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SOME FIXED POINT RESULTS OF ALMOST GENERALIZED CONTRACTIVE MAPPINGS IN ORDERED METRIC SPACES

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Abstract: In the present paper we prove some coincidences and common fixed point theorems for different weaker forms of compatibility satisfying an almost generalized contractive condition in ordered metric spaces. Our results generalize and unify some well-known previous results.

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Key Words: common fixed point, almost contraction, ordered metric spaces

1. Introduction and Preliminaries

The Banach contraction mapping is one of the pivotal results of functional analysis. It is widely considered as the source of metric fixed point theory. Also its significance lies in its vast applicability in a number of branches of mathematics. In 1968, Kannan [17] proved a fixed point theorem for a map satisfying a contractive condition that did not require continuity at each point. This paper was a origin for a multitude of fixed point theorems over the next two decades. On the other hand Sessa [21] introduced the notion of weakly commuting maps in metric spaces which are the generalization of commuting

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maps. Jungck [14] enlarged this concept of weakly commutativity by introducing compatible maps. In 1993, Jungck, Murthy and Cho [16] generalized the concept of compatible mappings into compatible mappings of type (A) and also Jungck [15] generalized the notion of compatible maps by introducing the notion of weakly compatible maps.

Definition 1.1. Let (X, d) be metric space. A map T : X → X is called an almost contraction with respect to a mapping S : X → X if there exist a constant $\delta \in]0, 1[$ and some L ≥ 0 such that

$$d(Tx, Ty) \leq \delta d(Sx, Sy) + Ld(Sy, Tx),$$

for all x, y ∈ X.
If we choose S = I_x, I_x is the identity map on X, we obtain the definition of almost contraction, the concept introduced by Berinde ([4, 5]).

This concept by Berinde in [4] was called as 'weak contraction', but in [5], Berinde renamed it as 'almost contraction' which is appropriate. Berinde [4] proved some fixed point theorems for almost contractions in complete metric spaces. Then many authors have studied this problematic and obtained significance results([3], [6]-[13], [18], [19]).
It was shown in [4] that any strict contraction, the Kannan [17] and Zamfirescu [22] mappings, as well as a large class of quasi-contractions, are all most contractions.

Definition 1.2. Let E be a subset of a metric space (X, d). Let S and T be two self maps of a metric space(X, d), T is said to be S-contraction if there exists $k \in [0, 1)$ such that $d(Tx, Ty) \leq kd(Sx, Sy)$ for all $x, y \in E$.

In 2006, Al-Thagafi and Shahzad [1] proved the following theorem which is the generalization of many known results.

Theorem 1.3 (1, Theorem 2.1). *Let E be a subset of a metric space (X, d) and S and T be self maps of E and $T(E) \subseteq S(E)$. Suppose that if S and T are weakly compatible, T is S-contraction and $T(E)$ is complete. Then S and T have a unique common fixed point in E.*

Recently Babu et al. [2] considered the class of mappings that satisfy 'condition (B)'. Let (X, d) be a metric space. A map $T : X \to X$ is said to satisfy 'condition (B)' if there exist a constant $\delta \in]0, 1[$ and some $L \geq 0$ such that

$$d(Tx, Ty) \leq \delta d(x, y) + L \min \{d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\},$$

for all $x, y \in X$.
They proved the existance of fixed point theorem for such mappings on complete

metric spaces. They also discussed in detail about quasi-contraction, almost contraction and the class of mappings that satisfy condition (B).

Definition 1.4. Let (X, d)be a metric space and S, T be mappings from X into itself. S and T are said to be compatible of type (A) if

$$\lim_{n\to\infty} d(TSx_n, SSx_n) = 0 \text{ and } \lim_{n\to\infty} d(STx_n, TTx_n) = 0,$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$.

Proposition 1.5. *Let (X, d) be a metric space and S, T* $: X \to X$ *be mappings. If S and T are compatible mappings of type (A) and* $\lim_{n\to\infty} Sx_n = lim_{n\to\infty} Tx_n = z$ *for some z* $\in$ *X. Then we have*
(1) $lim_{n\to\infty} TSx_n = Sz$ *if S is continuous at z.*
(2) STz = TSz and Sz = Tz if S and T are continuous at z.

Definition 1.6. A pair (S, T) of self-mappings on X is said to be weakly compatible if S and T commute at their coincidence point (i.e.$STx = TSx$, $x \in X$ whenever $Sx = Tx$). A point $y \in X$ is called a point of coincidence of two self-mappings S and T on X if there exists a point $x \in X$ such that$y = Tx = Sx$.

Definition 1.7. Let X be a nonempty set. Then $(X, d, \leq)$ is called an ordered metric space iff:
(i) (X, d) is a metric space,
(ii) $(X, \leq)$ is partial ordered.

Definition 1.8. Let S and T be two self maps of a metric space (X, d).They are said to satisfy almost generalized contractive condition if there exists $\delta \in [0, 1)$ and $L \geq 0$ such that

$$\begin{aligned} d(Sx, Ty) \leq\ & \delta \max\left\{d(x, y), d(x, Sx), d(y, Ty), \frac{d(x, Ty) + d(y, Sx)}{2}\right\} \\ & + L \min\left\{d(x, Sx), d(y, Ty), d(x, Ty), d(y, Sx)\right\}, \qquad (1.1) \end{aligned}$$

for all x, y $\in$ X.

2. Main Results

Theorem 2.1. *Let* $(X, \leq)$ *be a partially ordered set and suppose that there exists a metric d on X such that the metric* (X, d) *is complete. Let*

$A, B, S, T : X \to X$ be four mappings with respect to $\leq$ satisfying the following:

(i) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$,

(ii) one of $A(X)$, $B(X)$, $S(X)$ or $T(X)$ is a complete subspace of X,

(iii) there exists $\delta \in [0, 1[$ and $L \geq 0$ such that

$$d(Ax, By) \leq \delta \max\left\{d(Sx, Ty), d(Sx, Ax), d(Ty, By), \frac{d(Sx, By) + d(Ty, Ax)}{2}\right\} + L \min\left\{d(Sx, Ax), d(Ty, By), d(Sx, By), d(Ty, Ax)\right\}, \quad (2.1)$$

for all comparable elements x, $y \in X$. Then:

($\star$) A and S have a coincidence,

($\star\star$) B and T have a coincidence.

Proof. Suppose $x_0 \in X$ is arbitrary. Let us construct a sequence $\{y_n\}$ in X such that

$$\begin{aligned} y_{2n} &= Ax_{2n} = Tx_{2n+1} \text{ and} \\ y_{2n+1} &= Bx_{2n+1} = Sx_{2n+2}, \quad \text{for all } n \geq 0. \end{aligned}$$

Now

$$\begin{aligned} M(x_{2n}, x_{2n+1}) &= \max\Big\{d(Sx_{2n}, Tx_{2n+1}), d(Sx_{2n}, Ax_{2n}), d(Tx_{2n+1}, Bx_{2n+1}), \\ &\qquad \frac{d(Sx_{2n}, Bx_{2n+1}) + d(Tx_{2n+1}, Ax_{2n})}{2}\Big\} \\ &= \max\Big\{d(y_{2n-1}, y_{2n}), d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1}), \\ &\qquad \frac{d(y_{2n-1}, y_{2n+1}) + d(y_{2n}, y_{2n})}{2}\Big\} \\ &= \max\Big\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1}), \\ &\qquad \frac{d(y_{2n-1}, y_{2n+1})}{2}\Big\} \\ &\leq \max\Big\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1}), \\ &\qquad \frac{d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1})}{2}\Big\} \\ &= \max\left\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1})\right\}. \end{aligned}$$

Therefore $M(x_{2n}, x_{2n+1}) \leq \max\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1})\}$.

Since x_n and x_{n+1} are comparable then by taking y_{2n} for x and y_{2n+1} for y in (2.1), it follow that

$$\begin{aligned} d(y_{2n}, y_{2n+1}) &= d(Ax_{2n}, Bx_{2n+1}) \\ &\leq \delta M(x_{2n}, x_{2n+1}) + L\min\{d(Sx_{2n}, Ax_{2n}), d(Tx_{2n+1}, Ax_{2n+1}), \\ &\quad d(Sx_{2n}, Bx_{2n+1}), d(Tx_{2n+1}, Ax_{2n})\} \\ &\leq \delta \max\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1})\} \\ &\quad + L\min\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1}), \\ &\quad d(y_{2n-1}, y_{2n+1}), d(y_{2n}, y_{2n})\} \\ &\leq \delta \max\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1})\} \\ &\quad + L\min\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1}), d(y_{2n-1}, y_{2n+1}), 0\}. \end{aligned}$$

Thus

$$d(y_{2n}, y_{2n+1}) \leq \delta \max\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1})\}.$$

If max $\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1})\} = d(y_{2n-1}, y_{2n})$, then

$$d(y_{2n}, y_{2n+1}) \leq \delta d(y_{2n-1}, y_{2n}).$$

In case max$\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n+1})\} = d(y_{2n}, y_{2n+1})$ for some n, we have

$$d(y_{2n}, y_{2n+1}) \leq \delta d(y_{2n}, y_{2n+1}).$$

A contradiction!

Therefore we have

$$d(y_{2n}, y_{2n+1}) \leq \delta d(y_{2n-1}, y_{2n}).$$

Similarly, it can be proved that

$$d(y_{2n+1}, y_{2n+2}) \leq \delta d(y_{2n+1}, y_{2n}).$$

So

$$d(y_n, y_{n+1}) \leq \delta d(y_{n-1}, y_n) \leq \delta^2 d(y_{n-2}, y_{n-1}) \leq ... \leq \delta^n d(y_0, y_1),$$

for all $n \geq 1$. Now, for any positive integer m and n with $m \geq n$ we have

$$d(y_m, y_n) \leq d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + ... + d(y_{m-1}, y_m) \leq \frac{\delta^n}{1-\delta} d(y_0, y_1).$$

Hence we conclude that $\{y_n\}$is a Cauchy sequence.

Now suppose S(X) is complete, then the subsequence $\{y_{2n}\}$ being contaied in S(X) has a limit in S(X), call it u. Let v $\in S^{-1}$, then Sv = u. Note that the subsequences $\{y_{2n-1}\}$, Ax_{2n}, Sx_{2n}, Bx_{2n+1} and Tx_{2n-1} also converges to u.

Putting x = v and y = x_{2n+1} in (2.1), we have

$$\begin{aligned} d(Av, Bx_{2n+1}) \ \leq \ & \delta \max \{d(Sv, Tx_{2n+1}), d(Sv, Av), d(Tx_{2n+1}, Bx_{2n+1}), \\ & \left.\frac{d(Sv, Bx_{2n+1}) + d(Tx_{2n+1}, Av)}{2}\right\} \\ & + L \min \{d(Sv, Av), d(Tx_{2n+1}, Bx_{2n+1}), \\ & d(Sv, Bx_{2n+1}), d(Tx_{2n+1}, Av)\}, \end{aligned}$$

Taking the limit n $\to \infty$, we have

$$\begin{aligned} d(Av, u) \ \leq \ & \delta \max \left\{ d(u,u), d(u,Av), d(u,u), \frac{d(u,u) + d(u,Av)}{2} \right\} \\ & + L \min \{d(u,Av), d(u,u), d(u,u), d(u,Av)\} \\ = \ & \delta d(u, Av), \ \textit{a contradiction.} \end{aligned}$$

Therefore Av = u. Thus Av = u = Sv. This proves ($\star$).

Since A(X) $\subseteq$ T(X), there is an element w in X such that Av = Tw, i.e. Tw = u. Putting x = x_{2n} and y = w in (2.1), we have

$$\begin{aligned} d(Ax_{2n}, Bw) \ \leq \ & \delta \max \{d(Sx_{2n}, Tw), d(Sx_{2n}, Ax_{2n}), d(Tw, Bw), \\ & \left.\frac{d(Sx_{2n}, Bw) + d(Tw, Ax_{2n})}{2}\right\} \\ & + L \min \{d(Sx_{2n}, Ax_{2n}), d(Tw, Bw), \\ & d(Sx_{2n}, Bw), d(Tw, Ax_{2n})\}, \end{aligned}$$

Taking the limit n $\to \infty$, we have

$$\begin{aligned} d(u, Bw) \ \leq \ & \delta \max \left\{ d(u,u), d(u,u), d(u,Bw), \frac{d(u,Bw) + d(u,u)}{2} \right\} \\ & + L \min \{d(u,u), d(u,Bw), d(u,Bw), d(u,u)\} \\ = \ & \delta d(u, Bw). \end{aligned}$$

A contradiction!

Therefore Bw = u. Thus Bw = u = Tw. This proves ($\star\star$).

If we suppose that T(X) is complete then analogous argument establishes ($\star$) and ($\star\star$). If B(X) (resp A(X)) is complete, then u $\in$ B(X) $\subset$ S(X)(resp u $\in$ A(X) $\subset$ T(X)), and the argument establishes ($\star$) and ($\star\star$). □

Theorem 2.2. *Let $(X, \leq)$ be a partially ordered set and suppose that there exists a metric d on X such that the metric (X, d) is complete. Let $A, B, S, T : X \to X$ be four mappings with respect to $\leq$ satisfying the following:*

(i) A(X) $\subseteq$ T(X) and B(X) $\subseteq$ S(X),

(ii) the pairs $\{A, S\}$ and $\{B, T\}$ are compatible of type (A),

(iii) one of A, B, S and T is continuous,

(iv) there exists $\delta \in [0, 1[$ and L $\geq$ 0 such that

$$\begin{aligned} d(Ax, By) \leq\ & \delta \max \{d(Sx, Ty), d(Sx, Ax), d(Ty, By), \\ & \left.\frac{d(Sx, By) + d(Ty, Ax)}{2}\right\} \\ & + L \min \{d(Sx, Ax), d(Ty, By), d(Sx, By), d(Ty, Ax)\}, \end{aligned} \tag{2.2}$$

for all comparable elements x, y $\in$ X. Then A, B, S and T have a unique common fixed point in X.

Proof. On the lines of proof of Theorem 2.1, we conclude that $\{y_n\}$ is a Cauchy sequence. Since X is complete, the sequence $\{y_n\}$ converges to a point z in X and subsequences $\{Ax_{2n}\}$, $\{Sx_{2n}\}$, $\{Bx_{2n-1}\}$ and $\{Tx_{2n+1}\}$ also converges to z.

Now suppose that T is continuous. Since B ant T are compatible of type (A), then by Proposition 1.5, we have

$$BTx_{2n+1}, TTx_{2n+1} \to Tz \text{ as } n \to \infty.$$

Putting x = x_{2n} and y = Tx_{2n+1} in (2.2), we have

$$\begin{aligned} d(Ax_{2n}, BTx_{2n+1}) \leq\ & \delta \max\{d(Sx_{2n}, TTx_{2n+1}), d(Sx_{2n}, Ax_{2n}), \\ & d(TTx_{2n+1}, BTx_{2n+1}), \\ & \left.\frac{d(Sx_{2n}, BTx_{2n+1}) + d(TTx_{2n+1}, Ax_{2n})}{2}\right\} \\ & + L \min \{d(Sx_{2n}, Ax_{2n}), d(TTx_{2n+1}, BTx_{2n+1}), \\ & d(Sx_{2n}, BTx_{2n+1}), d(TTx_{2n+1}, Ax_{2n})\}. \end{aligned}$$

Taking the limit $n \to \infty$, we get

$$d(z, Tz) \leq \delta \max \left\{ d(z, Tz), d(z, z), d(Tz, Tz), \frac{d(z, Tz) + d(Tz, z)}{2} \right\}$$

$$
\begin{aligned}
& +L\ min\,\{d(z,z), d(Tz,Tz), d(z,Tz), d(Tz,z)\} \\
= \ & \delta d(z,Tz),
\end{aligned}
$$

which implies that Tz = z. Again by replacing x by x_{2n} and y by z in (2.2), we have

$$
\begin{aligned}
& d(Ax_{2n}, Bz) \leq \\
& \delta \max\{d(Sx_{2n}, Tz), d(Sx_{2n}, Ax_{2n}), d(Tz, Bz), \frac{d(Sx_{2n}, Tz) + d(Tz, Ax_{2n})}{2}\} \\
& \qquad + L\ min\,\{d(Sx_{2n}, Ax_{2n}), d(Tz, Bz), d(Sx_{2n}, Tz), d(Tz, Ax_{2n})\}\,.
\end{aligned}
$$

Taking the limit $n \to \infty$ and Tz = z we get

$$
\begin{aligned}
d(z, Bz) \ & \leq \ \delta \max\left\{d(z,z), d(z,z), d(z,Bz), \frac{d(z,Bz) + d(z,z)}{2}\right\} \\
& \quad +L\ min\,\{d(z,z), d(z,Bz), d(z,Bz), d(z,z)\} \\
& = \ \delta d(z,Bz),
\end{aligned}
$$

which implies that Bz = z. Since B(X) $\subseteq$ S(X), there exists a point w in X such that Bz = Sw = z. Again by (2.2), we have

$$
\begin{aligned}
d(Aw, Bz) \ & \leq \ \delta \max\{d(Sw,Tz), d(Sw,Aw), d(Tz,Bz), \\
& \quad \frac{d(Sw,Tz) + d(Tz,Aw)}{2}\} \\
& \quad +L\ min\,\{d(Sw,Aw), d(Tz,Bz), d(Sw,Tz), d(Tz,Aw)\} \\
& = \ \delta \max\{d(z,z), d(z,Aw), d(z,z), \frac{d(z,z) + d(z,Aw)}{2}\} \\
& \quad +L\ min\,\{d(z,Aw), d(z,z), d(z,z), d(z,Aw)\} \\
& = \ \delta d(z,Aw)
\end{aligned}
$$

which implies that Aw = z. Since A and S are compatible of type (A), and Aw = Sw = z, then by Proposition 1.5, we have

$$Az = ASw = SAw = Sz.$$

By using (2.2) again, we have Az = z.

Therefore Az = Bz = Sz = Tz = z, that is z is a common fixed point of A, B, S and T. For uniqueness, let z' be another common fixed point such that $z \neq z'$. Then

$$d(z, z') \ = \ d(Az, Bz')$$

$$\begin{aligned} &\leq \delta \max\{d(Sz,Tz'), d(Sz,Az), d(Tz',Bz'), \frac{d(Sz,Bz')+d(Tz',Az)}{2}\} \\ &\quad +L\ min\left\{d(Sz,Az), d(Tz',Bz'), d(Sz,Bz'), d(Tz',Az)\right\} \\ &= \delta d(z,z') \end{aligned}$$

which means that z = z'. Thus z is a unique common fixed point of A, B, S and T. □

Corollary 2.3. *Let $(X, \leq)$ be a partially ordered set and suppose that there exists a metric d on X such that the metric (X, d) is complete. Let $A, B, S, T : X \to X$ be mappings with respect to $\leq$ satisfying the following:*

(i) A(X) $\subseteq$ T(X) and B(X) $\subseteq$ S(X),

(ii) the pairs $\{A, S\}$ and $\{B, T\}$ are compatible of type (A),

(iii) one of A, B, S and T is continuous,

(iv) there exists $\delta \in$ [0, 1[and L $\geq$ 0 such that

$$d(Ax,By) \leq \delta \max\left\{d(Sx,Ty), \frac{d(Sx,Ax)+d(Ty,By)}{2}, \frac{d(Sx,By)+d(Ty,Ax)}{2}\right\} + L \min\left\{d(Sx,Ax), d(Ty,By), d(Sx,By), d(Ty,Ax)\right\}, \quad (2.3)$$

for all comparable elements x, y $\in$ X. Then A, B, S and T have a unique common fixed point in X.

Proof. As the inequality (2.3) is a special case of (2.2), the result follows from Theorem 2.2. □

Corollary 2.4. *Let $(X, \leq)$ be a partially ordered set and suppose that there exists a metric d on X such that the metric (X, d) is complete. Let $A, B, S, T : X \to X$ be four mappings with respect to $\leq$ satisfying the following*
(i) A(X) $\subseteq$ T(X) and B(X) $\subseteq$ S(X),
(ii) the pairs $\{A, S\}$ and $\{B, T\}$ are compatible of type (A),
(iii) one of A, B, S and T is continuous,
(iv) there exists $\delta \in$ [0, 1[and L $\geq$ 0 such that

$$\begin{aligned} d(Ax,By) &\leq \delta d(Sx,Ty) \\ &\quad +L \min\left\{d(Sx,Ax), d(Ty,By), d(Sx,By), d(Ty,Ax)\right\}, \quad (2.4) \end{aligned}$$

for all comparable elements x, y $\in$ X. Then A, B, S and T have a unique common fixed point in X.

Proof. As the inequality (2.4) is a special case of (2.2), the result follows from Theorem 2.2. □

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