CONTROL DESIGN APPLIED TO A NON-IDEAL STRUCTURAL SYSTEM WITH BEHAVIOR CHAOTIC

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Abstract: In this paper we study the behavior of a structure vulnerable to excessive vibrations caused by an non-ideal power source. To perform this study, the mathematical model is proposed, derive the equations of motion for a simple plane frame excited by an unbalanced rotating machine with limited power (non-ideal motor). The non-linear and non-ideal dynamics in system is demonstrated with a chaotic behavior. We use a State-Dependent Riccati Equation Control technique for regulate the chaotic behavior, in order to obtain a periodic orbit small and to decrease its amplitude. The simulation results show the identification by State-Dependent Riccati Equation Control is very effective.

AMS Subject Classification: 34H10, 49N05

Key Words: state-dependent Riccati equation control, chaos control, non-ideal system

1. Introduction

Most of the phenomena that occur have dynamic character, i.e., time-varying.

The representation of all these dynamic phenomena in the field of structural engineering is a big challenge. The great advancement of technology, large areas have been growing and improving, as the materials and construction techniques. What has made it possible to develop buildings, more slender and flexible.

In this paper, we approach the behavior of a structure vulnerable (linear dynamic model) the excessive excitation caused by a dynamic loading. This dynamic loading is a unbalanced rotating machine with limited power (non-ideal motor), which characterizes the non-ideal system.

A system is said ideal when the excitation does not influence the response of the system, i.e., the ideal excitation has an ideal power source. On the other hand, when the excitation is influenced by the response of the system, the excitation is said to be non-ideal

The first characteristic of system non-ideal is that it presents an equation that describes the interaction of the energy source with the ideal dynamic system and the second characteristic, is observed that the dynamics of a non-ideal system approaches of the ideal case as the supplied potency becomes sufficiently big and vice-versa. Several contributions to the study of non-ideal problems were presented in the current literature, [1]-[5] and recently [6]-[9].

Among the various ways to minimize vibrations and reduce the oscillatory motion caused by non-ideal power source in the structure is discussed [13] in this paper the application of the structural control.

One technique that has shown considerable promitient in simulations involves the approximate solutions of a nonlinear, state-dependent Riccati equation for the dynamical system in discrete time steps to calculate a feedback control law that is optimized around the system state-tracking control, in which the cost function to be minimized is quadratic in the difference between the actual or estimated state and a commanded state trajectory. This technique is called State-Dependent Riccati Equation (SDRE) control [10, 11] techniques can be applied to solve a wide range of problems [10, 12].

The aim of this paper is to propose the application of the State-Dependent Riccati Equation Control [18] to control the chaotic movement of a non-ideal structural system.

We organize the paper as it follows: in Section 2, we demonstrated the mathematical model and we showed of the non-ideal dynamics of the model. In Section 3, we discussed a control design problem for non-ideal model. In Section 4, we made the concluding remarks of this paper. Finally, we listed out the bibliographic references which we used in this paper.

2. Non-Ideal Structural System

An introduction to mathematical problem that is proposed, derive the equations of motion for a simple plane frame under excitation in vertical direction, as shown in Figure 1. where $x_1 = a + S + q_1$ and $x_2 = b + S + q_2$, where S is the

Figure 1: Structural System

non-ideal excitation.

The total kinetic energy (T) of the system is:

$$T = 1/2[m_1(\dot{S} + \dot{q}_1)^2 + m_2(\dot{S} + \dot{q}_2)^2] \quad (1)$$

The total potential energy (V) of the system is:

$$V = 1/2[k_1 q_1^2 + k_2 (q_2 - q_1)^2] \quad (2)$$

The Lagrangian (L=T-V) is:

$$L = 1/2[m_1(\dot{S} + \dot{q}_1)^2 + m_2(\dot{S} + \dot{q}_2)^2 - k_1 q_1^2 - k_2 (q_2 - q_1)^2] \quad (3)$$

The Lagrange equation for the generalized coordinate q_1 is:

$$m_1(\ddot{q}_1 + \ddot{S}) + k_1 q_1 - k_2 (q_2 - q_1) = -c_1 \dot{q}_1 \quad (4)$$

The Lagrange equation for the generalized coordinate q_2 is:

$$m_2(\ddot{q}_2 + \ddot{S}) + k_2 (q_2 - q_1) + c_2 (q_2 - q_1) = -c_1 \dot{q}_1 \quad (5)$$

Thus, the system can be modeled by the equations:

$$m_1\ddot{q}_1 + k_1q_1 - k_2(q_2 - q_1) + c_1\dot{q}_1 = m_1\ddot{S} \quad (6)$$

$$m_2\ddot{q}_2 + k_2(q_2 - q_1) + c_2(\dot{q}_2 - \dot{q}_1) = m_2\ddot{S} \quad (7)$$

$$\ddot{q}_1 + \frac{k_1}{m_1}q_1 - \frac{k_2}{m_1}(q_2 - q_1) + \frac{c_1}{m_1}\dot{q}_1 = -\ddot{S} \quad (8)$$

$$\ddot{q}_2 + \frac{k_2}{m_2}(q_2 - q_1) + \frac{c_2}{m_2}(\dot{q}_2 - \dot{q}_1) = -\ddot{S} \quad (9)$$

being $\omega_1^2 = \frac{k_1+k_2}{m_1}; \omega_2^2 = \frac{k_2}{m_2}$ has

$$\ddot{q}_1 + \omega_1^2q_1 - \frac{k_2}{m_1}q_2 + \frac{c_1}{m_1}\dot{q}_1 = -\ddot{S} \quad (10)$$

$$\ddot{q}_2 + \omega_2^2(q_2 - q_1) + \frac{c_2}{m_2}(\dot{q}_2 - \dot{q}_1) = -\ddot{S} \quad (11)$$

where S is the non-ideal excitation (unbalanced rotating machine with limited power)

$$\ddot{z} + b\dot{z} = r\ddot{q}_1 \sin z + a \quad (12)$$

$$S = p(\dot{z}^2 \cos z + \ddot{z} \sin z) \quad (13)$$

Where z is non-ideal excitation response, a , b , p and r are dimensionless constant positives. The term S is due interaction between the dynamical system and an energy source, for example, a DC motor with limited power supply. S is responsible to the Non-idealization of the system. Note also that in the equation (12) we may observe an interaction term. The parameter a is a constant depending on initial conditions and b is an internal damping in Dc Motor.

Making $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$, $x_4 = \dot{q}_2$, $x_5 = z$ and $x_6 = \dot{z}$, has

$$\dot{x}_1 = x_2 \quad (14)$$

$$\dot{x}_2 = -\omega_1^2x_1 + \frac{k_2}{m_1}x_3 - \frac{c_1}{m_1}x_2 - (p(x_6^2 \cos x_5 + x_6 \sin x_5)) \quad (15)$$

$$\dot{x}_3 = x_4 \quad (16)$$

$$\dot{x}_4 = \omega_2^2(x_1 - x_3) + \frac{c_2}{m_2}(x_2 - x_4) - (p(x_6^2 \cos x_5 + x_6 \sin x_5)) \quad (17)$$

$$\dot{x}_5 = x_6 \quad (18)$$

$$\dot{x}_6 = -bx_6^2 + rx_2 \sin x_5 + a \quad (19)$$

Figure 2: (a) Dynamical behavior of the time history (x_1), (b) FFT for (x_1), (c) Time history (x_2), (d) FFT (x_2), (e) Time history (x_3), (f) FFT (x_3), (g) Time history (x_4), and (h) FFT (x_4).

The Figure 2 illustrate the dynamics behavior of the adopted dynamics model, by using numerical values, for the chosen parameters $\omega_1 = 1$, $m_1 = 7$, $c_1 = 0.6$, $k_2 = 0.8$, $\omega_2 = 2$, $c_2 = 0.7$, $m_2 = 7$ to the structural system and $a = 2.3$, $b = 2.0$, $r = 0.5$ and $p = 0.5$ to the non-ideal excitation.

The Figure 3 show the portrait phase for the chaotic behavior.

Figure 3: Phase Portrait. (a) x_1 and x_2 projections and (b) x_3 and x_4 projections

Figure 4 illustrates the four Lyapunov exponents, $\lambda_1 = 0.077975$; $\lambda_2 = -0.039748$; $\lambda_3 = -0.047754$; $\lambda_4 = -0.052616$; $\lambda_5 = -0.0393 - 4$ and $\lambda_6 = -0.11427$, demonstrating the presence of the chaos with the one Lyapunov exponent positive. The total time for Lyapunov exponents computation was the $\Delta\tau = 100,000$ with time-step of $\tau = 0.5$.

Figure 4: Dynamics of Lyapunov exponents for system (a) Exponents (b) Positive Exponent

Aiming to minimize vibrations and reduce the oscillatory motion caused in the system in the following section proposes the application of State-Dependent Riccati Equation (SDRE) to reduce this chaotic motion, see Figure 4, to a small

stable orbit.

3. Control Design

An alternative to minimize vibrations unstable presented is the structural control. The structural control, basically promotes changes in stiffness and damping of the structure, either by adding external devices, either by the action of external forces. We can adopt various control models, such as the passive control, active control, hybrid control, and semi-active control. In this work we adopt the State-Dependent Riccati Equation Control proposed by Pearson (1962) and later expanded by Wernli and Cook [14], was independently studied by Mracek and Cloutier [15] and alluded to by Friedland [16]. Use of the State-Dependent Riccati Equation (SDRE) to control nonlinear system has seen increasing application [17].

$$J = \frac{1}{2} \int_{t_0}^{\infty} x^T Q(x) x + u^T R(x) u dt \quad (20)$$

with respect to the state x and control u subject to the nonlinear differential constraint

$$\dot{x} = f(x) + g(x)u \quad (21)$$

where $x \in R^n$, $y \in R^n$, $f(x) \in C^k$, $g(x) \in C^k$, $Q(x) \in C^k$, $R(x) \in C^k$, $k \geq 1$ and where $Q(x) = C^T(x)C(x) \geq 0$, and $R(x) > 0$ for all x . Here it is assumed that $f(0) = 0$ and $g(x) \neq 0$ for all x . It may also be desirable to select $Q(x)$ and $R(x)$ such that the performance index $J(x, u)$ in (20) is globally convex. We seek stabilizing approximate solutions of problem (20)-(21) of the form $u = \phi(x)$ where ϕ is a nonlinear function of x .

3.1. State-Dependent Coefficient Form

It is well-know that the nonlinear dynamics (22) can be represented by the following linear structure having state-dependent coefficients:

$$\dot{x} = A(x)x + B(x)u \quad (22)$$

where

$$f(x) = A(x)x$$

$$B(x) = g(x) \quad (23)$$

It is also known [7] that, in the multivariable case, there are an infinite number of ways to bring the nonlinear system to SDC form. Associated with the SDC form, we have the following definitions.

$A(x)$ is an observable (detectable) parameterization of the nonlinear system (in region Ω) if the pair $\{C(x), A(x)\}$, is pointwise observable (detectable) in the linear sense for all $x \in \Omega$.

$A(x)$ is a controllable (stabilizable) parameterization of the nonlinear system (in a region Ω) if the pair $\{B(x), A(x)\}$ is pointwise controllable (stabilizable) in the linear sense for all $x \in \Omega$.

3.2. Application of the Control Design Theory to a Non-Idel Model

The non-linear state model (21) can be written in a linear state dependent coefficient (SDC) form of (24), where the vector $x = [x_1 x_2 x_3 x_4 x_5 x_6]^T$ represents the system states the time dependent, $\dot{x} \in \mathbb{R}^2$ is the vector of first derivatives of the state, $x \in \mathbb{R}^1$ is the control function. U is the force applied to control. Considering the initial and end conditions as: $x(t_0) = x_0, x(\infty) = 0$ The state dependent coefficients are given by:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & -0.0857 & 0.1142 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 4 & 0.1 & -4 & -0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0.4207 & 0 & 0 & 0.0027 & -4 \end{bmatrix}$$

$$B(x) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$Q(x) = S^T(x)S(x) \quad (24)$$

$$S(x) = \text{diag} \{ \sqrt{q_i} \}_{i=1, \dots, 6} \quad (25)$$

Rewritten (22),

$$\dot{x} = A(x), x + \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} u \quad (26)$$

being x a vector.

The state x and u the control are given for function $f(x) = A(x), b(x) = B(x)$ e $d(x) = S(x)x$ [18].

Considering the functional to be minimized is

$$J = \frac{1}{2} \int_{t_0}^{\infty} x^T Q(x)x + u^T R(x)u dt \quad (27)$$

where

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and $R=[1]$.

Construct the nonlinear feedback controller via

$$u = -R^{-1}(x)B^T(x)P(x)x \quad (28)$$

Solve the state-dependent Riccati equation (SDRE)

$$A^T(x)P + PA(x) - PB(x)R^{-1}(x)B^T(x)P + Q(x) = 0 \quad (29)$$

Mracek and Cloutier [15], in the region Ω about the origin of the SDRE method provides a solution in an asymptotically stable limit cycle. In the case of a scalar solution, the method achieves an optimal solution to functional (28), even when Q and R are functions of x .

The SDRE control meet the first and second necessary condition of optimism, $H_u = 0$ (H is the Hamiltonian of the problem (24)-(28)). The Hamiltonian for the control is:

$$H(x, u, \lambda) = \frac{1}{2}(x^T Qx + u^T Ru) + \lambda^T (A(x)x + B(x)u) \quad (30)$$

The conditions necessary for the control are [18]:

$$\begin{aligned}\dot{\lambda} &= -Qx - \frac{1}{2} \left[\frac{\partial A(x)x}{\partial x} \right]^T \lambda - \left[\frac{\partial (B(x)u)}{\partial x} \right]^T \lambda, \\ \dot{x} &= A(x)x + B(x)u, \\ 0 &= Ru + B(x)\lambda.\end{aligned}\tag{31}$$

Here

$$u = -R^{-1}(x)B^T(x)\lambda\tag{32}$$

Assuming a co-state, it is known that:

$$\lambda = P(x)x\tag{33}$$

Substituting (33) in (34) are obtained by SDRE control (29).

The trajectories of the system with control may be seen, through Figures 5-14.

4. Conclusions

In this work, a dynamics of the Non-ideal Structural System with behavior chaotic is proposed and investigated.

We proposed the use of an SDRE control strategy to reduce the chaotic movement of this system to a small stable orbit.

The Figures 5 and 6 illustrated the effectiveness of the control strategy taken to these problems.

Figure 5: Dynamical behavior of the time history (a) x_1 , (b) x_2 , (c) x_3 , (d) x_4 , (e) x_5 and (f) x_6 .

Figure 6: Control Phase Portrait. (a) x_1 and x_2 projections, (b) x_3 and x_4 projections and (c) x_5 and x_6 projections.

Acknowledgments

The author thanks Conselho Nacional de Pesquisas (CNPq) for a financial supports (Proc.n. 301769/2012-5).

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