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SOME NEW GENERALIZED DIFFERENCE SEQUENCE SPACES ON SEMINORMED SPACE DEFINED BY ORLICZ FUNCTION

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Abstract: In this paper, we define the sequence space $\ell_M(\Delta^m, u, p, q, s)$ on seminormed complex linear space by using an orlicz function.we give various properties and some inclusion relations involving $\ell_M(\Delta^m, u, p, q, s)$. This study generalized some results of Bektaş and Altin [1].

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1. Introduction and Preliminaries

Let ℓ_∞, c and c_0 be the linear spaces of bounded, convergent and null sequence $x = (x_k)$ with complex number, respectively, normed by $\|x\|_\infty = \sup_k |x_k|$, where $k \in \mathbb{N}$. Throughout this paper $\omega(X), \ell_\infty(X), c(X)$ and $c_0(X)$ denote the class of all bounded, convergent and null X-valued sequences, where (X,q) is a seminormed space, seminormed by q. The zero sequence is denoted by $\bar{\theta} = (\theta, \theta.......)$, where θ is the zero element of X. The spaces are seminormed spaces seminormed by $g(x) = \sup_{k\in\mathbb{N}} q(x_k)$. For $X = \mathbb{C}$, the set of complex numbers, these represent the above corresponding scalar valued sequence spaces.

The idea of difference sequence sets was introduced by Kizmaz [7] and this

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subject was generalized by Et and Çolak [4]. After then the difference sequence spaces have been studied by various author such as Et [3], Et and Nurary [5], Malkowsky and Prasar [13], Mursaleen [14] Tripathy [18], [19], Tripathy et.al [20].

The study of orlicz sequence spaces was initiated with a certain specific purpose in Banach space theory. Indeed Lindberg [9] got interested in orlicz spaces in connection with finding Banach spaces with symmetric Schauder bases having complementary subspaces isomorphic to c_0 or ℓ_∞ $(1 \leq p_k \leq \infty)$.

Prashar and Choudhry [16] have introduced and discussed some properties of the four sequence spaces defined by using an orlicz function M, which generalized the sequence space ℓ_M and strongly summable sequence spaces $[C,1,p], [C,1,p]_0$ and $[C,1,p]_\infty$. Later on different types of sequence spaces were introduced by using an orlicz function by Bektas and Altin [1] Tripathy [19], Tripathy et al.[20]. The orlicz sequence spaces are the special cases of orlicz spaces introduced in [8]. Orlicz sequence spaces find a number of useful application in theory of nonlinear integral equation. Where as the orlicz sequence spaces are the generalizations of ℓ_p-spaces, the $\mathbf{L}_p$-spaces find themselves enveloped in orlicz spaces.

The main purpose of this paper is to introduce and study the sequence space $\ell_M(\Delta^m, u, p, q, s)$ which arises from the notion of generalized difference operator Δ^m and the concept of an orlicz function.

In this section, using the generalized difference operatorΔ^m and concept of an orlicz function, we generalized the sequence space $\ell_M(p)$ which was introduced by Prashar and Choudhry [16].

The difference sequence space, $Z(\Delta) = \Big\{x = (x_k) : \Delta x \in Z\Big\}$, where $X = \ell_\infty$, c and c_0, were studied by Kizmaz [7]. The notion of difference sequence spaces was generalized by Et and Çolak [4] as follow

$$Z(\Delta^m) = \Big\{x = (x_k) : \Delta^m x_k \in Z\Big\}$$

for $Z = \ell_\infty, c$ and c_0, where $m \in \mathbb{N}$, $\Delta^m x_k = (\Delta^{m-1}x_k - \Delta^{m-1}x_{k+1})$ for all $k \in \mathbb{N}$.

The generalized difference operator has the following binomial representation,

$$\Delta^m x_k = \sum_{v=0}^{m} (-1)^v \begin{bmatrix} m \\ v \end{bmatrix} x_{k+v}, \text{ for all } \ k \in \mathbb{N}.$$

These sequence spaces are BK-spaces with the norm $\|x\|_\Delta = \sum_{i=1}^m |x_i| + \|\Delta^m x\|_\infty$.

An orlicz function is a function $M : [0,\infty) \to [0,\infty)$ which is continuous, non-decreasing and convex with $M(0) = 0,\ M(x) > 0$ for $x > 0$ and $M(x) \to \infty$, as $x \to \infty$, see [8]

If the convexity of orlicz function M is replaced by $M(x+y) \leq M(x)+M(y)$ then this function is called modulus function introduced by Nakano [15] and further investigated by Ruckle [17], Maddox [11], Bilgin [2] and other.

Lindenstrauss and Tzafriri [10] defined the sequence space ℓ_M such as

$$\ell_M = \Big\{x \in \omega : \sum_{k=1}^{\infty} M(\frac{|x_k|}{\rho}) < \infty,\ \text{for some } \rho > 0\Big\}.$$

The space ℓ_M with the norm $\|x\| = \inf\Big\{\rho > 0 : \sum_{k=1}^{\infty} M(\frac{|x_k|}{\rho}) \leq 1)\Big\}$ becomes a Banach space which is called an orlicz sequence space. For $M(t) = t^p$, $1 \leq p \leq \infty$ the space ℓ_M is closely related to the sequence space ℓ_p.

Definition 1. Let X be a complex linear with zero element θ and (X, q) be the seminored space with norm q. By $\omega(X)$ we denote the linear space of all sequences and $x = (x_k)$ with $x_k \in X$ and the usual coordinate wise operations:

$$\alpha x = (\alpha x_k) \text{ and } x + y = (x_k + y_k)$$

for each $\alpha \in \mathbb{C}$ where $\mathbb{C}$ denotes the set of complex numbers. If $\lambda = (\lambda_k)$ is a scalar sequence and $x \in \omega(X)$ then we shall write $\lambda x = (\lambda_k x_k)$. Let U be the set of all sequences $u = (u_k)$ such that $u_k \neq 0$ and complex for all $k = 1, 2, 3...$ Let $p = (p_k)$ be sequence of positive real numbers and M be an orlicz function. Given $u \in U$. Let $s \geq 0$. Then we define the sequence space.

$$\ell_M(\Delta^m, u, p, q, s) = \Big\{x \in \omega(X) : \sum_{k=1}^{\infty} k^{-s}\big[M\big(q(\frac{u_k\Delta^m x_k}{\rho})\big)\big]^{p_k} < \infty,\ \text{for some } \rho > 0\Big\}.$$

The following inequalities will be used throughout this paper. Let $p = (p_k)$ be a sequence of strictly positive real numbers with $0 \leq p_k \leq \sup_{p_k} = G$ and let $D = \max(1, 2^{G-1})$. Then for all k and $a_k, b_k \in \mathbb{C}$, we have

$$\big|a_k + b_k\big|^{p_k} \leq D\big\{|a_k|^{p_k} + |b_k|^{p_k}\big\} \quad [12] \tag{1}$$

A sequence space E is said to be solid (or normal) if $(\alpha_k x_k) \in E$ whenever $(x_k) \in E$ for all sequences (α_k) of scalar with $|\alpha_k| \leq 1$.

Remark 1.1. If M is convex function and $M((0) = 0$,then $M(\lambda x) \leq \lambda M(x)$ for all λ with $0 \leq \lambda \leq 1$.

2. Main Results

Theorem 2.1. *The sequence space $\ell_M(\Delta^m, u, p, q, s)$ is a linear space over the field $\mathbb{C}$ complex number.*

Proof. Let $x, y \in \ell_M(\Delta^m, u, p, q, s)$ and $\alpha, \beta \in \mathbb{C}$. Then there exist some positive numbers ρ_1, ρ_2 such that

$$\sum_{k=1}^{\infty} k^{-s}\big[M\big(q(\frac{u_k\Delta^m x_k}{\rho_1}))\big]^{p_k} < \infty$$

and

$$\sum_{k=1}^{\infty} k^{-s}\big[M\big(q(\frac{u_k\Delta^m y_k}{\rho_2}))\big]^{p_k} < \infty$$

Define $\rho_3 = \max(2|\alpha|\rho_1,\ 2|\beta|\rho_2)$. Since M is non-decreasing and convex, q is seminorm and Δ^m is linear we have

$$\sum_{k=1}^{\infty} k^{-s}\big[M\big(q(\frac{u_k\Delta^m(\alpha x_k + \beta y_k)}{\rho_3}))\big]^{p_k}$$

$$\leq \sum_{k=1}^{\infty} k^{-s}\big[M\big(q(\frac{u_k\Delta^m \alpha x_k}{\rho_3})) + q(\frac{u_k\Delta^m \beta y_k}{\rho_3})\big]^{p_k}$$

$$\leq \sum_{k=1}^{\infty} \frac{1}{2^{p_k}} k^{-s}\big[M\big(q(\frac{u_k\Delta^m x_k}{\rho_1})) + M\big(q(\frac{u_k\Delta^m y_k}{\rho_2}))\big]^{p_k} \tag{2}$$

$$\leq \sum_{k=1}^{\infty} k^{-s}\big[M\big(q(\frac{u_k\Delta^m x_k}{\rho_1})) + M\big(q(\frac{u_k\Delta^m y_k}{\rho_2}))\big]^{p_k}$$

$$\leq D\sum_{k=1}^{\infty} k^{-s}\big[M\big(q(\frac{u_k\Delta^m x_k}{\rho_1}))\big]^{p_k} + D\sum_{k=1}^{\infty} k^{-s}\big[M\big(q(\frac{u_k\Delta^m y_k}{\rho_2}))\big]^{p_k} < \infty$$

This proves that $\ell_M(\Delta^m, u, p, q, s)$ is linear.

Theorem 2.2. *The sequence space $\ell_M(\Delta^m, u, p, q, s)$ is paranormed (not necessarily totaly paranormed) with*

$$g_u(x) = \inf\Big\{\rho^{\frac{p_n}{H}} : \Big(\sum_{k=1}^{\infty} k^{-s}\big[M\big(q(\frac{\Delta^m u_k x_k}{\rho}))\big]^{p_k}\Big)^{\frac{1}{H}} \leq 1 \ \ n = 1, 2, 3..\Big\},$$

where $H = \max(1, \sup_k p_k)$.

Proof. Clearly $g_u(x) = g_u(-x))$.The subadditivity of g_u follows from equation(2) on taking $\alpha = 1$ and $\beta = 1$. Since $q(\theta) = 0$ and $M(0) = 0$, we get $\inf\left\{\rho^{\frac{p_n}{H}}\right\} = 0$ for $x = \theta$.

Finally, we prove that the scalar multiplication is continuous. Let λ be any number.

By definition,

$$g_u(\lambda x) = \inf\left\{\rho^{\frac{p_n}{H}} : \left(\sum_{k=1}^{\infty} k^{-s}\Big[M\Big(q(\frac{\Delta^m \lambda u_k x_k}{\rho})\Big)\Big]^{p_k}\right)^{\frac{1}{H}} \leq 1 \;\; n = 1, 2, 3..\right\}$$

Then

$$g_u(\lambda x) = \inf\left\{(\lambda r)^{\frac{p_n}{H}} : \left(\sum_{k=1}^{\infty} k^{-s}\Big[M\Big(q(\frac{\Delta^m u_k x_k}{r})\Big)\Big]^{p_k}\right)^{\frac{1}{H}} \leq 1 \;\; n = 1, 2, 3..\right\}$$

Where $r = \frac{\rho}{\lambda}$. Since $|\lambda|^{p_k} \leq \max(1, |\lambda|)^H$, then $|\lambda|^{\frac{p_k}{H}} \leq \Big(\max(1, |\lambda|^H)\Big)^{\frac{1}{H}}$

Hence

$$g_u(\lambda x) \leq \max(1, |\lambda|)^H \inf\left\{(r)^{\frac{p_n}{H}} : \left(\sum_{k=1}^{\infty} k^{-s}\Big[M\Big(q(\frac{\Delta^m u_k x_k}{r})\Big)\Big]^{p_k}\right)^{\frac{1}{H}} \leq 1 \;\; n = 1, 2, 3..\right\}$$

and therefore $g_u(\lambda x)$ converges to zero where $g_u(x)$ converges to zero in

$$\ell_M(\Delta^m, u, p, q, s).$$

Now suppose that $\lambda_n \to 0$ and $x \in \ell_M(\Delta^m, u, p, q, s)$. For arbitrary $\epsilon > 0$. Let $\mathbb{N}$ be a positive integer such that

$$\sum_{k=N+1}^{\infty} k^{-s}\Big[M\Big(q(\frac{\Delta^m u_k x_k}{\rho})\Big)\Big]^{p_k} < (\frac{\epsilon}{2})^H$$

for some $\rho > 0$. This implies that

$$\left(\sum_{k=N+1}^{\infty} k^{-s}\Big[M\Big(q(\frac{\Delta^m u_k x_k}{\rho})\Big)\Big]^{p_k}\right)^{\frac{1}{H}} \leq \frac{\epsilon}{2}$$

Let $0 < |\lambda| < 1$, then using **Remark 1.1** we get

$$\sum_{k=N+1}^{\infty} k^{-s}\Big[M\Big(q(\frac{\Delta^m \lambda u_k x_k}{\rho})\Big)\Big]^{p_k} < \sum_{k=N+1}^{\infty} k^{-s}\big[|\lambda| M\big(q(\frac{\Delta^m u_k x_k}{\rho})\big)\big]^{p_k} < (\frac{\epsilon}{2})^H.$$

Since M is continuous everywhere in $[0,\infty)$, then

$$f(t)=\sum_{k=1}^{N}k^{-s}\Big[M\big(q(\frac{\Delta^m t u_k x_k}{\rho}))\Big]^{p_k}$$

is continuous at 0. So there is $1>\delta>0$, such that $|f(t)|<\frac{\epsilon}{2}$ for $0<t<\delta$. Let K be such that $|\lambda_n|<\delta$ for $n>K$, then for $n>K$ we have

$$\Big(\sum_{k=1}^{\infty}k^{-s}\Big[M\big(q(\frac{\Delta^m\lambda_n u_k x_k}{\rho}))\Big]^{p_k}\Big)^{\frac{1}{H}}<\frac{\epsilon}{2}$$

Since $0<\epsilon<1$ we have,

$$\Big(\sum_{k=1}^{\infty}k^{-s}\Big[M\big(q(\frac{\Delta^m\lambda_n u_k x_k}{\rho}))\Big]^{p_k}\Big)^{\frac{1}{H}}<1 \quad \text{, for} \quad n>K$$

If we take limit on inf $\Big\{\rho^{\frac{p_n}{H}}\Big\}$ we get $g_u(\lambda x)\to 0$.

3. Some Particular Cases

We get the following sequence spaces from $\ell_M(\Delta^m,p,u,q,s)$ on giving particular values to p,q,s. Taking $p_k=1$ for all $k\in\mathbb{N}$

$$\ell_M(\Delta^m,u,q,s)=\Big\{x\in\omega(X):\sum_{k=1}^{\infty}k^{-s}\big[M\big(q(\frac{u_k\Delta^m x_k}{\rho}))\big]<\infty,\ s\geq 0\ \text{ for some } \rho>0\Big\}.$$

If s=0, then we have

$$\ell_M(\Delta^m,u,p,q)=\Big\{x\in\omega(X):\sum_{k=1}^{\infty}\big[M\big(q(\frac{u_k\Delta^m x_k}{\rho}))\big]^{p_k}<\infty,\ \text{ for some } \rho>0\Big\}.$$

If $p_k=1$ for all $k\in\mathbb{N}$, and $s=0$, then we have

$$\ell_M(\Delta^m,u,q)=\Big\{x\in\omega(X):\sum_{k=1}^{\infty}\big[M\big(q(\frac{u_k\Delta^m x_k}{\rho}))\big]\ <\infty,\ \text{ for some } \rho>0\Big\}.$$

If we take $s = 0$, $q(x) = |x|$ and $X = \mathbb{C}$,then we have

$$\ell_M(\Delta^m, u, p) = \Big\{x \in \omega(X) : \sum_{k=1}^{\infty}[M(\frac{|u_k\Delta^m x_k|}{\rho})]^{p_k} < \infty, \text{ for some } \rho > 0\Big\}.$$

In addition to the above sequence spaces, we have $\ell_M(\Delta^m, u, p, q, s) = \ell_M(p)$ due to Prashar and Choudhry [16], on taking $u_k = 1$ for all $k \in \mathbb{N}, m = 0, s = 0,$ and $q(x) = |x|$ and $X = \mathbb{C}$. If we take $u_k = 1$ for all $k \in \mathbb{N}$, and $m = 0$ we have $\ell_M(\Delta^m, p, u, q, s) = \ell_M(p, q, s)$, see [1].

Theorem 3.1. *Let $0 \le p_k \le t_k < \infty$ for each $k \in \mathbb{N}$. Then:*

(i) $\ell_M(\Delta^m, p, u, q) \subseteq \ell_M(\Delta^m, t, u, q)$

(ii) $\ell_M(\Delta^m, u, q) \subseteq \ell_M(\Delta^m, u, q, s)$

(iii) $\ell_M(\Delta^m, p, u, q) \subseteq \ell_M(\Delta^m, p, u, q, s)$

Proof. Let $x \in \ell_M(\Delta^m, p, u, q, s)$. Then there exist some $\rho > 0$ such that

$$\sum_{k=1}^{\infty}\big[M\big(q(\frac{u_k\Delta^m x_k}{\rho})\big)\big]^{p_k} < \infty.$$

This implies that $\big[M\big(q(\frac{u_i\Delta^m x_i}{\rho})\big)\big] \le 1$ for sufficiently large values of i, says that $i \ge k_0$ for some fixed $k_o \in \mathbb{N}$. Since M is non-decreasing, we get

$$\sum_{k=1}^{\infty}\big[M\big(q(\frac{u_k\Delta^m x_k}{\rho})\big)\big]^{t_k} < \infty,$$

since

$$\sum_{k\ge k_0}^{\infty}\big[M(q(\frac{u_k\Delta^m x_k}{\rho}))\big]^{t_k} \le \sum_{k\ge k_0}^{\infty}\big[M(q(\frac{u_k\Delta^m x_k}{\rho}))\big]^{p_k} < \infty.$$

Hence $x \in \ell_M(\Delta^m, u, t, q)$.

The proof of *(ii)* and *(iii)* is trivial.

Theorem 3.2. *Let $0 < p_k \le t_k < \infty$ for each k. Then $\ell_M(\Delta^m, u, p) \subseteq \ell_M(\Delta^m, u, t)$.*

Proof. Proof can be given by the same way as Theorem 3.1(i)

Theorem 3.3.

(i) If $0 < p_k \le 1$ for all $k \in \mathbb{N}$, then $\ell_M(\Delta^m, u, p, q) \subseteq \ell_M(\Delta^m, u, q)$

(*ii*) *If* $p_k \geq 1$ *for all* $k \in \mathbb{N}$, *then* $\ell_M(\Delta^m, u, q) \subseteq \ell_M(\Delta^m, u, p, q)$

Proof. (*i*) If we take $t_k = 1$ for all $k \in \mathbb{N}$, in Theorem 3.1(i), then $\ell_M(\Delta^m, u, p, q) \subseteq \ell_M(\Delta^m, u, q)$

(*ii*) f we take $p_k = 1$ for all $k \in \mathbb{N}$, in Theorem 3.1(i), then $\ell_M(\Delta^m, u, q) \subseteq \ell_M(\Delta^m, u, p, q)$

Proposition 3.4. *For any two sequence* $p = (p_k)$ *and* $t = (t_k)$ *of positive real numbers and any two seminorms* q_1 *and* q_2 *we have* $\ell_M(\Delta^m, u, p, q_1, r) \cap \ell_M(\Delta^m, u, t, q_2, s) \neq \emptyset$ *for all* $m, n \in \mathbb{N}$ $r, s > 0$.

Proof. Since the Zero element belongs to $\ell_M(\Delta^m, u, p, q_1, r)$ and $\ell_M(\Delta^m, u, t, q_2, s)$, thus the intersection is nonempty.

Theorem 3.5. *The sequence space* $\ell_M(\Delta^m, u, p, q, s)$ *is solid.*

Proof. Let $(x_k) \in \ell_M(\Delta^m, u, p, q, s)$, i.e,

$$\sum_{k=1}^{\infty} k^{-s}\big[M\big(q(\frac{\Delta^m u_k x_k}{\rho})\big)\big]^{p_k} < \infty.$$

Let(α_k) be sequence of scalar such that $|\alpha_k| \leq 1$, for all $k \in \mathbb{N}$. Then we have

$$\sum_{k=1}^{\infty} k^{-s}\big[M\big(q(\frac{\Delta^m \alpha_k u_k x_k}{\rho})\big)\big]^{p_k} \leq \sum_{k=1}^{\infty} k^{-s}\big[M\big(q(\frac{\Delta^m u_k x_k}{\rho})\big)\big]^{p_k} < \infty.$$

Hence $(\alpha_k x_k) \in \ell_M(\Delta^m, u, p, q, s)$ for all sequences of scalar (α_k) with $|\alpha_k| \leq 1$ for all $k \in \mathbb{N}$, whenever $(x_k) \in \ell_M(\Delta^m, u, p, q, s)$. Therefore the space $\ell_M(\Delta^m, u, p, q, s)$ is solid sequence space.

Corollary 3.6.

(*i*) *Let* $|u_k| \leq 1$ *for all* $k \in \mathbb{N}$. *Then* $\ell_M(\Delta^m, p, q, s) \subseteq \ell_M(\Delta^m, u, p, q, s)$

(*ii*) *Let* $|u_k| \geq 1$ *for all* $k \in \mathbb{N}$. *Then* $\ell_M(\Delta^m, u, p, q, s) \subseteq \ell_M(\Delta^m, p, q, s)$

Proof. Proof is trivial.

Theorem 3.7. *Let* M, M_1*and* M_2 *be an orlicz function and* s, s_1*and* s_2 *be non negative real numbers. Then we have:*

(*i*) *Let* $\ell_{M_1}(\Delta^m, u, p, q, s) \cap \ell_{M_2}(\Delta^m, u, p, q, s) \subseteq \ell_{M_1+M_2}(\Delta^m, u, p, q, s)$

(*ii*) *If* $s_1 \leq s_2$, *then* $\ell_M(\Delta^m, u, p, q, s_1) \subseteq \ell_M(\Delta^m, u, p, q, s_2)$

Proof.(i) From (1) we have

$$k^{-s}[(M_1+M_2)(q(\frac{\Delta^m u_k x_k}{\rho}))]^{p_k}$$

$$= k^{-s}[M_1(q(\frac{\Delta^m u_k x_k}{\rho})) + M_2(q(\frac{\Delta^m u_k x_k}{\rho}))]^{p_k}$$

$$\leq Dk^{-s}[M_1(q(\frac{\Delta^m u_k x_k}{\rho}))]^{p_k} + Dk^{-s}[M_2(q(\frac{\Delta^m u_k x_k}{\rho}))]^{p_k}$$

Let $x \in \ell_{M_1}(\Delta^m, u, p, q, s) \cap \ell_{M_2}(\Delta^m, u, p, q, s)$, when adding the above inequality from $k = 1$ to ∞, we get $x \in \ell_{M_1+M_2}(\Delta^m, u, p, q, s)$.

(ii) Let $s_1 \leq s_2$ and $x \in \ell_M(\Delta^m, u, p, q, s_1)$. Since $k^{-s_2} \leq k^{-s_1}$, we have $x \in \ell_M(\Delta^m, u, p, q, s_2)$

This completes the proof.

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