International Journal of Pure and Applied Mathematics
Volume 84 No. 5 2013, 477-496
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v84i5.3

ijpam.eu

ENUMERATION TECHNIQUE FOR SOLVING LINEAR FRACTIONAL FUZZY SET COVERING PROBLEM

Ratnesh Rajan Saxena[1], Rashmi Gupta[2 §]

[1]Department of Mathematics
Deen Dayal Upadhyaya College
University of Delhi
Delhi, 110015, INDIA

[2]Department of Mathematics
University of Delhi,
Delhi, 110007, INDIA

Abstract: In this paper an enumeration technique for solving linear fractional fuzzy set covering problem is defined. Set covering problems belong to the class of 0-1 integer programming problems that are NP-complete. Many applications arises having the set covering problems, switching theory, testing of VLSI circuits and line balancing often take on a set covering structure. Linear fractional set covering problems involving coefficients in the objective function with some lack of precision are usual. To solve them several approaches have been proposed. In this paper a solution algorithm to fuzzy linear fractional set covering problem is suggested, in order to defuzzify the problem the concept of vector ranking function is presented further for obtaining efficient solution to the problem, a lexicographic approach is used. A linearization technique is used to obtain the optimal solution for crisp linear fractional set covering problem. An illustrative example is included to demonstrate the correctness of the proposed solution algorithm.

AMS Subject Classification: 05C72
Key Words: set covering, fuzzy number, vector ranking function

Received: February 18, 2012

§Correspondence author

1. Introduction

The set covering problem is one of the most famous problems in complexity and approximation theory. Given a set $\mathcal{C} = \{c_1, \ldots, c_n\}$ of elements and a collection $\mathcal{S} = \{s_1, \ldots, s_n\}$ of subsets of $\mathcal{C}$, the goal is to find a subset $S' \subset S$ of minimum cardinality such that $\bigcup_{S_j \in S'} S_j = \mathcal{C}$.

The fuzzy set theory has been applied in many disciplines such as operations research, managemental sciences, control theory, artificial intelligence etc. The growth of applications of the fuzzy set theory has been accumulating. At the turn of the century, reducing complex real world system into precise mathematical model is the main trend in science and engineering. In the middle of century, operations research began to be applied to real-world decision-making problems and this became one of the most important fields in science and engineering. Unfortunately, real world situations are often not so deterministic. Thus precise mathematical models are not enough to tackle all practical problems. In this paper the concept of optimality for linear fractional fuzzy set covering problems with fuzzy parameters is obtained.

Recently, a great interest has arisen in Fractional Programming Problems which is another subclass of non-linear programming, where the constraints are linear and objective function is the ratio of linear or non linear functions. In the year 1977, Arora and Puri [10] discussed the enumeration technique for the set covering problem with linear fractional function as its objective functions. Extending their work, in the year 1998, Saxena and Arora [11] discussed a cutting plane technique for multiobjective fractional set covering problem.

There are number of applications of set covering problems with fractional objective function. Some of them are 'air line crew scheduling', 'truck routing', 'political districting', 'information retrieval', etc. For example, suppose an airline company has m flights to operate upon and n crews at its disposal, it being understood that a crew can handle atleast one flight. Let $c_j > 0$ be the cost paid by the company when its jth crew is operated and let d_j be the commission that the company receives from the authorities when it employs its jth crew. Let $\alpha > 0$ be the fixed amount paid to the company. Now the company is interested in scheduling its crew in such a way that the cost is minimized and at the same time the profit is maximized, i.e., it is interested in determining a set of crews which would cover all the flights and for which $\dfrac{\sum c_j}{\sum d_j + \alpha}$ is minimum.

The aim of this paper is to present a method by using a vector ranking function. In fact solution for linear fractional set covering problem with fuzzy parameters based on multiobjective linear fractional set covering programming problem technique.

The paper has the following structure. In Section 2, linear fractional set covering problems, fuzzy linear fractional set covering problems and some definitions are presented. In Section 3, a vector ranking function is presented to convert linear fractional fuzzy set covering problem to a multiobjective linear fractional set covering problem. In Section 4, some basic concepts of optimality for multiobjective linear integer programming problems are given. In Section 5, an algorithm is developed to solve fuzzy fractional set covering problem using lexicographic approach. In Section 6, numerical example is being given in support of the algorithm. Section 7, concludes the paper.

2. Theoretical Development

Linear fractional set covering Problems (FCP). Consider a set $I = \{1, 2, \ldots, m\}$ and a set $P = \{P_1, P_2, \ldots, P_n\}$ where $P_j \subseteq I$, $j \in J = [1, 2, \ldots, n]$. A subset J^* of J is said to be a cover of I if $\bigcup_{j \in J^*} P_j = I$. Let a cost $c_j > 0$ be associated with every $j \in J$. The total cost of the cover J^* is equal to $\sum_{j \in J^*} c_j$. The linear fractional set covering problem (FCP) is to find a cover of minimum cost subject to the condition that atleast one of the utility is satisfied and objective function is a linear fractional function. Mathematically, the problem is

$$\text{(CP)} \quad \min z = \frac{\sum_{j=1}^{n} c_j x_j}{\sum_{j=1}^{n} d_j x_j + \alpha}$$

$$\text{subject to} \sum_{j=1}^{n} a_j x_j \geq 1, \quad i \in 1 \tag{2.1}$$

$$x_j = 0 \text{ or } 1, \quad j \in J.$$

where $$x_j = \begin{cases} 1 & \text{if } j \text{ is in the cover} \\ 0 & \text{otherwise} \end{cases}$$

and $$a_{ij} = \begin{cases} 1 & \text{if } i \in P_j \\ 0 & \text{otherwise} \end{cases}.$$

It is assumed that c_j's and d_j's are non-negative and α is a scalar such that $\sum(d_j x_j + \alpha) > 0$.

In matrix form, (FCP) can be written as

$$\min z = Cx/Dx + \alpha$$
$$\text{subject to} \quad Ax \geq b$$

where $x^T = (x_1, x_2, \ldots, x_n)$ with $x_j = 0$ or 1, $j = 1, 2, \ldots, n$. Here $C = (c_1, c_2, \ldots, c_n) \in R^n$ and $D = (d_1, d_2, \ldots, d_n) \in R^n$ are row vectors. A is an $m \times n$ matrix of zeros and ones and $b^T = (1, 1, \ldots, 1)$ is a row vector of ones, and α is a scalar such that $\sum(Dx + \alpha) > 0$.

Fuzzy Linear Fractional Set Covering Problem (FFCP). If the coefficient in the objective function of problem (CP) (2.1) becomes fuzzy in nature then problem (CP) (2.1) translates to fuzzy set covering problem (FFCP) i.e.

$$\text{(FFCP)} \quad \min z = \frac{\sum_{j=1}^{n} \widetilde{c}_j x_j}{\sum_{j=1}^{n} \widetilde{d}_j x_j + \alpha}$$
$$\text{subject to} \quad \sum_{j=1}^{n} a_{ij} x_j \geq 1, \quad i \in I \tag{2.2}$$
$$x_j = 0 \text{ or } 1, \quad j \in J$$

where $\quad x_j = \begin{cases} 1 & \text{if } j \text{ is in the cover} \\ 0 & \text{otherwise} \end{cases}$

and $\quad a_{ij} = \begin{cases} 1 & \text{if } i \in P_j \\ 0 & \text{otherwise} \end{cases}.$

Definitions.

2.1. **Cover Solution:** A solution X which satisfies (2.1) is said to be a cover solution.

2.2. **Redundant Cover:** For any cover J, a column $j^* \in J$ is said to be redundant if $J - \{j^*\}$ is also a cover.

If a cover contains one or more redundant columns, it is called a redundant cover. Column j^* is redundant with respect to the cover J iff $\sum_{j \in J} a_{ij} \geq 2$ for all $i \in P_j$.

2.3. **Prime Cover**: A cover j^* is said to be a prime cover, if none of the columns corresponding to $j \in j^*$ is redundant. A solution corresponding to the prime cover is called a prime cover solution.

2.4. Let $x = (x_1, x_2, \ldots, x_n)^T$, $y = (y_1, y_2, \ldots, y_n)^T \in R^n$ be two vectors. Then we write $x \leq y$ iff $x_i \leq y_i$ for all i belong to $N = \{1, 2, \ldots, n\}$; $x < y$ iff $x_i \leq y_i$ for all $i \in N; x \neq y$ iff $x_i \neq y_i$ for some $i \in N$.

2.5. A fuzzy set $\widetilde{A}$ on $\mathbb{R}$ is called a fuzzy number if it satisfies the following two conditions:

(i) It's membership function is upper semi continuous.

(ii) There exists three intervals $[a, b]$, $[b, c]$ and $[c, d]$ such that membership function of $\widetilde{A}$ is increasing on $[a, b]$, equal to 1 on $[b, c]$, decreasing on $[c, d]$ and equal to 0 anywhere else.

Notation. $F(\mathbb{R})$ denotes the set of all fuzzy numbers.

3. Ordering Elements of $F(\mathbb{R})$

In order to compare the fuzzy numbers the ordering of fuzzy numbers is being given.

A simple method for ordering the elements of $F(\mathbb{R})$ consists in the definition of a ranking function. Which maps each fuzzy number into a point of the real line, where a natural order already exists.

Remark. More than one ranking function can be defined (Bortolan and Degani, 1985; Cadenas and Verdegay, 1997).

Based on the decision maker's preferences, assume there exists k important attributes associated to fuzzy number $\widetilde{A}$ such that ith of them can be characterized by the ranking function $R_i : F(\mathbb{R}) \to R$. In this case, a crisp k-dimensional vector, $R(\widetilde{A})$, to $\widetilde{A}$ is considered as follows:

$$R(\widetilde{A}) = (R_1(\widetilde{A}), R_2(\widetilde{A}), \ldots, R_k(\widetilde{A}))^T .$$

Definition 3.1. The vector function $R(.)$, defined as above, is called a vector ranking function. Moreover let $\widetilde{A}$ and $\widetilde{B}$ belong to $F(\mathbb{R})$, then:

$$\widetilde{A} \underset{R}{\leq} \widetilde{B} \quad \text{iff} \quad R(\widetilde{A}) \leq R(\widetilde{B})$$

$$\widetilde{A} \underset{R}{<} \widetilde{B} \quad \text{iff} \quad R(\widetilde{A}) < R(\widetilde{B})$$

$$\widetilde{A} \underset{R}{=} \widetilde{B} \quad \text{iff} \quad R(\widetilde{A}) \neq R(\widetilde{B})$$

$$\widetilde{A} \underset{R}{\neq} \widetilde{B} \quad \text{iff} \quad R(\widetilde{A}) \neq R(\widetilde{B})$$

Similarly

$$\widetilde{A} \underset{R}{\geq} \widetilde{B} \quad \text{iff} \quad \widetilde{B} \underset{R}{\leq} \widetilde{A} \quad \text{and} \quad \widetilde{A} > \widetilde{B} \quad \text{iff} \quad \widetilde{B} \underset{R}{<} \widetilde{A}$$

Vector Ranking Functions for $k = 1, 2, 3$.

(i) Let $\widetilde{A}$ be a fuzzy number. Roubens ranking function is a vector ranking function with $k = 1$ when the decision maker choose $R_1(.)$ as the only characteristic for ordering. It is defined as:

$$R(\widetilde{A}) = \frac{1}{2} \int_0^1 \left(\inf \widetilde{A}_r + \sup \widetilde{A}_r \right) dr$$

where $\widetilde{A}_r$ is an r-cut of $\widetilde{A}$ i.e. $\widetilde{A}_r = \{x \in R \,|\, \widetilde{A}(X) \geq r\}$, $0 < r \leq 1$.

(ii) For $k = 2$, $R(\widetilde{A}) = (E(\widetilde{A}), -\operatorname{var}(\widetilde{A}))^T$ where $E(\widetilde{A})$, and $\operatorname{var}(\widetilde{A})$ are the expectation and variance of the density function associated with $(\widetilde{A})$ (Delgado *et al.* [7], 1998a).

(iii) For $k = 3$, $R(\widetilde{A}) = (V(\widetilde{A}),\ A(\widetilde{A}),\ F(\widetilde{A}))^T$ where $V(\widetilde{A})$, $A(\widetilde{A})$ and $F(\widetilde{A})$ are value, ambiguity and fuzziness of $\widetilde{A}$, respectively, which are defined as:

$$V(\widetilde{A}) = \int_0^1 r[L_{\widetilde{A}}(r) + R_{\widetilde{A}}(r)]dr$$

$$A(\widetilde{A}) = \int_0^1 r[R_{\widetilde{A}}(r) - L_{\widetilde{A}}(r)]dr$$

$$F(\widetilde{A}) = \int_0^{1/2} [R_{\widetilde{A}}(r) - L_{\widetilde{A}}(r)]dr + \int_{1/2}^1 [L_{\widetilde{A}}(r) - R_{\widetilde{A}}(r)]dr$$

where both $L_{\widetilde{A}}(.)$ and $R_{\widetilde{A}}(.)$ are from [0, 1] to $\mathbb{R}$ and defined as:

$$L_{\widetilde{A}}(r) = \begin{cases} \inf\{x|x \in \widetilde{A}_r\} & \text{if } \ r \in (0,1] \\ \inf\{x|x \in \sup(\widetilde{A})\} & \text{if } \ r = 0 \end{cases}$$

$$R_{\widetilde{A}}(r) = \begin{cases} \sup\{x|x \in \widetilde{A}_r\} & \text{if } r \in (0,1] \\ \sup\{x|x \in \sup(\widetilde{A})\} & \text{if } r = 0 \end{cases}$$

and

$$\sup(\widetilde{A}) = \{x \in R|\widetilde{A}(x) > 0\}\,.$$

4. Multiobjective Linear Integer Programming Problem

In this section some of the basis of multiobjective linear integer programming problem have been reviewed.

Consider the model:

$$\begin{aligned} \text{(MIP)} \quad & \min z(x) = Cx \\ & \text{subject to } Ax \geq b \\ & \qquad\qquad x = 0 \text{ or } 1 \end{aligned} \tag{4.1}$$

where C is a $k \times n$ matrix of coefficients of the linear objective function, $A \in R^{m \times n}$ and $b \in R^m$ and $x \in R^n$, is called a multiobjective linear integer programming problem (MIP).

Let $X = \{x \in R^n | Ax \geq b, x = 0 \text{ or } 1\}$ be a feasible set of (MFP).

Definitions.

4.1. A point $x^* \in X$ is called a complete optimal solution for (MIP) iff $z(x^*) \leq z(x) \; \forall \; x \in X$.

4.2. A point $x^* \in X$ is called a Pareto optimal solution for (MIP) iff there does not exist another $x \in X$ such that $z(x) \leq z(x^*)$ and $z(x) \neq z(x^*)$.

4.3. A point $x^* \in X$ is called a weak pareto optimal solution for (MIP) iff there does not exists another $x \in X$ such that $z(x) < z(x^*)$.

Let E^c, E^p and E^{wp} denote the set of all complete optimal solution, pareto optimal and weak pareto optimal solutions for (MIP) respectively, then $E^c \subseteq E^p \subseteq E^{wp}$.

5. Algorithm: Fuzzy Fractional Set Covering Problem

In this section a method is investigated to enumerate a linear fractional set covering problem with fuzzy parameters. Moreover, suppose that R is a given vector ranking function

Then mathematically Fuzzy Fractional Multiobjective linear set covering problem is:

$$\begin{aligned} \text{(FFMCP)} \quad & \min \widetilde{z} \underset{R}{=} \frac{\widetilde{c}x}{\widetilde{d}x + \alpha} \\ & \text{subject to } x \in X \end{aligned} \tag{5.1}$$

where $\widetilde{c} = (\widetilde{c}_1, \widetilde{c}_2, \ldots, \widetilde{c}_n)$ and $\widetilde{d} = (\widetilde{d}_1, \widetilde{d}_2, \ldots, \widetilde{d}_n) \in \mathbb{R}^n$, α is some constant so that $(\widetilde{d}x + \alpha) > 0$.

Definitions.

5.1. A point $x^* \in X$ is called an R-optimal solution for (FFMCP) (5.1) iff $\dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha} \underset{R}{\leq} \dfrac{\widetilde{c}x}{\widetilde{d}x + \alpha}$ $\forall\, x \in X$.

5.2. A point $x^* \in X$ is called an R-efficient solution for (FFMCP) (5.1) iff there does not exist another $x \in X$ such that $\dfrac{\widetilde{c}x}{\widetilde{d}x + \alpha} \underset{R}{\leq} \dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha}$ and $\dfrac{\widetilde{c}x}{\widetilde{d}x + \alpha} \underset{R}{\neq} \dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha}$.

5.3. A point $x^* \in X$ is called an R-weak efficient solution for (FFMCP) (5.1) iff there does not exist another $x \in X$ such that $\dfrac{\widetilde{c}x}{\widetilde{d}x + \alpha} \underset{R}{<} \dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha}$.

We denote R-optimal, R-efficient and R-weak efficient solution sets by X^{ro}, X^{re}, X^{rw} respectively, then $X^{ro} \subseteq X^{re} \subseteq X^{rw}$.

Now associated with the (FFMCP) (5.1), now consider the following (FFM CP) problemes.

$$\begin{aligned} &\min\{z(x) = R\left(\frac{\widetilde{c}x}{\widetilde{d}x + \alpha}\right) | x \in X\}, \\ &\text{where } R(.) = (R_1(.), R_2(.), \ldots, R_k(.)\} \end{aligned} \tag{5.2}$$

Next some theorems are presented which will provide the relationship between the optimal solution of the (FFMCP) (5.2) and (FFMCP) (5.1).

Theorem 5.1. *A point $x^* \in X$ is an R-optimal solution for the (FFMCP) (5.1) iff x^* is a complete optimal solution of FFMCP (5.2).*

Proof. ($\Rightarrow$) Assume that $x^* \in X$ is an R-optimal solution of the (FFMCP) (5.1), then for any $x \in X$, $\dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha} \underset{R}{\leq} \dfrac{\widetilde{c}x}{\widetilde{d}x + \alpha}$. Using Definition 3.1 of $\underset{R}{\leq}$, $R\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha}\right) \leq R\left(\dfrac{\widetilde{c}x}{\widetilde{d}x + \alpha}\right)$, for all $x \in X$, hence $z(x^*) \leq z(x)$ $\forall\, x \in X$.
Therefore, x^* is a complete optimal solution for (FFMCP) (5.2).

($\Leftarrow$) Assume x^* is a complete optimal solution for (FFMCP) (5.2), then for all $x \in X$, $z(x^*) \leq z(x)$, hence $R\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha}\right) \leq R\left(\dfrac{\widetilde{c}x}{\widetilde{d}x + \alpha}\right)$. Therefore $\dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha} \underset{R}{\leq} \dfrac{\widetilde{c}x}{\widetilde{d}x + \alpha}$ for all $x \in X$. This implies x^* is an R-optimal solution for (FFMCP) (5.1). $\square$

Theorem 5.2. *A point $x^* \in X$ is an R-efficient solution for (FFMCP)* (5.1) *iff x^* is a pareto optimal solution for (FFMCP)* (5.2).

Proof. ($\Rightarrow$) Let $x^* \in X$ be an R-efficient solution for (FFMCP) (5.1). On the contrary, suppose that x^* is not a pareto optimal solution for (FFMCP) (5.2), i.e., there exists $\bar{x} \in X$ such that $z(\bar{x}) \leq z(x^*)$ and $z(\bar{x}) \neq z(x^*)$, hence $R\left(\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x} + \alpha}\right) \leq R\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha}\right)$ and $R\left(\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x} + \alpha}\right) \neq R\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha}\right)$. This implies $\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x} + \alpha} \underset{R}{\leq} \dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha}$ and $\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x} + \alpha} \neq \dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha}$ which contradicts that $x^* \in X$ is an efficient solution for the (FFMCP) (5.1).

($\Leftarrow$) Let $x^* \in X$ be a pareto optimal solution for (FFMCP) (5.2). On the contrary, suppose that $x^* \in X$ is not an R-efficient solution for the (FFMCP) (5.1), i.e. there exists an $\bar{x} \in X$ such that $\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x} + \alpha} \underset{R}{\leq} \dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha}$ and $\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x} + \alpha} \underset{R}{\neq} \dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha}$ this implies and $R\left(\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x} + \alpha}\right) \leq R\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha}\right)$ and $R\left(\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x} + \alpha}\right) \neq R\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha}\right)$.

This means $z(\bar{x}) \geq z(x^*)$ and $z(\bar{x}) \neq z(x^*)$ which contradicts that $x^* \in X$ is a pareto optimal solution for (FFMCP) (5.2). $\square$

Theorem 5.3. *A point $x^* \in X$ is an R-weak efficient solution for (FFMCP)* (5.1) *iff x^* is a weak pareto optimal solution for (FFMCP)* (5.2).

Proof. ($\Rightarrow$) Let $x^* \in X$ be a R-weak efficient solution for (FFMCP) (5.1). On the contrary suppose x^* is not a weak pareto optimal solution for (FFMCP) (5.2) this implies there exist another $x \in X$ such that $z(x) < z(x^*)$ hence $\left(\dfrac{\widetilde{c}x}{\widetilde{d}x + \alpha}\right) < R\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha}\right)$ this implies $\dfrac{\widetilde{c}x}{\widetilde{d}x + \alpha} \underset{R}{<} \dfrac{\widetilde{c}x^*}{\widetilde{d}x^* + \alpha}$ which contradicts that $x^* \in X$ is an R-weak efficient solution for (FFMCP) (5.1).

($\Leftarrow$) On the other hand let $x^* \in X$ be a weak pareto optimal solution for (FFMCP) (5.2). On the contrary, suppose that $x^* \in X$ is not an R-weak efficient solution for the (FFMCP) (5.1) i.e. there exists an $\bar{x} \in X$ such that

$\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x}+\alpha} < \dfrac{\widetilde{c}x^*}{\widetilde{d}x^*+\alpha}$ this implies $R\left(\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x}+\alpha}\right) < R\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^*+\alpha}\right)$. This means $z(\bar{x}) < z(x^*)$ which contradicts that $x^* \in X$ is a weak pareto optimal solution for (FFMCP) (5.2). $\square$

Lexicographic approach. This ranking function approach is being used for solving fuzzy linear fractional programming problem.

Suppose that the decision maker orders the components of the vector ranking function based on some preferences, by starting with the most important component. In this case a lexicographic ranking function to a fuzzy number is associated.

Definitions.

5.4. A vector x is called lexicographically negative, denoted by $x \prec 0$ if the following two conditions hold:

(a) x is not identically zero

(b) The first non-zero component of x is negative.

A lexicographically non-positive vector, denoted by $\preceq 0$, is either the zero vector or a lexicographically negative vector.

5.5. A vector x is lexicographically less than y, denoted by $x \prec y$, if and only if $x - y \prec 0$.

Also x is said to be lexicographically less than or equal to y , denoted by $x \preceq y$, if and only if $x - y \preceq 0$.

5.6. The vector ranking function $\mathcal{R}$ whose component are ordered on the basis of the decision maker's preferences, starting with the most important is called a lexicographic vector ranking function. Moreover, let $\widetilde{a}$ and $\widetilde{b}$ belong to $F(R)$, then define

- $\widetilde{a} \underset{R}{\leq} \widetilde{b}$ if and only if $\mathcal{R}(\widetilde{a}) \preceq \mathcal{R}(\widetilde{b})$
- $\widetilde{a} \underset{R}{<} \widetilde{b}$ if and only if $\mathcal{R}(\widetilde{a}) \prec \mathcal{R}(\widetilde{b})$
- $\widetilde{a} \underset{R}{=} \widetilde{b}$ if and only if $\mathcal{R}(\widetilde{a}) = \mathcal{R}(\widetilde{b})$
- $\widetilde{a} \underset{R}{\neq} \widetilde{b}$ if and only if $\mathcal{R}(\widetilde{a}) \neq \mathcal{R}(\widetilde{b})$

Now return to the (FFMCP) (5.1) and apply lexicographic vector ranking function to solve it. In (FFMCP) (5.1) set $R = \mathcal{R}$,i.e., the vector raking function is a lexicographic vector ranking function, thus (FFMCP) (5.1) changes to (FFMCP) (5.3) as:

$$\begin{aligned} &(\text{FFMCP}) \quad \min \widetilde{z} \underset{\mathcal{R}}{=} \frac{\widetilde{c}x}{\widetilde{d}x + \alpha} \\ &\qquad\qquad\quad \text{s.t. } x \in X \end{aligned} \tag{5.3}$$

Setting $R = \mathcal{R}$ in definition (5.1), (5.2) and (5.3) gives the definition of $\mathcal{R}$-optimal solution, $\mathcal{R}$-efficient solution and $\mathcal{R}$-weak efficient solution for (FFMCP) (5.3), respectively.

As mentioned earlier associated with the (FFMCP) (5.3) considered the following MOFP problem

$$\begin{aligned} &\text{MOFP} \quad \min \left(\mathcal{R}_1\left(\frac{\widetilde{c}x}{\widetilde{d}x + \alpha}\right), \mathcal{R}_2\left(\frac{\widetilde{c}x}{\widetilde{d}x + \alpha}\right), \ldots, \mathcal{R}_K\left(\frac{\widetilde{c}x}{\widetilde{d}x + \alpha}\right)\right) \\ &\qquad\qquad \text{s.t. } x \in X\,. \end{aligned} \tag{5.4}$$

For solving (MOFP) (5.4), by using lexicographic minimization process (Steuer, 1986), recursively construct the following reduced feasible region.

$$\begin{aligned} S_0 &= X \\ S_1 &= \left\{ y \in S_0 : \mathcal{R}_1\left(\frac{\widetilde{c}y}{\widetilde{d}y + \alpha}\right) = \min\left(\mathcal{R}_1\left(\frac{\widetilde{c}x}{\widetilde{d}x + \alpha}\right), x \in S_0\right)\right\}, \\ S_2 &= \left\{ y \in S_1 : \mathcal{R}_2\left(\frac{\widetilde{c}y}{\widetilde{d}y + \alpha}\right) = \min\left(\mathcal{R}_2\left(\frac{\widetilde{c}x}{\widetilde{d}x + \alpha}\right), x \in S_1\right)\right\}, \\ &\vdots \\ S_k &= \left\{ y \in S_{k-1} : \mathcal{R}_k\left(\frac{\widetilde{c}y}{\widetilde{d}y + \alpha}\right) = \min\left(\mathcal{R}_k\left(\frac{\widetilde{c}x}{\widetilde{d}x + \alpha}\right), x \in S_{k-1}\right)\right\} \end{aligned}$$

The process begins by minimizing objective function $\mathcal{R}_1\left(\frac{\widetilde{c}x}{\widetilde{d}x + \alpha}\right)$ which is bounded over $S_0 = X$. By constraining S_0 to the minimal value of the first criterion, S_1 is obtained, Then, the second objective function $\mathcal{R}_2\left(\frac{\widetilde{c}x}{\widetilde{d}x + \alpha}\right)$ which is bounded over S_1 is minimized. By constraining S_1 to the minimal value of the second criterion, S_2 is obtained. The process continues until either

$S_k \neq \phi$ or for some ℓ $(1 \leq \ell \leq k)$, S_ℓ, $S_{\ell+1}, \ldots, S_k = \phi$. The later occurs when $\mathcal{R}_1\left(\dfrac{\widetilde{c}x}{\widetilde{d}x+\alpha}\right)$ is unbounded over $S_{\ell-1}$ (Steuer, 1986).

Following theorem provides one to one correspondence between the solutions of (FFMCP) (5.1) and (MOFP) (5.4):

Theorem 5.4. *Extreme points of S_j, $1 \leq j \leq k$, are extreme points of X.*

Proof. The set of extreme points of the optimal set min $\{cx \mid x \in X\}$ are extreme points of X. Thus if $\bar{x}$ is an extreme point of $S_1, \bar{x}$ is an extreme point of X. Similarly, if $\bar{x}$ is an extreme point of $S_2, \bar{x}$ is an extreme point of S_1, and so forth. In this way, the theorem is proved. □

Theorem 5.5. *Let $x^* \in S_k$, then x^* is an $\mathcal{R}$-efficient solution for the* (MOFP) (5.4).

Proof. Suppose x^* is not an $\mathcal{R}$-efficient solution for the (MOFP) (5.4) Then there exists an $\bar{x} \in X$ such that $\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x}+\alpha} \underset{\mathcal{R}}{\leq} \dfrac{\widetilde{c}x^*}{\widetilde{d}x^*+\alpha}$ and $\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x}+\alpha} \underset{R}{\neq} \dfrac{\widetilde{c}x^*}{\widetilde{d}x^*+\alpha}$. It follows that $\mathcal{R}\left(\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x}+\alpha}\right) \preceq \mathcal{R}\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^*+\alpha}\right)$ and $\mathcal{R}\left(\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x}+\alpha}\right) \neq \mathcal{R}\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^*+\alpha}\right)$. Hence $\mathcal{R}\left(\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x}+\alpha}\right) - \mathcal{R}\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^*+\alpha}\right) \preceq 0$ and $\mathcal{R}\left(\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x}+\alpha}\right) - \mathcal{R}\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^*+\alpha}\right) \neq 0$.

Now consider the following two cases:

Case 1. Let $\mathcal{R}_1\left(\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x}+\alpha}\right) < \mathcal{R}_2\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^*+\alpha}\right)$. Hence x^* does not belong to S_1. From $S_k \subseteq S_{k-1} \subseteq \ldots \subseteq S_0 = X$, it follows that x^* does not belong to S_k, which contradicts that $x^* \in S_k$.

Case 2. There exists a t $\in \{2, \ldots, k\}$ such that $\mathcal{R}_1\left(\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x}+\alpha}\right) - \mathcal{R}_t\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^*+\alpha}\right)$ < 0 and $\mathcal{R}_i\left(\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x}+\alpha}\right) - \mathcal{R}_i\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^*+\alpha}\right) = 0$ for all $i = \{1, 2, \ldots, t-1\}$. Since $\mathcal{R}_1\left(\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x}+\alpha}\right) - \mathcal{R}_1\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^*+\alpha}\right) = 0$ for all $i = \{1, 2, \ldots, t-1\}$ and $x^* \in S_k \subseteq S_{k-1} \subseteq \ldots \subseteq S_0 = X$. This implies, $\bar{x} \in S_i$, for all $i = \{1, 2, \ldots, t-1\}$. On the other hand, $\mathcal{R}_t\left(\dfrac{\widetilde{c}\bar{x}}{\widetilde{d}\bar{x}+\alpha}\right) < \mathcal{R}_t\left(\dfrac{\widetilde{c}x^*}{\widetilde{d}x^*+\alpha}\right)$ and $\bar{x} \in S_i, i \in \{1, 2, \ldots, t-1\}$ imply that x^* does not belong to S_i. This contradicts that $x^* \in S_k$. □

From Theorem 5.4, if $x^* \in S_k$, then x^* is an $\mathcal{R}$-efficient solution for the (MOFP) (5.4). A situation may happen that the set S_k is empty. It occurs when for an ℓ $(1 \leq \ell \leq k)\mathcal{R}_\ell\left(\dfrac{\widetilde{c}x}{\widetilde{d}x + \alpha}\right)$ is unbounded over $S_{\ell-1}$. In this case, the decision maker have the option of gaining the flexibility to change the order of components of vector ranking function corresponding with the remaining objective functions.

Hence one to one correspondences is obtained between the solutions of (FFMCP) (5.1) and (MOFP) (5.4). Now to solve (MOFP) (5.4) which is a crisp linear fractional set covering problem a linearlization technique has been used as below.

Step 1. Given Linear Fractional Set Covering Problem (LFP). Form the corresponding continuous program (LFP') by embedding the feasible region into R^n (a cube with n vertices). Let S be the feasible set for (LFP').

Step 2. Choose a feasible solution $X_0 \in S$ such that $\nabla f(X_0) \neq 0$. Form the corresponding linear program (LP)

$$\underset{X\in S}{\text{Minimize}}\, \nabla f(x_0)^T X \ \underset{X\in S}{\text{Minimize}} \sum_{j=1}^{n} c_j x_j \,.$$

Step 3. Arrange the cost coefficients c_j's of Step 2 in an ascending order and rename the variable according as y_u's to get the linear program (LP'). Go to step 4 to find the optimal solution of the program (LP').

Step 4. For $j = \{1, 2, \ldots, n\}$, define $Y_0 = (y_1, y_2, \ldots, y_n)$ as zero variable with $y_j = 0 \ \forall \ j \in J$ and Y_{ji} as j-variable; $i \in I_j = \{1, 2, \ldots, {}^{n+1-j}C_1\}$ and $j \in J$

Since none of the constraints is satisfied, modify Y_0 to 1-variable, Y_{1i}; $i \in I_1 = \{1, 2, \ldots, {}^{(n+1)-1}C_1\}$ by fixing variable y_j (individual) to 1 for $j \in J$.

Step 5. Suppose at rth iteration Y_{ri} is r-variable, $r \in J$, $i \in I_r = \{1, 2, \ldots, {}^{n+1-r}C_1\}$.

If Y_{ri} is feasible for some $i \in I_r$ go to Step 7, otherwise go to step 6.

Step 6. For each r-variable Y_{ri}, $r \in I_r$

Let $p =$ number of constraints of (LP) satisfied by Y_{ri}'s

$z =$ the value of the corresponding objective function for Y_{ri}'s

Choose that Y^*_{ri} for which 'p' is maximum and 'z' is minimum. Let it happen for $i = q$. Fix variable $y_q = 1$ in Y_{ri} which gets modified to $Y_{si}(s = r + 1)$ as s-variable with s one, $i \in I_s = \{1, 2, \ldots, {}^{n+1-s}C_1\}$.

Go to Step 5.

Step 7. Find the value of objective function for each Y_{ri}. Let minimum value be for $i = k$. The optimal solution of the (LP') is Y^*_{rk}.

Step 8. Rearrange the variable as done in Step 3 to get the corresponding solution X^*_{rk} of (LFP).

If $X^*_{rk} = X_0$ then X^*_{rk} will be optimal prime cover solution of (LFP), otherwise take $X_0 = X^*_{rk}$ and go to step 2.

6. Numerical Example

To illustrate the efficiency of the proposed method we consider the following example:

Example 6.1. Consider the following linear fractional fuzzy set covering problem

$$\begin{aligned} \min z(x) &= \frac{\widetilde{c}_1 x_1 + \widetilde{c}_2 x_2 + \widetilde{c}_3 x_3}{\widetilde{d}_1 x_1 + \widetilde{d}_2 x_2 + \widetilde{d}_3 x_3} \\ \text{subject to } & x_1 + x_2 \geq 1 \\ & x_2 + x_3 \geq 1 \\ & x_1 + x_3 \geq 1 \\ & x_l = 0 \text{ or } 1, \quad i = 1, 2, 3 \end{aligned} \tag{6.1}$$

where the membership functions of $\widetilde{c}_1$, $\widetilde{c}_2$, $\widetilde{c}_3$, $\widetilde{d}_1$, $\widetilde{d}_2$ and $\widetilde{d}_3$ are

$$\widetilde{c}_1(x) = \begin{cases} 0 & x < 5 \\ x - 5 & 5 \leq x < 6 \\ 1 & 6 \leq x \leq 7 \\ (20 - x)/13 & 7 < x \leq 20 \\ 0 & 20 < x \end{cases}$$

$$\widetilde{c}_2(x) = \begin{cases} 0 & x < 16 \\ x - 16 & 16 \leq x < 17 \\ 1 & 17 \leq x \leq 18 \\ (40 = x)/22 & 18 < x \leq 40 \\ 0 & 40 < x \end{cases}$$

$$\widetilde{c}_3(x) = \begin{cases} 0 & x < 24 \\ x - 24 & 24 \leq x < 25 \\ 1 & 25 \leq x \leq 26 \\ (40 = x)/22 & 26 < x \leq 50 \\ 0 & 50 < x \end{cases}$$

$$\widetilde{d}_1(x) = \begin{cases} 0 & x < 10 \\ x - 10 & 10 \leq x < 11 \\ 1 & 11 \leq x \leq 12 \\ (30 - x)/18 & 12 < x \leq 30 \\ 0 & 30 < x \end{cases}$$

$$\widetilde{d}_2(x) = \begin{cases} 0 & x < 15 \\ x - 15 & 15 \leq x < 16 \\ 1 & 16 \leq x \leq 17 \\ (20 - x)/13 & 17 < x \leq 30 \\ 0 & 30 < x \end{cases}$$

$$\widetilde{d}_3(x) = \begin{cases} 0 & x < 20 \\ x - 20 & 20 \leq x < 21 \\ 1 & 21 \leq x \leq 22 \\ (50 - x)/28 & 22 < x \leq 50 \\ 0 & 50 < x \end{cases}$$

where $V(\widetilde{A})$, $A(\widetilde{A})$ and $F(\widetilde{A})$ are as defined before: Note that

$$\begin{array}{lll}
V(\widetilde{c}_1) = 8.5, & V(\widetilde{c}_2) = 21, & V(\widetilde{c}_3) = 88/3 \\
V(\widetilde{d}_1) = 43/3, & V(\widetilde{d}_2) = 37/2, & V(\widetilde{c}_3) = 76/3 \\
A(\widetilde{c}_1) = 17/6, & A(\widetilde{c}_2) = 13/3 & A(\widetilde{c}_3) = 27/2 \\
A(\widetilde{d}_1) = 11/3, & A(\widetilde{d}_2) = 17/6, & A(\widetilde{d}_3) = 16/3 \\
F(\widetilde{c}_1) = 3.5, & F(\widetilde{d}_2) = 23/4, & F(\widetilde{c}_3) = 25/4 \\
F(\widetilde{d}_1) = 19/4, & F(\widetilde{c}_2) = 7/2, & F(\widetilde{d}_3) = 29/4
\end{array}$$

The following MLCP is associated with the problem (6.1):

$$\min z(x) =$$
$$\left\{\left(\frac{8.5x_1+21x_2+\frac{88}{3}x_3}{\frac{43}{3}x_1+\frac{37}{2}x_2+\frac{76}{3}x_3}\right)\left(\frac{\frac{17}{6}x_1+\frac{13}{3}x_2+\frac{27}{2}x_3}{\frac{11}{3}x_1+\frac{17}{6}x_2+\frac{16}{3}x_3}\right),\left(\frac{3.5x_1+\frac{23}{4}x_2+\frac{25}{4}x_3}{\frac{19}{4}x_1+\frac{7}{2}x_2+\frac{29}{4}x_3}\right)\right\}$$

$$\begin{aligned}
\text{subject to } & x_1 + x_2 \geq 1 \\
& x_2 + x_3 + \geq 1 \\
& x_1 + x_3 \geq 1 \\
& x_1 = 0 \text{ or } 1; \quad i = 1, 2, 3.
\end{aligned} \tag{6.2}$$

To solve the above problem, lexicographic approach is being used first minimize the first objective function to obtain S_1:

i.e.

$$\begin{aligned}
\min f(x) = & \frac{8.5x_1 + 21x_2 + \frac{88}{3}x_3}{\frac{43}{3}x_1 + \frac{37}{2}x_2 + \frac{76}{3}x_3} \\
\text{subject to } & x_1 + x_2 \geq 1 \\
& x_2 + x_3 \geq 1 \\
& x_1 + x_3 \geq 1 \\
x_i = 0 \text{ or } 1 \quad & \forall\, i = 1, 2, 3.
\end{aligned} \tag{6.3}$$

This is a crisp linear fractional set covering problem. Now to solve this problem linearlization technique is being used as defined before.

Step 1: The corresponding (FCP') is

$$\min f(x) = \frac{8.5x_1 + 21x_2 + \frac{88}{3}x_3}{\frac{43}{3}x_1 + \frac{37}{2}x_2 + \frac{76}{3}x_3}$$

$$\text{subject to } X = (x_1, x_2, x_3) \in X$$
$$= \{(x_1, x_2, x_3) | x_1 + x_2 \geq 1, x_2 + x_3 \geq 1,$$
$$x_1 + x_3 \geq 1, x_1, x_2, x_3 \geq 0\}$$

Step 2: Choose $X_0 = (1, 1, 1)$ as one of the feasible solution of (FCP') with $\nabla f(x_0) \neq 0$. The corresponding (LP) is

$$\min \nabla f(x_0^T)X = -0.103X_1 + 0.039X_2 + 0.063x_3, \quad X \in S$$

Step 3: Arrange the coefficients in increasing order by defining x_1 ad y_1, x_2 as y_2, x_3 as y_3. The new (LP') now is

$$\min -0.103x_1 + 0.039x_2 + 0.063x_3$$
$$\text{subject to } \; y_1 + y_2 \geq 1, y_2 + y_3 \geq 1, y_1 + y_3 \geq 1.$$

Step 4: Take $Y_0 = (0, 0, 0)$, a zero variable, obviously none of the constraint of (LP$'$) is satisfied, therefore Y_0 is not a feasible solution.

Step 5: Consider all possible one-variable solutions Y_{1i}, $i = I_1 = \{1, 2^{(3+1)-1}C_1\}$ as

$$Y_{11} = (1, 0, 0), \quad Y_{12} = (0, 1, 0), \quad Y_{13} = (0, 0, 1)$$

Action for	Y_{11}	Y_{12}	Y_{13}
No. of eqns. Satisfied (p)	2	2	2
Value of objective function (z)	-0.103	0.039	0.063

p is equal for all but the value of z is minimum for Y_{11}. Fix $y_1 = 1$ in Y_{1i}. Now, the two-variables $Y'_{2i}s$, $i \in I_2 = \{1, 2, \ldots, ^{3+1-2}C_1\}$ to act are as follows:

$$Y_{21} = (1, 1, 0)Y_{22} = (1, 0, 1)$$

Action for	Y_{21}	Y_{22}
No. of eqns. Satisfied (p)	3	3
Value of objective function (z)	-0.07	-0.043

'p' is maximum for all but the value of 'z' is minimum for Y_{21}. Thus the optimal solution for (LP$'$) is $(1, 1, 0)$.

Step 8: After rearranging the terms, the corresponding solution of FCP is $X_1^* = (1,1,0)$. Since $X_1^* = X_0$, take $X_1^* = X_0$ and go to *Step* 2.

Algorithm revisited.

Step 2: Here $X_0 = (1,1,0)$ as one of the feasible solution of (FCP') with $\nabla f(X_0) \neq 0$. The corresponding (LP) is

$$\min \nabla f(X_0)^T X = -0.133x_1 + 0.133x_2 + 0.200x_3, \quad X \in S$$

Step 3: Arrange the coefficients in increasing order by defining x_1 as y_1, x_2 as y_2, x_3 as y_3. The new (LP') now is

$$\begin{aligned} &\min 0.133y_1 + 0.133y_2 + 0.200y_3 \\ &\text{subject to } y_1 + y_2 \geq 1; y_2 + y_3 \geq 1; \\ &\qquad\qquad y_1 + y_3 \geq 1, y_1, y_2, y_3 \geq 0 \end{aligned}$$

Step 4: Take $Y_0 = (0,0,0)$, a zero variable. Obviously none of the constraint of (LP') is satisfied, therefore Y_0 is not a feasible solution.

Step 5: Consider all possible one-variable solutions $Y_{1i}, i \in I_1 = \{1, 2^{(3+1)-1}C_1\}$ as

$$Y_{11} = (1,0,0), \quad Y_{12} = (0,1,0), \quad Y_{13} = (0,0,1)$$

Action for Y_{11} Y_{12} Y_{13} No. of eqns. Satisfied (p) 2 2 2 Value of objective function (z) -0.133 0.133 0.200

p is equal for all but the value of z is minimum for Y_{11}. Fix $y_1 = 1$ in Y_{1i}. Now, the two-variables $Y_{2i}'s, i \in I_2 = \{1, 2, \ldots {}^{3+1-2}C_1\}$ to act are as follows:

$$Y_{21} = (1,1,0) \quad Y_{22} = (1,0,1)$$

Action for	Y_{21}	Y_{22}
No. of eqns. satisfied (p)	3	3
Value of objective function (z)	0	0.333

'p' is maximum for all but the value of 'z' is minimum for Y_{21}. Thus the optimal solution for (LP$'$) is $(1,1,0)$.

Step 8: After rearranging the terms, the corresponding solution of FCP is $X_1^* = (1,1,0)$. Since $X_1^* \neq X_0$, therefore this will be the optimal prime cover of (FCP) (6.3).

Thus $S_1 = \{(1,1,0)\}$, therefore this will be the optimal solution for (6.2) and hence for the problem (6.1).

Conclusion

In this paper a linear fractional set covering problem with fuzzy parameters in the objective function is considered. The approach for solving this problem is to use lexicographic vector ranking function. To improve the draw back of using a single characteristic, a k-dimensional vector ranking function to a fuzzy number is associated.

References

[1] R. Bellman, L.A. Zadeh, Decision making in fuzzy environment, *Management Sci. B*, **17** (1970), 141-164.

[2] G. Bortolan, R. Degani, A review of some methods for ranking fuzzy numbers, *Fuzzy Sets and Systems*, **15** (1985), 1-19.

[3] M. Zangiabadi, H.R. Maleki, A method for solving linear programming problems with fuzzy parameters based on multiobjective linear programming technique, *Asia Pecific Journal of Operational Research*, **24**, No. 4 (2007), 557-573.

[4] R. Steuer, *Multiple Criteria Optimization: Theory, Computation and Application*, Wiley (1986).

[5] M. Delgado, J.L. Verdegay, M.A. Vila, Imprecise costs in mathematical programming problems, *Control and Cybernetics*, **16**, No. 2 (1987), 113-121.

[6] M. Delgado, J.L. Verdegay, M.A. Vila, A general model for fuzzy linear programming, *Fuzzy Sets and Systems*, **29** (1989), 21-29.

[7] M. Delgado, J.L. Verdegay, M.A. Vila, W. Voxman, A fuzziness measure for fuzzy numbers: Applications, *Fuzzy Sets and Systems*, **94** (1998), 205-216.

[8] Bazaraa et al., *Non Linear Programming Theory and Algorithms*, 2nd edition, John Wiley & Sons, 1993.

[9] J.M. Cadenas, J.L. Verdegay, Using fuzzy numbers in linear programming, *IEEE Transactiona on Systems, Man, and Cybernetics* (Part B), **27**, No. 6 (1997), 1016-1022.

[10] S.R. Arora, M.C. Puri, Enumeration technique for the set covering problem with linear fractional as its objectiuve function, *ZAMM*, **56** (1977), 181-186.

[11] R.R. Saxena, S.R. Arora, Cutting plane technique for the multi-objective set covering problem with linear fractional objective functions, *IJOMAS*, **14**, No. 1 (1998), 111-112.

[12] R.R. Saxena, S.R. Arora, A linearization technique for solving the quadratic set covering problem, *Optimization*, **39**, No. 1 (1996), 35-42.

[13] R.R. Saxena, S.R. Arora, Enumeration technique for solving multiobjective linear set covering problem, *APZOR*, **12** (1995), 87-97.

[14] R.R. Saxena, S.R. Arora, Cutting plane technique for the multi-objective set covering pronlem with linear fractional objective functions, sent after revision to the *APZOR*, New Zealand (1997).