NEW ROBUST EXPONENTIAL STABILITY CRITERIA FOR NEUTRAL SYSTEM WITH MIXED CONSTANT DELAYS

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Abstract: In this paper, we propose new sufficient conditions guaranteeing exponential stability of linear neutral systems with mixed constant delays. With a change of variable and bounded techniques along with constructed Lyapunov-Krasovskii functionals, the sufficient conditions are formulated without adding free matrices used. Three numerical examples are given to show effectiveness of the proposed criteria by comparing upper bounds of the time-delays and rate of convergence with some recent works.

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1. Introduction

Neutral system is one of the important system and has received much attention to researchers since it can be used to describe practical applications in sciences

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and engineering such as aircraft stabilization, lossless transmission lines, population ecology, chemical processes, etc [1], [2]. Unlike other systems, the neutral has time-delay in both the state and derivative called discrete and neutral delays, respectively. It is well-known that time-delay in the system often be a source of instability or poor system performance. Thus researchers have investigated such systems with time-delay by proposing sufficient criteria for the systems to be stable (see for examples [1], [2], [3], [4], [5], [8], [9], [12], [13], [10], [14], [15], [16], [17] and the reference therein).

In practice, systems may be disturbed by a small amount of perturbation in time. Thus systems with some uncertainties are common modeled for practical uses. Thus the problem of robust stability analysis has been widely investigated [1], [2], [3], [4], [8], [9], [10], [11], [12], [14], [16].

In the past decade, researchers have introduced new sufficient conditions for the system with time-delay. Conditions developed in the past can be classified into two types; delay-dependent and delay-independent. The former condition can be considered as more appropriate since it provides stability to the system related to the delay's size. Therefore, an increasing attention has been focused on the delay-dependent condition which, in general, can be considered as less conservative than the delay-independent ones. In addition, many applications require fast speed of convergence, thus many researches paid more attention on exponential stability which offers faster speed of convergence than asymptotic stability [2], [4], [9].

To obtain stability criteria of such system, several techniques such as descriptor transformation, triple-integral term in Lyapunov-Krasovskii functional, matrix decomposition, or free-weighting matrices have been used in many works. However, these techniques lead to larger size of linear matrix inequality. In this paper, we use rather typical techniques base on Lyapunov-Krasovskii functionals, along with a change of variable, the Leibniz-Newton formula, and integral inequalities to guarantee (robust) exponential stability of the neutral systems with mixed constant delays. The proposed criteria are demonstrated their effectiveness in three numerical examples by comparing upper bounds of the time-delay and the rate of convergence with results in recent works.

2. Preliminaries

Throughout this article, $*$ represents the elements below the main diagonal of a symmetric matrix. The notation $A > 0$ means A is positive definite; A^T denotes the transpose of the matrix A . $\lambda(A)$ are eigenvalues of A ; $\lambda_{max}(A) :=$

$\max\{Re(\lambda) : \lambda \in \lambda(A)\}$. $x_t := \{x(t+s) : s \in [-\tau, 0]\}$; $\|x_t\| := \sup_{s \in [-\tau, 0]} \|x(t+s)\|$ and $\|\cdot\|$ denotes the Euclidean norm of given vector or matrix. $R^{n \times n}$ denotes real value matrix with dimension $n \times n$.

Consider the neutral system

$$\dot{x}(t) = (\bar{A} + \Delta A(t))x(t) + (\bar{B} + \Delta B(t))x(t - \tau) + \bar{C}\dot{x}(t - h), \quad (1)$$

where $t > 0, x(t) \in R^n$ is the state vector of the system. $\bar{A}, \bar{B}, \bar{C} \in R^{n \times n}$ are known constant matrices with $\|\bar{C}\| < 1$. The state and neutral time-delays, τ and h , are positive constants and $\mu = \max\{\tau, h\}$. We assume the initial condition is $x(t + \theta) = \phi(\theta)$, $\forall \theta \in [-\mu, 0]$ and $\phi(\cdot)$ is a differentiable vector-valued initial function.

The time-varying structured uncertainties $\Delta A(t)$ and $\Delta B(t)$ are of the form

$$[\Delta A(t), \Delta B(t)] = GF(t)[E_1, E_2], \quad (2)$$

where G, E_1, E_2 are known matrices with appropriated dimensions, and the time-varying matrix $F(t)$ satisfies $F^T(t)F(t) \leq I, t \geq 0$.

We next reintroduce the well-known definition and lemmas used in the main result.

Definition 1. System (1) is said to be exponentially stable if there exist scalars $K > 0$ and the rate of convergence $\alpha > 0$ such that

$$\|x(t, \phi)\| \leq Ke^{-\alpha t} \|\phi\|, \forall t > 0.$$

Lemma 2. (see [7]) Consider the time-delay system

$$\dot{x}(t) = f(t, x_t), x(t) = \phi(t), t \in [-h, 0].$$

If there exists a Lyapunov function $V(t, x_t)$ and $\lambda_1, \lambda_2 > 0$ such that for every solution $x(t)$ of the system, the following conditions hold:

- (i) $\lambda_1 \|x(t)\|^2 \leq V(t, x_t) \leq \lambda_2 \|x_t\|^2$,
- (ii) $\dot{V}(t, x_t) \leq 0$,

then the solution of the system is bounded; i.e., there exists $N > 0$ such that $\|x(t, \phi)\| \leq N \|\phi\|, \forall t \geq 0$.

Lemma 3. (see [11]) Let $a(\gamma) \in R^n$ and $b(\gamma) \in R^n$ and $\gamma \in \Omega$. Then, for any symmetric positive definite matrix $X \in R^{n \times n}$ and any matrix $M \in R^{n \times n}$, the following holds:

$$-2 \int_{\Omega} b^T(\gamma) a(\gamma) d\gamma \leq \int_{\Omega} \begin{bmatrix} a(\gamma) \\ b(\gamma) \end{bmatrix}^T W \begin{bmatrix} a(\gamma) \\ b(\gamma) \end{bmatrix} d\gamma,$$

where

$$W = \begin{bmatrix} X & XM \\ M^T X & (XM + I)^T X^{-1} (XM + I) \end{bmatrix}.$$

Lemma 4. (see [6]) For any symmetric positive definite matrix $M \in \mathbb{R}^{n \times n}$, scalar $\sigma \geq 0$ and vector function $w : [0, \sigma] \rightarrow \mathbb{R}^n$ such that the integrations concerned are well defined, then

$$\left(\int_0^\sigma w(s) ds \right)^T M \left(\int_0^\sigma w(s) ds \right) \leq \sigma \int_0^\sigma w^T(s) M w(s) ds.$$

3. Main Results

Theorem 5. For given scalars $\alpha, \varepsilon > 0$, time-delays $\tau, h \geq 0$, the neutral system (1) is robustly exponentially stable with convergent rate α if there exist symmetric and positive-definite matrices $P, Q_1, Q_2, Q_3, N, W, R$ such that the following inequality holds:

$$\Pi = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} & \tau(W + P)^T \\ * & Z_{22} & Z_{23} & Z_{24} & Z_{25} & 0 \\ * & * & Z_{33} & Z_{34} & Z_{35} & 0 \\ * & * & * & Z_{44} & Z_{45} & 0 \\ * & * & * & * & Z_{55} & 0 \\ * & * & * & * & * & -N \end{bmatrix} < 0, \quad (3)$$

where

$$\begin{aligned} Z_{11} &= P(A + B) + (A + B)^T P + WB + B^T W + Q_1 + Q_2 \\ &\quad + (BA)^T N B A + A^T Q_3 A + (\tau^2 + h^2) A^T R A - 2R + \varepsilon E_1^T E_1, \\ Z_{12} &= (BA)^T N B B + A^T Q_3 B - WB + (\tau^2 + h^2) A^T R B + R + \varepsilon e^{\alpha\tau} E_1^T E_2 \\ Z_{13} &= (BA)^T N B C + A^T Q_3 C + PC + (\tau^2 + h^2) A^T R C, \\ Z_{14} &= - (BA)^T N B D - A^T Q_3 D - PD - (\tau^2 + h^2) A^T R D + R, \\ Z_{15} &= (BA)^T N B G + A^T Q_3 G + PG + (\tau^2 + h^2) A^T R G, \\ Z_{22} &= (BB)^T N B B + B^T Q_3 B + (\tau^2 + h^2) B^T R B - Q_1 - R - \varepsilon e^{2\alpha\tau} E_2^T E_2 \\ Z_{23} &= (BB)^T N B C + B^T Q_3 C + (\tau^2 + h^2) B^T R C \\ Z_{24} &= - (BB)^T N B D - B^T Q_3 D - (\tau^2 + h^2) B^T R D \end{aligned}$$

$$\begin{aligned}
Z_{25} &= (BB)^T N B G + B^T Q_3 G + (\tau^2 + h^2) B^T R G \\
Z_{33} &= (BC)^T N B C + C^T Q_3 C + (\tau^2 + h^2) C^T R C - Q_3 \\
Z_{34} &= - (BC)^T N B D - C^T Q_3 D - (\tau^2 + h^2) C^T R D \\
Z_{35} &= (BC)^T N B G + C^T Q_3 G + (\tau^2 + h^2) C^T R G \\
Z_{44} &= (BD)^T N B D + D^T Q_3 D + (\tau^2 + h^2) D^T R D - Q_2 - R \\
Z_{45} &= - (BD)^T N B G - D^T Q_3 G - (\tau^2 + h^2) D^T R G \\
Z_{55} &= (BG)^T N B G + G^T Q_3 G + (\tau^2 + h^2) G^T R G - \varepsilon.
\end{aligned}$$

Proof. We begin by changing variable

$$z(t) = e^{\alpha t} x(t), \alpha > 0. \quad (4)$$

With the form of uncertainties in equation (2), the equation (1) becomes

$$\dot{z}(t) = A z(t) + B z(t - \tau) + C \dot{z}(t - h) - D z(t - h) + G y(t), \quad (5)$$

where $A = \bar{A} + \alpha I$, $B = e^{\alpha \tau} \bar{B}$, $C = e^{\alpha h} \bar{C}$, $D = \alpha e^{\alpha h} \bar{C}$ and $y(t) = F(t) [E_1 z(t) + e^{\alpha \tau} E_2 z(t - \tau)]$.

Next consider a Lyapunov-Krasovskii functional candidate,
 $V(t, z_t) = V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7$ where

$$\begin{aligned}
V_1(z_t) &= z^T(t) P z(t), \quad V_2(z_t) = \int_{-\tau}^0 \int_{t+\beta}^t \dot{z}^T(s) B^T X B \dot{z}(s) ds d\beta, \\
V_3(z_t) &= \int_{t-\tau}^t z^T(s) Q_1 z(s) ds, \quad V_4(z_t) = \int_{t-h}^t z^T(s) Q_2 z(s) ds, \\
V_5(z_t) &= \int_{t-h}^t \dot{z}^T(s) Q_3 \dot{z}(s) ds, \quad V_6(z_t) = \tau \int_{-\tau}^0 \int_{t+\beta}^t \dot{z}^T(s) R \dot{z}(s) ds d\beta, \\
V_7(z_t) &= h \int_{-h}^0 \int_{t+\beta}^t \dot{z}^T(s) R \dot{z}(s) ds d\beta.
\end{aligned}$$

The derivatives of $V_i, i = 1, 2, \dots, 7$ along the trajectory (5) are

$$\dot{V}_1 = 2z^T(t) P \dot{z}(t)$$

Utilizing the Leibniz-Newton formula, $x(t) - x(t - \tau) = \int_{t-\tau}^t \dot{x}(s) ds$, equation (5) becomes

$$\dot{z}(t) = (A + B) z(t) - B \int_{t-\tau}^t \dot{z}(s) ds + C \dot{z}(t - h) - D z(t - h) + G y(t).$$

Then we have

$$\dot{V}_1 = 2z^T(t)P[(A+B)z(t) - B \int_{t-\tau}^t \dot{z}(s)ds + C\dot{z}(t-h) - Dz(t-h) + Gy(t)]$$

Apply Lemma 3, with $a(\gamma) = B\dot{z}(s)$, $b(\gamma) = Pz(t)$ and the Leibniz-Newton formula, we obtain

$$\begin{aligned} -2 \int_{t-\tau}^t z^T(t)PB\dot{z}(s)ds &\leq \int_{t-\tau}^t \dot{z}^T(s)B^TXB\dot{z}(s)ds + 2z^T(t)PM^TXB \\ &\quad * \int_{t-\tau}^t \dot{z}(s)ds + \tau z^T(t)P(M^TX + I)X^{-1}(M^TX + I)Pz(t). \end{aligned}$$

Let $W = PM^TX$, $N = \tau X$. We then obtain

$$\begin{aligned} \dot{V}_1 &\leq z^T(t) \left[2P(A+B)z(t) + 2WBz(t) - 2WBz(t-\tau) \right. \\ &\quad + \tau^2(W+P)N^{-1}(W+P)^Tz(t) + 2PC\dot{z}(t-h) \\ &\quad \left. - 2PDz(t-h) + 2PGy(t) \right] + \int_{t-\tau}^t \dot{z}^T(s)B^TXB\dot{z}(s)ds \end{aligned} \quad (6)$$

$$\begin{aligned} \dot{V}_2 &= \tau \dot{z}^T(t)B^TXB\dot{z}(t) - \int_{t-\tau}^t \dot{z}^T(s)B^TXB\dot{z}(s)ds \\ &= \tau \left[Az(t) + Bz(t-\tau) + C\dot{z}(t-h) - Dz(t-h) + Gy(t) \right]^T \\ &\quad * B^TXB \left[Az(t) + Bz(t-\tau) + C\dot{z}(t-h) - Dz(t-h) + Gy(t) \right] \\ &\quad - \int_{t-\tau}^t \dot{z}^T(s)B^TXB\dot{z}(s)ds \end{aligned} \quad (7)$$

$$\dot{V}_3 = z^T(t)Q_1z(t) - z^T(t-\tau)Q_1z(t-\tau) \quad (8)$$

$$\dot{V}_4 = z^T(t)Q_2z(t) - z^T(t-h)Q_2z(t-h) \quad (9)$$

$$\begin{aligned} \dot{V}_5 &= \dot{z}^T(t)Q_3\dot{z}(t) - \dot{z}^T(t-h)Q_3\dot{z}(t-h) \\ &= \left[Az(t) + Bz(t-\tau) + C\dot{z}(t-h) - Dz(t-h) + Gy(t) \right]^T Q_3 \\ &\quad * \left[Az(t) + Bz(t-\tau) + C\dot{z}(t-h) - Dz(t-h) + Gy(t) \right] \\ &\quad - \dot{z}^T(t-h)Q_3\dot{z}(t-h) \end{aligned} \quad (10)$$

$$\dot{V}_6 = \tau^2 \dot{z}^T(t)R\dot{z}(t) - \tau \int_{t-\tau}^t \dot{z}^T(s)R\dot{z}(s)ds$$

$$\dot{V}_7 = h^2 \dot{z}^T(t) R \dot{z}(t) - h \int_{t-h}^t \dot{z}^T(s) R \dot{z}(s) ds$$

Apply Lemma 4 and the Newton-Leibniz formula to $\dot{V}_6$ and $\dot{V}_7$, we have

$$\begin{aligned} \dot{V}_6 &\leq \tau^2 \dot{z}^T(t) R \dot{z}(t) - z^T(t) R z(t) + 2z^T(t) R z(t - \tau) - z^T(t - \tau) R z(t - \tau) \\ &\leq \tau^2 [Az(t) + Bz(t - \tau) + C\dot{z}(t - h) - Dz(t - h) + Gy(t)]^T \\ &\quad * R [Az(t) + Bz(t - \tau) + C\dot{z}(t - h) - Dz(t - h) + Gy(t)] \\ &\quad - z^T(t) R z(t) + 2z^T(t) R z(t - \tau) - z^T(t - \tau) R z(t - \tau) \end{aligned} \quad (11)$$

$$\begin{aligned} \dot{V}_7 &\leq h^2 [Az(t) + Bz(t - \tau) + C\dot{z}(t - h) - Dz(t - h) + Gy(t)]^T \\ &\quad * R [Az(t) + Bz(t - \tau) + C\dot{z}(t - h) - Dz(t - h) + Gy(t)] \\ &\quad - z^T(t) R z(t) + 2z^T(t) R z(t - h) - z^T(t - h) R z(t - h) \end{aligned} \quad (12)$$

With the uncertainties satisfying equation (2), it follows that for any $\varepsilon > 0$, we have

$$\varepsilon [E_1 z(t) + e^{\alpha\tau} E_2 z(t - \tau)]^T [E_1 z(t) + e^{\alpha\tau} E_2 z(t - \tau)] \geq \varepsilon y^T(t) y(t) \quad (13)$$

Combining Eqs. (6) - (13), we obtain

$$\begin{aligned} \dot{V} &\leq z^T(t) [2P(A + B) + 2WB + (BA)^T N B A + \tau^2(W + P)N^{-1}(W + P)^T \\ &\quad + Q_1 + Q_2 + A^T Q_3 A + (\tau^2 + h^2)A^T R A - 2R + \varepsilon E_1^T E_1] z(t) \\ &\quad + z^T(t) [2(BA)^T N B B + 2A^T Q_3 B - 2WB + 2(\tau^2 + h^2)A^T R B \\ &\quad + 2R + 2\varepsilon e^{\alpha\tau} E_1^T E_2] z(t - \tau) + z^T(t) [2(BA)^T N B C + 2A^T Q_3 C + 2PC \\ &\quad + 2(\tau^2 + h^2)A^T R C] \dot{z}(t - h) + z^T(t) [2R - 2(BA)^T N B D - 2A^T Q_3 D \\ &\quad - 2PD - 2(\tau^2 + h^2)A^T R D] z(t - h) + z^T(t) [2(BA)^T N B G + 2A^T Q_3 G \\ &\quad + 2PG + 2(\tau^2 + h^2)A^T R G] y(t) + z^T(t - \tau) [(BB)^T N B B + B^T Q_3 B \\ &\quad + (\tau^2 + h^2)B^T R B - Q_1 - R - \varepsilon e^{2\alpha\tau} E_2^T E_2] z(t - \tau) \\ &\quad + z^T(t - \tau) [2(BB)^T N B C + 2B^T Q_3 C + 2(\tau^2 + h^2)B^T R C] \dot{z}(t - h) \\ &\quad + z^T(t - \tau) [-2(BB)^T N B D - 2B^T Q_3 D - 2(\tau^2 + h^2)B^T R D] z(t - h) \end{aligned}$$

$$\begin{aligned}
& + z^T(t-\tau) \left[2(BB)^T N B G + 2B^T Q_3 G + 2(\tau^2 + h^2) B^T R G \right] y(t) \\
& + \dot{z}^T(t-h) \left[(BC)^T N B C + C^T Q_3 C + (\tau^2 + h^2) C^T R C - Q_3 \right] \dot{z}(t-h) \\
& + \dot{z}^T(t-h) \left[-2(BC)^T N B D - 2C^T Q_3 D - 2(\tau^2 + h^2) C^T R D \right] z(t-h) \\
& + \dot{z}^T(t-h) \left[2(BC)^T N B G + 2C^T Q_3 G + 2(\tau^2 + h^2) C^T R G \right] y(t) \\
& + z^T(t-h) \left[(BD)^T N B D + D^T Q_3 D + (\tau^2 + h^2) D^T R D - Q_2 - R \right] z(t-h) \\
& + z^T(t-h) \left[-2(BD)^T N B G - 2D^T Q_3 G - 2(\tau^2 + h^2) D^T R G \right] y(t) \\
& + y^T(t) \left[(BG)^T N B G + G^T Q_3 G + (\tau^2 + h^2) G^T R G - \varepsilon \right] y(t) \\
& = \Omega^T \Pi^* \Omega,
\end{aligned}$$

where $\Omega = [z(t), z(t-\tau), \dot{z}(t-h), z(t-h), y(t)]^T$ and

$$\Pi^* = \begin{bmatrix} Z_{11}^* & Z_{12} & Z_{13} & Z_{14} & Z_{15} \\ * & Z_{22} & Z_{23} & Z_{24} & Z_{25} \\ * & * & Z_{33} & Z_{34} & Z_{35} \\ * & * & * & Z_{44} & Z_{45} \\ * & * & * & * & Z_{55} \end{bmatrix}, \quad (14)$$

where $Z_{11}^* = 2P(A+B) + 2WB + (BA)^T N B A + \tau^2(W+P)N^{-1}(W+P)^T + Q_1 + Q_2 + A^T Q_3 A + (\tau^2 + h^2)A^T R A - 2R + \varepsilon E_1^T E_1$.

By Schur compliment, one can see that the non-linear matrix Π^* is equivalent to the linear matrix Π as defined in (3). Since $\Pi < 0$, it implies that $\Pi^* < 0$, thus, $\dot{V} < 0$.

With the Lyapunov-Krasovskii functionals defined above, we have

$$\lambda_1 \|z(t)\|^2 \leq V(t, z_t) \leq \lambda_2 \|z_t\|^2,$$

where $\lambda_1 = \lambda_{\min}(P)$ and $\lambda_2 = \lambda_{\max}(P) + \tau^2 \lambda_{\max}(B^T X B) + \tau \lambda_{\max}(Q_1) + h \lambda_{\max}(Q_2) + h \lambda_{\max}(Q_3) + (\tau^3 + h^3) \lambda_{\max}(R)$. By Lemma 2, there exists $K_1 > 0$ such that $\|z(t, \phi)\| \leq K_1 \|\phi\|, \forall t \geq 0$. With the change of variable as in equation (4), this implies that the solution $x(t, \phi)$ satisfies $\|x(t, \phi)\| \leq K \|\phi\| e^{-\alpha t}, \forall t \geq 0$. Thus, the robust exponentially stability criterion of equation (1) is obtained. The proof is complete. $\square$

Next consider a linear neutral system without uncertainties of the form:

$$\dot{x}(t) = \bar{A}x(t) + \bar{B}x(t-\tau) + \bar{C}\dot{x}(t-h). \quad (15)$$

One can easily obtain an exponential stability criterion of the linear neutral (15) as in the following corollary.

Corollary 6. *For given $\alpha > 0$ and time-delays $\tau, h \geq 0$, the neutral system (15) is exponentially stable with convergent rate α if there exist symmetric and positive-definite matrices $P, Q_1, Q_2, Q_3, N, W, R$ such that the following inequality holds:*

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \tau(W + P)^T \\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & 0 \\ * & * & \Sigma_{33} & \Sigma_{34} & 0 \\ * & * & * & \Sigma_{44} & 0 \\ * & * & * & * & -N \end{bmatrix} < 0 \quad (16)$$

where

$$\begin{aligned} \Sigma_{11} &= P(A + B) + (A + B)^T P + WB + B^T W + Q_1 + Q_2 \\ &\quad + (BA)^T N B A + A^T Q_3 A + (\tau^2 + h^2) A^T R A - 2R, \\ \Sigma_{12} &= (BA)^T N B B + A^T Q_3 B - WB + (\tau^2 + h^2) A^T R B + R, \\ \Sigma_{13} &= (BA)^T N B C + A^T Q_3 C + PC + (\tau^2 + h^2) A^T R C, \\ \Sigma_{14} &= - (BA)^T N B D - A^T Q_3 D - PD - (\tau^2 + h^2) A^T R D + R, \\ \Sigma_{22} &= (BB)^T N B B + B^T Q_3 B + (\tau^2 + h^2) B^T R B - Q_1 - R \\ \Sigma_{23} &= (BB)^T N B C + B^T Q_3 C + (\tau^2 + h^2) B^T R C \\ \Sigma_{24} &= - (BB)^T N B D - B^T Q_3 D - (\tau^2 + h^2) B^T R D \\ \Sigma_{33} &= (BC)^T N B C + C^T Q_3 C + (\tau^2 + h^2) C^T R C - Q_3 \\ \Sigma_{34} &= - (BC)^T N B D - C^T Q_3 D - (\tau^2 + h^2) C^T R D \\ \Sigma_{44} &= (BD)^T N B D + D^T Q_3 D + (\tau^2 + h^2) D^T R D - Q_2 - R. \end{aligned}$$

Proof. Followed the proof of the theorem 5 with $\Delta A = \Delta B = 0, \varepsilon = 0$. This yields $G = E_1 = E_2 = 0$. Then the proof is complete. $\square$

Remark 1. Our both proposed criteria are delay-dependent with rate of convergence α . The formulations are based on a change of variable and typical Lyapunov-Krasovskii functionals except V_2 which is chosen to complement the integral term arisen from $\dot{V}_1$. Moreover, both criteria do not require addition of any free weight matrix. This keeps the size of LMI rather small comparing to other works.

Remark 2. Case $\alpha = 0$, it easily to show that the zero solutions of (1) and (15) are asymptotically stable.

4. Numerical Examples

To demonstrate the effectiveness of our proposed criteria, we compare the upper bounds on time-delays and convergent rate with other works. Here we solve the LMIs (3) and (16) using LMI control toolbox in MATLAB with transformations of $A = \bar{A} + \alpha I, B = e^{\alpha\tau}(\bar{B} - \alpha\bar{C}), C = e^{\alpha\tau}\bar{C}$ as defined earlier.

Example 1. Consider the linear neutral delay system (1) **with** uncertainties described in [1], [10], [12] with

$$\bar{A} = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \bar{B} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \bar{C} = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}$$

$$0 \leq |c| < 1, G = I, E_1 = E_2 = 0.2I.$$

Clearly that $\|\bar{C}\| = c < 1$ for all c . Here we compare the upper bounds τ with bounds from [1], [10], [12] for the case of asymptotic stability (A.S.). Solving the LMI (3), in case $\tau = h$, allows the upper bounds τ ranging from 2.69($c = 0.4$) to 4.83($c = 0.05$). Obviously one can see that our obtained upper bounds are larger than results from the others for all values c (see Table 1). Next we investigate the rate of convergence of our criterion for the same upper bounds as obtained by [1] which are the largest bounds available. Our criterion offers the convergent rates ranging from 0.28 to 0.09 for various c (see Table 1).

Then we investigate the stability in case $\tau \neq h$. Results from criteria [1], [10] are the same as in the case of $h = \tau$ since the upper bounds from those criteria are $\max\{h, \tau\}$. Thus, we only compare our upper bounds with [12]. Solving LMI (3), in the case of A.S., we obtain larger upper bounds. Then, we examine the rates of convergence when the same upper bounds τ are given as those from [12]. The convergent rates vary from 0.47($c = 0.4$) to 0.5($c = 0.05$). From both cases, one can see that our robust exponential stability criterion (3) is less conservative than the existing works.

Example 2. Consider the linear neutral delay system (15) **without** uncertainties as studied in [15] with

$$\bar{A} = \begin{bmatrix} -1 & 0 \\ 0.1 & -2 \end{bmatrix}, \bar{B} = \begin{bmatrix} -0.5 & 0.1 \\ 0.3 & -1 \end{bmatrix}, \bar{C} = \begin{bmatrix} -0.3 & 0 \\ 0.1 & 0.2 \end{bmatrix}.$$

case A.S. with $h = \tau$ and c varies								
c	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4
[1] [*]	2.62	2.55	2.47	2.38	2.27	2.14	2.00	1.85
[10]	2.26	2.01	1.79	1.59	1.40	1.23	1.08	0.94
[12]	2.17	1.93	1.72	1.52	1.35	1.19	1.04	0.91
our (3) $\alpha = 0$	4.83	4.53	4.22	3.91	3.60	3.4	2.99	2.69
case E.S.	α for the same τ given in [1] [*]							
our α from (3)	0.28	0.23	0.19	0.16	0.13	0.12	0.10	0.09

Table 1: Upper bounds on time-delay τ guaranteeing stability of (1) with $h = \tau$ and various c in Example 1

c	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4
[12] [*] ($\alpha = 0$)	2.29	2.15	2	1.85	1.7	1.54	1.38	1.21
our (3) $\alpha = 0$	4.83	4.53	4.22	3.91	3.60	3.4	2.99	2.69
case E.S.	α for the same τ given in [12] [*]							
our α from (3)	0.5	0.49	0.48	0.47	0.47	0.47	0.47	0.47

Table 2: Upper bounds on time-delay τ guaranteeing stability of (1) with $h = 0.1$ and various c in Example 1

Here $\|\bar{C}\| = 0.325 < 1$. In case of h and τ are varied, we compare the rates of convergence with [15] for different values of h and τ . Results are shown in Table 3. One can clearly see that our criterion (16) allows faster convergence rate than the condition given in [15] for all cases.

Example 3. Consider the linear neutral delay system (15) **without** uncertainties studied in [5], [12], [13], [16] with

$$\bar{A} = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}, \bar{B} = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix}, \bar{C} = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}.$$

Here $\|\bar{C}\| = 0.292 < 1$. We first consider stability in the case of $\tau = h$. Note that criteria from [5], [12], [13], [16] are asymptotically stable. Thus, we compare the upper bound in case of A.S. with recent works [5], [12], [13], [16] and list some upper bounds in case of E.S. (see Table 4) Our criterion (16) allows delay to be infinite (for A.S.) and 16.08, 2.26, 1.16, 0.83 with convergent rates $\alpha = 0.1, 0.7, 1.3, 1.7$, respectively.

In case of $\tau \neq h$, our criterion (16) allows the upper bounds of delay τ to be infinite when h is fixed (for A.S.). Then, we investigate the rates of

h		0.5	1	1.5	2	2.5
$\tau = 0.5$	[15]	0.84	0.76	0.59	0.47	0.39
	our (16)	1.23	0.87	0.67	0.54	0.45
$\tau = 1.5$	[15]	0.31	0.29	0.27	0.25	0.22
	our (16)	2.38	1.10	0.74	0.57	0.46
$\tau = 2$	[15]	0.22	0.21	0.20	0.19	0.18
	our (16)	2.40	1.17	0.77	0.58	0.46

Table 3: Comparing rate of convergence α for the same delays given in [15] as discussed in Example 2

convergence when delays h and τ are given to be the same values as provided in [12] (for A.S.). Results allow convergent rates varying from 0.91 to 5.1 as listed in Table 5. From both cases, it is clearly that our criterion (16) provides larger upper bounded of the delay τ with faster speed of convergence.

case $h = \tau$									
A.S.					E.S.				
$\alpha = 0$					$\alpha =$	0.1	0.7	1.3	1.7
[5]	[12]	[13]	[16]	our (16)					
1.04	1.72	0.70	3.99	∞		16.08	2.26	1.16	0.83

Table 4: Upper bounds on time-delay τ with various α in Example 3

	upper bounds h, τ for $\alpha = 0$ as given in [12]							
h	0.1	0.3	0.5	0.7	0.9	1.1	1.3	1.72
τ	1.83	1.80	1.77	1.74	1.72	1.72	1.72	1.72
our α	5.1	4.43	3.17	2.28	1.76	1.43	1.21	0.91

Table 5: Upper bounds of decay rate α for the same upper bounds of delays given by [12] in Example 3

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