrometeorology, 1991). Other offshore strips could be used as a shallow area model. Proceeding from the depth of the Gulf of Taganrog H≤5 m, the segment of the water area will measure with the following geometric dimensions: 30 m length; 10 m vertical extent from the bottom (see Figure 1). The length of the surface wave will be: λ ≈15 m, horizontal step size will be h_x =0.05 m, vertical one h_z =0.05 m. Initial values of shallow-water

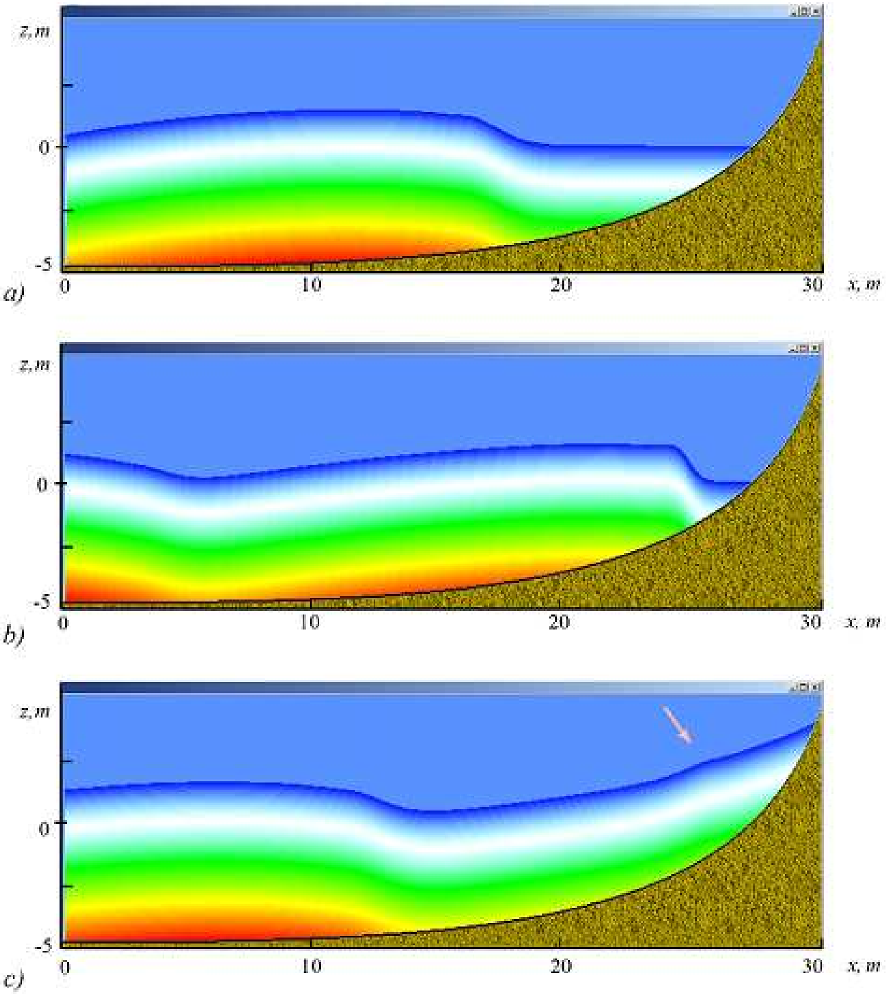

Figure 3: Dynamics of change in the profile of the surface gravity wave arriving steep coastal slope with initial parameters: $f = 0.32$Hz; $\lambda = 15$m; $c = 4.8$m/s; $H = 5$m; $a = 1$m; $kH = 2.1$; $\varepsilon = 0.2$; time a) $t = 2.7$s; b) $t = 3.8$s; c) $t = 5.1$s

parameter will be within $1 \leq kH \leq 10$. For the shallow water simulation grid size will 600200 cells, vertical water level is 100 cells.

Figure 3 shows the nonlinear gravity wave run-up onto the steep coastal slope in dynamics. We give a harmonic gust as a pressure pulse on the left side boundary. Approaching the coastline the gulf depth is getting shallow, the

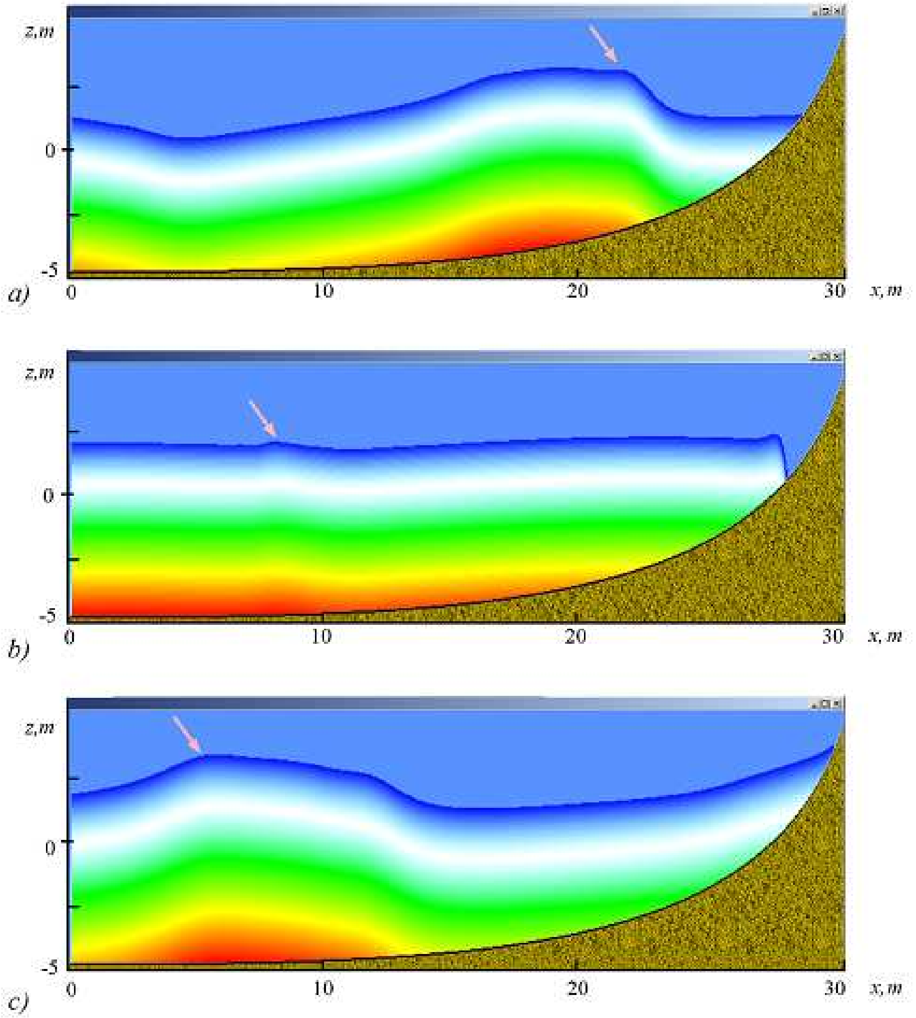

Figure 4: Further superposition stages of surface gravity wave run-up and run back

wave "feels" the bottom, nonlinear effects increase. That results in steepening of leading edge of the surface water crest (see Figure 3, *b*), it washes almost vertical slope. Next, the breaking wave flows down the slope, the run back takes place. The generated run back in Figure 3, *c*is marked with an arrow.

The flowing back wave meets the next crest amplifying the steepening of its leading edge. The superposition of incoming and reflected waves results in their interference. This process is shown in Figure 4, the position of reflected run back is marked with an arrow. Figure 4, *b* demonstrates relative extinction of waves caused by their superposition, that results in generation of a continuous flow with one leading edge. On the next stages the run back propagates farther lifting the initial level of basin filling.

It is noteworthy to say that the run-up is not accompanied by breaking of wave crest if the initial wave steepness lessens. With wave length lessening nonlinear profile distortion decreases, the run-up results in the constant water flow.

To check the validity the simulation results of nonlinear surface gravity wave run-up onto the coastal slopes were compared with the existing numerical Kawasaki, 2010, and experimental data Kimmoun, Branger, 2007. The comparison it was stated that obtained simulation results were in good agreement with basic run-up stages of surface waves. In conclusion, we can note that the suggested model makes possible to describe the process of surface wave run-up onto the coastal slope as well as the following run back and superposition.

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