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ON THE CONVERGENCE OF ITERATIVE PROCESSES FOR NONLINEAR OPERATOR EQUATIONS

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Abstract: In this paper, a nonlinear operator equation involving a strictly pseudocontractive mapping is investigated based on an explicit iterative method. Weak and strong convergence theorems of fixed points are established in real Hilbert spaces.

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1. Introduction

The theory of iterative processes for nonlinear operator equations is a cross subject between nonlinear functional analysis, and operation research. It has emerged as an effective and powerful tool for studying a wide class of problems which arise in economics, finance, image reconstruction, ecology, transportation,

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network, elasticity and optimization; see [5], [12], [15], [16], [19], [21], [23], [27] and the references therein. For the existence of solutions of nonlinear operator equations, we refer authors to [1], [3], [9], [10], [13], [14] and the references therein. However, from the standpoint of real world applications it is not only to know the existence of solutions, but also to be able to construct an iterative process to approximate their solutions. The computation of solutions is important in the study of many real world problems. For instance, in computer tomography with limited data, each piece of information implies the existence of a convex set in which the required solution lies. The problem of finding a point in the intersection of these convex sets is some positive integer, is then of crucial interest, and it cannot be directly solved. Therefore, an iterative process must be used to approximate such point. Recently, many convergence theorems of solutions of nonlinear operator equations are established based on iterative methods; see, for example, [2], [6]-[8], [17], [20], [22] and [26].

In this paper, a nonlinear operator equation involving a strictly pseudocontractive mapping is investigated based on an explicit iterative method. Weak and strong convergence theorems of fixed points are established in real Hilbert spaces.

2. Preliminaries

In what follows, we always assume that H is a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\|\cdot\|$, and C is a nonempty closed convex subset of H.

Let $S : C \to C$ be a mapping. In this paper, we use $F(S)$ to stand for the set of fixed points. Recall that the mapping S is said to be *nonexpansive* if

$$\|Sx - Sy\| \leq \|x - y\|, \quad \forall x, y \in C.$$

If C is a bounded closed convex subset of H, then fixed point sets of nonexpansive mappings are not empty closed convex; see [9] and the reference therein.

The mapping S is said to be *strictly pseudocontractive* if there exists a constant $\kappa \in [0, 1)$ such that

$$\|Sx - Sy\|^2 \leq \|x - y\|^2 + \kappa\|(I - S)x - (I - S)y\|^2, \quad \forall x, y \in C.$$

For such a case, S is also said to be a *κ-strictly pseudocontractive mapping*. The class of strictly pseudocontractive mappings was introduced by Brower and Petryshyn [4]; see [4] for more details. It is clear that every nonexpansive mapping is a 0-strictly pseudocontractive mapping.

In order to prove our main results, we need the following lemmas.

Lemma 2.1. ([11]) *In a real Hilbert space, the following inequality holds*

$$\|ax + (1-a)y\|^2 = a\|x\|^2 + (1-a)\|y\|^2 - a(1-a)\|x-y\|^2, \quad \forall a \in [0,1],\ x, y \in H.$$

Lemma 2.2. ([18]) *Let C be a nonempty closed convex subset of a real Hilbert space H and $S : C \to C$ be a κ-strictly pseudocontractive mapping. Then the mapping $I - S$ is demiclosed at zero, that is, if $\{x_n\}$ is a sequence in C such that $x_n \to x$ weakly and $x_n - Sx_n \to 0$, then $x \in F(S)$.*

Lemma 2.3. ([24]) *Let H be a Hilbert space and $0 < p \le t_n \le q < 1$ for all $n \ge 1$. Suppose that $\{x_n\}$ and $\{y_n\}$ are sequences in H such that*

$$\limsup_{n\to\infty} \|x_n\| \le r, \quad \limsup_{n\to\infty} \|y_n\| \le r$$

and

$$\lim_{n\to\infty} \|t_n x_n + (1-t_n)y_n\| = r$$

hold for some $r \ge 0$. Then $\lim_{n\to\infty} \|x_n - y_n\| = 0$.

Lemma 2.4. ([25]) *Let $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ be three nonnegative sequences satisfying the following condition:*

$$a_{n+1} \le (1+b_n)a_n + c_n, \quad \forall n \ge n_0,$$

where n_0 is some nonnegative integer, $\sum_{n=1}^{\infty} b_n < \infty$ and $\sum_{n=1}^{\infty} c_n < \infty$. Then $\lim_{n\to\infty} a_n$ exists.

3. Main Results

Now, we prove our main results.

Theorem 3.1. *Let C be a nonempty closed convex subset of a real Hilbert space H and $S : C \to C$ be a κ-strictly pseudocontractive mapping with a nonempty fixed point set. Let $\{x_n\}$ be a sequence generated in the following iterative process*

$$\begin{cases} x_1 \in C, \quad \text{arbitrarily chosen}, \\ x_{n+1} = \alpha_n x_n + \beta_n y_n + \gamma_n e_n, \\ y_n = \delta_n x_n + (1-\delta_n)Sx_n, \quad n \ge 1, \end{cases} \tag{3.1}$$

where $\{e_n\}$ is a bounded sequence in C, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ and $\{\delta_n\}$ are real number sequences in $(0,1)$ satisfying the following restrictions:

(a) $\alpha_n + \beta_n + \gamma_n = 1$;

(b) $0 < a \le \beta_n \le b < 1$, *where a and b are real constants*;

(c) $\kappa \le \delta_n \le c < 1$, *where c is some constant*;

(d) $\sum_{n=1}^{\infty} \gamma_n < \infty$.

Then the sequence $\{x_n\}$ converges weakly to some point x, which satisfies that $Sx = x$.

Proof. First, we show the sequence $\{x_n\}$ generated in the above iterative process is bounded. Fix $p \in F(S)$ and $S_n = \delta_n I + (1-\delta_n)S$. It follows from Lemma 2.1 and the restriction (c) that

$$\begin{aligned}
&\|S_n x - S_n y\|^2 \\
&= \delta_n \|x-y\|^2 + (1-\delta_n)\|Sx - Sy\|^2 \\
&\quad - \delta_n(1-\delta_n)\|(x-y) - (Sx - Sy)\|^2 \\
&\le \|x-y\|^2 - (\delta_n - \kappa)\|(x-y) - (Sx-Sy)\|^2 \\
&\le \|x-y\|^2.
\end{aligned}$$

This shows that S_n is nonexpansive, and $F(S_n) = F(S)$ for each $n \ge 1$. Note that

$$\begin{aligned}
\|x_{n+1} - p\| &\le \alpha_n \|x_n - p\| + \beta_n \|S_n x_n - p\| + \gamma_n \|e_n - p\| \\
&\le \|x_n - p\| + \gamma_n \|e_n - p\|.
\end{aligned}$$

It follows from Lemma 2.4 that $\lim_{n\to\infty} \|x_n - p\|$ exists. This implies in turn that $\{x_n\}$ is bounded. Let $\lim_{n\to\infty} \|x_n - p\| = d > 0$. Notice that

$$\|x_n - p + \gamma_n(e_n - x_n)\| \le \|x_n - p\| + \gamma_n \|e_n - x_n\|.$$

It follows that

$$\limsup_{n\to\infty} \|x_n - p + \gamma_n(e_n - x_n)\| \le d. \tag{3.2}$$

Notice that

$$\begin{aligned}
\|y_n - p + \gamma_n(e_n - x_n)\| &\le \|S_n x_n - p\| + \gamma_n \|e_n - x_n\| \\
&\le \|x_n - p\| + \gamma_n \|e_n - x_n\|.
\end{aligned}$$

It follows that

$$\limsup_{n\to\infty} \|y_n - p + \gamma_n(e_n - x_n)\| \le d. \tag{3.3}$$

On the other hand, we have

$$\begin{aligned}
&\lim_{n\to\infty} \|x_{n+1} - p\| \\
&= \lim_{n\to\infty} \|(1-\beta_n)(x_n - p + \gamma_n(e_n - x_n)) \\
&\qquad + \beta_n(y_n - p + \gamma_n(e_n - x_n))\| \\
&= d.
\end{aligned} \tag{3.4}$$

In view of (3.2), (3.3) and (3.4), we obtain from Lemma 2.3 that

$$\lim_{n\to\infty} \|y_n - x_n\| = 0.$$

That is,

$$\lim_{n\to\infty} \|S_n x_n - x_n\| = 0.$$

Next, we show that $\lim_{n\to\infty} \|Sx_n - x_n\| = 0$. Notice that

$$\begin{aligned}
&\|x_n - Sx_n\| \\
&\leq \|x_n - S_n x_n\| + \|\big(\delta_n x_n + (1-\delta_n)Sx_n\big) - Sx_n\| \\
&\leq \|x_n - S_n x_n\| + \delta_n \|x_n - Sx_n\|.
\end{aligned} \tag{3.5}$$

This implies from the restriction (c) and (3.5) that

$$\lim_{n\to\infty} \|Sx_n - x_n\| = 0. \tag{3.6}$$

Since $\{x_n\}$ is bounded, we see that there exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$, which converges weakly to x. It follows from Lemma 2.2 that $x \in F(S)$. Assume that there exists another subsequence $\{x_{n_j}\}$ of $\{x_n\}$, which converges weakly to x' $(\neq x)$. It follows from Lemma 2.2 that $x' \in F(S)$. Since $\lim_{n\to\infty} \|x_n - p\|$ exists for any $p \in F(S)$, by virtue of Opial's property of H, it follow that have

$$\begin{aligned}
\lim_{n\to\infty} \|x_n - x\| &= \liminf_{i\to\infty} \|x_{n_i} - x\| < \liminf_{i\to\infty} \|x_{n_i} - x'\| \\
&= \lim_{n\to\infty} \|x_n - x'\| = \liminf_{j\to\infty} \|x_{n_j} - x'\| \\
&< \liminf_{j\to\infty} \|x_{n_j} - x\| = \lim_{n\to\infty} \|x_n - x\|.
\end{aligned}$$

This is a contradiction. Hence, we have $x = x'$. This shows that the sequence $\{x_n\}$ converges weakly to x, where x satisfies that $x = Sx$. This completes the proof. □

In the light of Theorem 3.1, we have the following results.

Corollary 3.2. *Let C be a nonempty closed convex subset of a real Hilbert space H and $S : C \to C$ be a nonexpansive mapping with a nonempty fixed point set. Let $\{x_n\}$ be a sequence generated in (3.1), where $\{e_n\}$ is a bounded sequence in C, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ and $\{\delta_n\}$ are real number sequences in $(0, 1)$ satisfying the following restrictions:*

(a) $\alpha_n + \beta_n + \gamma_n = 1$;

(b) $0 < a \leq \beta_n \leq b < 1$, *where a and b are real constants*;

(c) $0 \leq \delta_n \leq c < 1$, *where c is some constant*;

(d) $\sum_{n=1}^{\infty} \gamma_n < \infty$.

Then the sequence $\{x_n\}$ converges weakly to some point x, which satisfies that $Sx = x$.

Proof. Since every nonexpansive mapping is a 0-strictly pseudocontractive mapping, we can immediately deduce from Theorem 3.1 the desired conclusion. This completes the proof. □

Corollary 3.3. *Let C be a nonempty closed convex subset of a real Hilbert space H and $S : C \to C$ be a nonexpansive mapping with a nonempty fixed point set. Let $\{x_n\}$ be a sequence generated in the following iterative process*

$$\begin{cases} x_1 \in C, \quad \text{arbitrarily chosen,} \\ x_{n+1} = \alpha_n x_n + \beta_n S x_n + \gamma_n e_n, \quad n \geq 1, \end{cases} \tag{3.7}$$

where $\{e_n\}$ is a bounded sequence in C, $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ are real number sequences in $(0, 1)$ satisfying the following restrictions:

(a) $\alpha_n + \beta_n + \gamma_n = 1$;

(b) $0 < a \leq \beta_n \leq b < 1$, *where a and b are real constants*;

(c) $\sum_{n=1}^{\infty} \gamma_n < \infty$.

Then the sequence $\{x_n\}$ converges weakly to some point x, which satisfies that $Sx = x$.

Proof. Letting $\delta_n = \kappa = 0$, we can immediately deduce from Theorem 3.1 the desired conclusion. This completes the proof. □

Recall that a mapping $S : C \to C$ is *semicompact* if any bounded sequence $\{x_n\}$ in C satisfying $\lim_{n\to\infty} \|x_n - Sx_n\| = 0$ has a convergent subsequence.

Next, we give strong convergence theorems with the aid of semicompactness.

Theorem 3.4. *Let C be a nonempty closed convex subset of a real Hilbert space H and $S : C \to C$ be a κ-strictly pseudocontractive mapping with a nonempty fixed point set. Let $\{x_n\}$ be a sequence generated in the* (3.1), *where $\{e_n\}$ is a bounded sequence in C, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ and $\{\delta_n\}$ are real number sequences in $(0, 1)$ satisfying the following restrictions:*

(a) $\alpha_n + \beta_n + \gamma_n = 1$;

(b) $0 < a \le \beta_n \le b < 1$, *where a and b are real constants*;

(c) $\kappa \le \delta_n \le c < 1$, *where c is some constant*;

(d) $\sum_{n=1}^{\infty} \gamma_n < \infty$.

If S is semicompact, then the sequence $\{x_n\}$ converges strongly to some point x, which satisfies that $Sx = x$.

Proof. Since S is semicompact, we see that there exits a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ converges strongly to $x \in C$. Notice that

$$\|x - Sx\| \le \|x - x_{n_i}\| + \|x_{n_i} - Sx_{n_i}\| + \|Sx_{n_i} - Sx\|.$$

Since S is Lipshcitz continuous, we obtain from (3.6) that $x \in F(S)$. Notice that we have obtain that $\lim_{n\to\infty} \|x_n - p\|$ exists for every $p \in F(S)$. Therefore, we can obtain the desired conclusion immediately. □

In the light of Theorem 3.4, we have the following results.

Corollary 3.5. *Let C be a nonempty closed convex subset of a real Hilbert space H and $S : C \to C$ be a nonexpansive mapping with a nonempty fixed point set. Let $\{x_n\}$ be a sequence generated in the* (3.1), *where $\{e_n\}$ is a bounded sequence in C, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ and $\{\delta_n\}$ are real number sequences in $(0, 1)$ satisfying the following restrictions:*

(a) $\alpha_n + \beta_n + \gamma_n = 1$;

(b) $0 < a \le \beta_n \le b < 1$, *where a and b are real constants*;

(c) $0 \le \delta_n \le c < 1$, *where c is some constant*;

(d) $\sum_{n=1}^{\infty} \gamma_n < \infty$.

If S is semicompact, then the sequence $\{x_n\}$ converges strongly to some point x, which satisfies that $Sx = x$.

Proof. Since every nonexpansive mapping is a 0-strictly pseudocontractive mapping, we can immediately deduce from Theorem 3.4 the desired conclusion. This completes the proof. □

Corollary 3.6. *Let C be a nonempty closed convex subset of a real Hilbert space H and $S : C \to C$ be a nonexpansive mapping with a nonempty fixed point set. Let $\{x_n\}$ be a sequence generated in the (3.7), where $\{e_n\}$ is a bounded sequence in C, $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ are real number sequences in $(0, 1)$ satisfying the following restrictions:*

(a) $\alpha_n + \beta_n + \gamma_n = 1$;

(b) $0 < a \leq \beta_n \leq b < 1$, *where a and b are real constants*;

(c) $\sum_{n=1}^{\infty} \gamma_n < \infty$.

If S is semicompact, then the sequence $\{x_n\}$ converges strongly to some point x, which satisfies that $Sx = x$.

Proof. Letting $\delta_n = \kappa = 0$, we can immediately deduce from Theorem 3.4 the desired conclusion. This completes the proof. □

Finally, we study the classical variational inequality based on the iterative process (3.1).

Let $A : C \to H$ be a mapping. A is said to be *monotone* if

$$\langle Ax - Ay, x - y \rangle \geq 0, \quad \forall x, y \in C.$$

The mapping A is said to be *inverse-strongly monotone* if there exists a constant $\alpha > 0$ such that

$$\langle Ax - Ay, x - y \rangle \geq \alpha \|Ax - Ay\|^2, \quad \forall x, y \in C.$$

For such a case, A is also said to be *α-inverse-strongly monotone.* It is known that $I - rA$ is nonexpansive, where r is a positive real number with $r \leq 2\alpha$.

Recall that the classical variational inequality is to find a point $x \in C$ such that

$$\langle Ax, y - x \rangle \geq 0, \quad \forall y \in C.$$

Notice that the variational inequality is equivalent to a fixed point problem the operator $P_C(I - rA)$, where P_C stands for the metric projection, I stands for the identity mapping and r stands for some positive constant. It is not hard to see that $F(P_C(I - rA)) = VI(C, A)$, where $VI(C, A)$ stands for the set of solutions of the variational inequality.

Theorem 3.7. *Let C be a nonempty closed convex subset of a real Hilbert space H and $A : C \to H$ be a α-inverse-strongly monotone mapping. Let $\{x_n\}$ be a sequence generated in the following iterative process*

$$\begin{cases} x_1 \in C, \quad \textit{arbitrarily chosen}, \\ x_{n+1} = \alpha_n x_n + \beta_n y_n + \gamma_n e_n, \\ y_n = \delta_n x_n + (1 - \delta_n) P_C(x_n - rAx_n), \quad n \geq 1, \end{cases}$$

where $\{e_n\}$ is a bounded sequence in C, r is a positive real number with $r \leq 2\alpha$, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ and $\{\delta_n\}$ are real number sequences in $(0, 1)$ satisfying the following restrictions:

(a) $\alpha_n + \beta_n + \gamma_n = 1$;

(b) $0 < a \leq \beta_n \leq b < 1$, *where a and b are real constants*;

(c) $0 \leq \delta_n \leq c < 1$, *where c is some constant*;

(d) $\sum_{n=1}^{\infty} \gamma_n < \infty$.

Then the sequence $\{x_n\}$ converges weakly to some point x, which satisfies that

$$\langle Ax, y - x \rangle \geq 0, \quad \forall y \in C.$$

Proof. Since $P_C(I - rA)$ is a 0-strictly pseudocontractive mapping, we can immediately deduce from Theorem 3.1 the desired conclusion. This completes the proof. □

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