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ON WEAKLY CONCIRCULAR SYMMETRIES OF THREE-DIMENSIONAL TRANS-SASAKIAN MANIFOLDS

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Abstract: The purpose of this paper is to study weakly concircular symmetric and weakly concircular Ricci symmetric three-dimensional trans -Sasakian manifolds.

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1. Introduction

The notion of weakly symmetric manifolds was introduced by Tamassy and Binh[16]. A non flat Riemannian manifold $(M^n, g)(n > 2)$ is called weakly symmetric if its curvature tensor R of type $(0, 4)$ satisfies the condition

$$(\nabla_X R)(Y,Z,U,V) = A(X)R(Y,Z,U,V) + B(Y)R(X,Z,U,V) \\ +H(Z)R(Y,X,U,V) + D(U)R(Y,Z,X,V) + E(V)R(Y,Z,U,X) \quad (1)$$

for all vector fields X, Y, Z, U, $V \in \chi(M^n)$, where A, B, H, $D$and E are

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1-forms(not simultaneously zero) and ∇ denotes the operator of covariant differentiation with respect to the Riemannian metric g. The $1-$forms are called the associated $1-$forms of the manifold and an $n-$dimensional manifold of this kind is denoted by $(WS)_n$. In 1999 De and Bandyopadhyay[4] studied a $(WS)_n$ and proved that in such a manifold the associated $1-$forms $B = H$ and $D = E$. Hence (1.1) reduces to the following form:

$$(\nabla_X R)(Y,Z,U,V) = A(X)R(Y,Z,U,V) + B(Y)R(X,Z,U,V)$$
$$+B(Z)R(Y,X,U,V) + D(U)R(Y,Z,X,V) + D(V)R(Y,Z,U,X) \quad (2)$$

A transformation of an $3-$dimensional Riemannian manifold M, which transforms every geodesic circle of M into a geodesic circle, is called a concircular transformation[18] and is defined by

$$C(Y,Z)U = R(Y,Z)U - \frac{r}{6}[g(Z,U)Y - g(Y,U)Z] \quad (3)$$

where r is the scalar curvature of the manifold.
Recently Shaikh and Hui[14] introduced the notion of weakly concircular symmetric manifolds. A Riemannian manifold $(M^n, g)(n > 2)$ is called weakly concircular symmetric manifold if its concircular curvature tensor C satisfies the condition

$$(\nabla_X C)(Y,Z,U) = A(X)C(Y,Z,U) + B(Y)C(X,Z,U)$$
$$+H(Z)C(Y,X,U) + D(U)C(Y,Z,X) \quad (4)$$

for all vector fields X, Y, Z, $U \in \chi(M^3)$, where A, B, H, and $D$are 1-forms(not simultaneously zero) and 3 dimensional manifold of this kind is denoted by $(WCS)_3$. Also it is shown thatin a $(WCS)_3$ the associated $1-$forms $B = H$, and hence the defining condition (1.4) of a $(WCS)_3$ reduces to the following form:

$$(\nabla_X C)(Y,Z,U) = A(X)C(Y,Z,U) + B(Y)C(X,Z,U)$$
$$+B(Z)C(Y,X,U) + D(U)C(Y,Z,X) \quad (5)$$

A, B and D are 1-forms(not simultaneously zero).

Again Tamassy and Binh[17] introduced the notion of weakly Ricci symmetric manifolds. A Riemannian manifold $(M^n, g)(n > 2)$ is called weakly Ricci symmetric manifold if its Ricci tensor S of type $(0,2)$ is not identically zero and satisfies the condition

$$(\nabla_X S)(Y,Z) = A(X)S(Y,Z) + B(Y)S(X,Z) + D(Z)S(Y,X) \quad (6)$$

where A, B and D are three non-zero $1-$forms, called the associated $1-$forms of the manifold , and ∇ denotes the operator of covariant differentiation with respect to the metric g. Such an $3-$dimensional manifold is denoted by $(WRS)_3$.

Let e_1, e_2, e_3 be an orthonormal basis of the tangent space at each point of the manifold and let

$$P(Y,V) = \sum_{i=1}^{3} C(Y, e_i, e_i, V), \tag{7}$$

then from (1.3), we get

$$P(Y,V) = S(Y,V) - \frac{r}{3} g(Y,V). \tag{8}$$

The tensor P is called the concircular Ricci symmetric tensor [5], which is a symmetric tensor of type $(0,2)$. In[5] De and Ghosh introduced the notion of weakly concircular Ricci symmetric manifolds. A Riemannian manifold $(M^n, g)(n > 2)$ is called weakly concircular Ricci symmetric manifold [5]if its concircular Ricci tensor P of type $(0,2)$ is not identically zero and satisfies the condition

$$(\nabla_X P)(Y,Z) = A(X)P(Y,Z) + B(Y)P(X,Z) + D(Z)P(Y,X) \tag{9}$$

where A, B and D are 1-forms(not simultaneously zero).

In 1985, Oubina[11] introduced the notion of trans-Sasakian manifolds, which contains both the class of Sasakian and cosympletic structures, closely related to the locally conformal Kähler manifolds. Trans-Sasakian manifolds of type $(0,0)$, $(\alpha,0)$ and $(0,\beta)$ are the cosympletic, $\alpha-$Sasakian and $\beta-$Kenmotsu manifolds respectively. In particular, if $\alpha = 1$, $\beta = 0$:and $\alpha = 0$, $\beta = 1$, then trans-Sasakian manifold reduces to a Sasakian and Kenmotsu manifolds respectively. Thus trans-Sasakian structures provide a large class of generalized quasi-Sasakian structures. The Local structure of trans-Sasakian manifolds of dimension $n \geq 5$ has been completely characterized by J. C. Marrero[9] . He proved that a trans-Sasakian manifold of dimension $n \geq 5$ is either cosymplectic or $\beta-$Kenmotsu or $\alpha-$Sasakian manifold. But when $n > 3$ trans-Sasakian manifold does not exist In this paper we consider the three dimensional trans-Sasakian manifold.

In 2002, Kim et al. [10] studied generalized Ricci recurrent trans-Sasakian manifolds. In [8] De and Tripathi studied Ricci semi-symmetric trans-Sasakian manifolds. Tamassy and Binh[17] studied weakly symmetric and weakly Ricci symmetric Sasakian manifolds and proved that in such a manifold the sum of the associated $1-$forms vanishes everywhere. Again Özgür[12] studied weakly symmetric weakly Ricci symmetric Kenmotsu manifolds and proved that in such a manifold the sum of the associated $1-$forms is zero everywhere and hence such a manifold does not exist unless the sum of the associated $1-$forms is everywhere zero. Weakly symmetric and weakly Ricci symmetric properties for trans-Sasakian manifolds, Lorentzian $\alpha-$Sasakian manifolds were studied in [13], [1] and [15] respectively.

The object of the present paper is to study weakly concircular symmetric and weakly concircular Ricci symmetric trans-Sasakian manifolds. Section 2 deals with preliminaries of trans-Sasakian manifolds. In Section 3 of the paper we have obtianed all the $1-$forms of weakly concircular symmetric three dimensional trans-Sasakian manifold and hence such a structure exist. In the last section we study weakly concircular Ricci symmetric three dimensional trans-Sasakian manifolds and obtained all the $1-$forms of weakly concircular Ricci symmetric three dimensional trans-Sasakian manifold and consequently such a structure exist.

2. Preliminaries

A $(2n+1)$-dimensional smooth manifold M is said to be an almost contact metric manifold [2]if it admits a $(1,1)$tensor field ϕ, a vector field ξ, a $1-$form η, and a Riemannian metric g, which satisfy

$$\phi\xi = 0, \quad \eta(\phi X) = 0, \quad \phi^2 X = -X + \eta(X)\xi, \tag{10}$$

$$g(\phi X, Y) = -g(X, \phi Y), \quad \eta(\xi) = 1, \quad g(X, \xi) = \eta(X), \tag{11}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{12}$$

for all vector fields X, Y on M.

An almost contact metric manifold is said to be trans-Sasakian manifold[11] if $(M \times R, J, G)$ belongs to the class W_4 of the Hermitian manifolds, where J is the almost complex structure on $M \times R$ defined by

$$J(Z, f\frac{d}{dt}) = (\phi Z - f\xi, \eta(Z)\frac{d}{dt})$$

for any vector field Z on M and smooth function f on $M \times R$ and G is product metric on $M \times R$. This may be stated by the condition[3]

$$(\nabla_X \phi)(Y) = \alpha\{\{g(X,Y)\xi - \eta(Y)X\} + \beta\{g(\phi X, Y)\xi - \eta(Y)\phi X\}, \tag{13}$$

where α, β are smooth functions on M and such a structure is said to be the trans-Sasakian structure of type (α, β). from (2.4) it follows that

$$\nabla_X \xi = -\alpha\phi X + \beta\{X - \eta(X)\xi\}, \tag{14}$$

$$(\nabla_X \eta)(Y) = -\alpha g(\phi X, \phi Y). \tag{15}$$

In a trans-Sasakian manifold $M^3(\phi, \xi, \eta, g)$ the following relations hold [11]:

$$\begin{aligned} R(X,Y)\xi &= (\alpha^2 - \beta^2)[\eta(Y)X - \eta(X)Y] - (X\alpha)\phi Y - (X\beta)\phi^2(Y) \\ &+ 2\alpha\beta[\eta(Y)\phi X - \eta(X)\phi Y] + (Y\alpha)\phi X + (Y\beta)\phi^2(X), \end{aligned} \tag{16}$$

$$\begin{aligned} \eta(R(X,Y)Z) &= (\alpha^2 - \beta^2)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)] \\ &- 2\alpha\beta[g(\phi X, Z)\eta(Y) - g(\phi Y, Z)\eta(X)] \\ &- (Y\alpha)g(\phi X, Z) - (X\beta)\{g(Y,Z) - \eta(Y)\eta(Z)\} \\ &+ (X\alpha)g(\phi Y, Z) - (Y\beta)\{g(X,Z) - \eta(X)\eta(Z)\}, \end{aligned} \tag{17}$$

$$S(X,\xi) = [2(\alpha^2 - \beta^2) - (\xi\beta)]\eta(X) - ((\phi X)\alpha) - (X\beta), \tag{18}$$
$$R(\xi, X)\xi = (\alpha^2 - \beta^2 - \xi\beta)[\eta(X)\xi - X], \tag{19}$$
$$S(\xi,\xi) = 2(\alpha^2 - \beta^2 - \xi\beta), \tag{20}$$
$$(\xi\alpha) + 2\alpha\beta = 0 \tag{21}$$
$$Q\xi = [2(\alpha^2 - \beta^2) - (\xi\beta)]\xi + \phi(grad\alpha) - (grad\beta), \tag{22}$$

where R is the curvature tensor of type$(1,3)$ of the manifold and Q is the symmetric endomorphism of the tangent space at each point of the manifold corresponding to the Ricci tensor S, that is , $g(QX,Y) = S(X,Y)$ for any vector fields X, Y on M .

3. Weakly Concircular Symmetric Three-Dimensional Trans-Sasakian Manifolds

Definition 1. A trans-Sasakian manifold $M^3(\phi, \xi, \eta, g)$ is said to be weakly concircular symmetric if its concircular curvature tensor C satisfies (1.5).

Setting $Y = V = e_i$ in (1.5) and taking summation over $i, 1 \le i \le 3$, we get

$$\begin{aligned} &(\nabla_X S)(Z,U) - \frac{dr(X)}{3}g(Z,U) = A(X)[S(Z,U) - \frac{r}{3}g(Z,U)] \\ &+B(Z)[S(X,U) - \frac{r}{3}g(X,U)] + D(U)[S(X,Z) - \frac{r}{3}g(X,Z)] \\ &-\frac{r}{6}[\{B(X) + D(X)\}g(Z,U) - B(Z)g(X,U) - D(U)g(Z,X)] \\ &+B(R(X,Z)U) + D(R(X,U)Z) \end{aligned} \tag{23}$$

Putting $X = Z = U = \xi$ in (3.1) and then using (2.7) and (2.11) we obtain

$$A(\xi) + B(\xi) + D(\xi) = \frac{gradF.\xi}{F} \tag{24}$$

where $F = 6(\alpha^2 - \beta^2 - \xi\beta) - r$
We can see that if $gradF$ is orthogonal to ξ then

$$A(\xi) + B(\xi) + D(\xi) = 0 \tag{25}$$

Since $A(X) = g(X, \rho), A(\xi) = B(\xi) = D(\xi) = g(\rho_i \xi_i)$
In view of (3.3) we obtain that $A(\xi) = B(\xi) = D(\xi) = 0$.

If $gradF$ and ξ are not inclined orthogonal then $gradF.\xi \neq 0$. Hence

$$A(\xi) + B(\xi) + D(\xi) \neq 0 \tag{26}$$

that is $A(\xi) = B(\xi) = D(\xi) \neq 0$.

Theorem 1. *In a weakly concircular trans-Sasakian manifold $M^3(\phi, \xi, \eta, g)$ the relation* (3.2) *holds.*

Next substituting X and Z by ξ in (3.1) and then using (2.9) and (2.12) we obtain

$$(\nabla_\xi S)(\xi, U) = [A(\xi) + B(\xi)]S(U, \xi) + [\alpha^2 - \beta^2 - (\xi\beta)][D(U) + \eta(U)D(\xi)] \tag{27}$$

Again we have

$$\begin{aligned}(\nabla_\xi S)(\xi, U) &= \nabla_\xi S(\xi, U) - S(\nabla_\xi \xi, U) - S(\xi, \nabla_\xi U) \\ &= [2\{2\alpha(\xi\alpha) - 2\beta(\xi\beta)\} - \xi(\xi\beta)]\eta(U) \\ &\quad -(U(\xi\beta)) - (\phi U(\xi\alpha)),\end{aligned} \tag{28}$$

where (2.9) has been used. In view of (3.5) and (3.6) we obtain from (3.2) that

$$\begin{aligned}D(U) = &\frac{[12\{2\alpha(\xi\alpha) - 2\beta(\xi\beta)\} - 6\xi(\xi\beta)]\,\eta(U)}{F} - \\ &\frac{6U[\xi\beta] + 6\phi U[\xi\alpha] + 2\xi[r]\eta(u)}{F} - \left\{\frac{\xi[F]}{F^2}\right\} \\ &\{\{6[2(\alpha^2 - \beta^2) - \xi\beta] - 2r\}\eta(U) - 6\{U[\beta] + \phi U[\alpha]\}\} + \\ &D(\xi)\left\{\frac{6\{(\alpha^2 - \beta^2)\eta(U) - U[\beta]\} - 6\phi U[\alpha] - r\eta(U)}{F}\right\}\end{aligned} \tag{29}$$

for any vector field U.

If $gradF$ and ξ are orthogonal then by virtue of (3.2) and (3.3) we get

$$\begin{aligned} D(U) &= \frac{[12\{2\alpha(\xi\alpha) - 2\beta(\xi\beta)\} - 6\xi(\xi\beta)]\,\eta(U)}{F} \\ &\quad - \frac{6U[\xi\beta] + 6\phi U[\xi\alpha] + 2\xi[r]\eta(u)}{F} \\ &\neq 0 \end{aligned}$$

If $gradF$ and ξ are not orthogonal then by virtue of (3.4) we get $D(U) \neq 0$ Next setting $X = U = \xi$ in (3.1) and proceeding in a similar manner as above we get

$$\begin{aligned} B(Z) = {} & \frac{[12\{2\alpha(\xi\alpha) - 2\beta(\xi\beta)\} - 6\xi(\xi\beta)]\,\eta(Z)}{F} - \\ & \frac{6Z[\xi\beta] + 6\phi Z[\xi\alpha] + 2\xi[r]\eta(Z)}{F} - \left\{\frac{\xi[F]}{F^2}\right\} \\ & \big\{\{6[2(\alpha^2 - \beta^2) - \xi\beta] - 2r\}\eta(Z) - 6\{Z[\beta] + \phi Z[\alpha]\}\big\} + \\ & B(\xi)\left\{\frac{6\{(\alpha^2 - \beta^2)\eta(Z) - Z[\beta]\} - 6\phi Z[\alpha] - r\eta(Z)}{F}\right\} \end{aligned} \tag{30}$$

for any vector field Z

If $gradF$ and ξ are orthogonal then by virtue of (3.2) and (3.3) we get

$$\begin{aligned} B(Z) &= \frac{[12\{2\alpha(\xi\alpha) - 2\beta(\xi\beta)\} - 6\xi(\xi\beta)]\,\eta(Z)}{F} - \\ &\quad \frac{6Z[\xi\beta] + 6\phi Z[\xi\alpha] + 2\xi[r]\eta(Z)}{F} \\ &\neq 0 \end{aligned}$$

If $gradF$ and ξ are not orthogonal then by virtue of (3.4) we get $B(Z) \neq 0$

Again setting $Z = U = \xi$ in (3.1) we get

$$\begin{aligned} (\nabla_\xi S)(\xi,\xi) - \frac{dr(x)}{3} &= A(X)[S(\xi,\xi) - \frac{dr(x)}{3}] + B(R(X,\xi)\xi) + D(R(X,\xi)\xi) \\ &+ [B(\xi) + D(\xi)]\left[S(X,\xi) - \frac{dr(x)}{3}\eta(X)\right] \\ &- \frac{r}{6}[B(X) + D(X) - B(\xi)\eta(X) - D(\xi)\eta(X)] \end{aligned} \tag{31}$$

Now we have

$$(\nabla_X S)(\xi,\xi) = \nabla_X S(\xi,\xi) - 2S(\nabla_X \xi,\xi),$$

which yields by using (2.5) and (2.9) that

$$\begin{aligned}(\nabla_\xi S)(\xi,\xi) &= 2[2\alpha(X\alpha) - 2\beta(X\beta) - (X(\xi\beta))] \\ &+ 2\alpha[(X\alpha) - \eta(X)(\xi\alpha) - ((\phi X)\beta)] \\ &+ 2\beta[(\phi X)\alpha + \{(X\beta) - (\xi\beta)\eta(X)\}] \qquad (32)\end{aligned}$$

using (2.10) (2.11) and (3.10) in $(3,9)$ we get

$$\begin{aligned}A(X) = \frac{X(F+r)}{F} + \frac{6\alpha\{(X\alpha) - \eta(X)(\xi\alpha) - (\phi X)\beta\}}{F} + \\ \frac{6\beta[(\phi X)[\alpha] + \{X[\beta] - \xi\beta\eta(X)\}] - X[r]}{F} - \\ \frac{[B(\xi) + D(\xi)][3\{(\alpha^2 - \beta^2)\eta(X) - X[\beta]\} - 3\phi X[\alpha] - \frac{r}{2}\eta(X)]}{F} - \\ \frac{[B(X) + D(X)][3(\alpha^2 - \beta^2) - \xi\beta\frac{r}{2}]}{F}\end{aligned} \qquad (33)$$

for any vector X,
If $gradF$ and ξ are orthogonal then by virtue of (3.2) and (3.3) we get

$$\begin{aligned}A(X) &= \frac{X(F+r)}{F} + \frac{6\alpha\{(X\alpha) - \eta(X)(\xi\alpha) - (\phi X)\beta\}}{F} \\ &+ \frac{6\beta[(\phi X)[\alpha] + \{X[\beta] - \xi\beta\eta(X)\}] - X[r]}{F} \\ &- \frac{[B(X) + D(X)][3(\alpha^2 - \beta^2) - \xi\beta\frac{r}{2}]}{F} \\ &\neq 0\end{aligned}$$

If $gradF$ and ξ are not orthogonal then by virtue of (3.4) we get $A(X) \neq 0$
This leads the following:

Theorem 2. *There exists no weakly Concircular symmetric trans-Sasakian manifold M^3, if $A + B + D$ is not everywhere zero.*

4. Weakly Concircular Ricci Symmetric Three-Dimensional Trans-Sasakian Manifolds

Definition 2. A trans-Sasakian manifold $M^3(\phi, \xi, \eta, g)$ is said to be weakly concircular Ricci symmetric if its concircular Ricci tensor P of type $(0, 2)$ satisfies (1.9).

In view of (1.8), (1.9) yields

$$(\nabla_X S)(Y,Z) - \frac{dr(X)}{3}g(Y,Z) = A(X)[S(Y,Z) - \frac{r}{3}g(Y,Z)] + B(Y)[S(X,Z) - \frac{r}{3}g(X,Z)] + D(Z)[S(X,Y) - \frac{r}{3}g(X,Y)] \tag{34}$$

Setting $X = Y = Z = \xi$ in(4.1), we get the relation (3.2) and hence we can state the following:

Theorem 3. *In a weakly concircular Ricci symmetric trans-Sasakian manifold $M^3(\phi, \xi, \eta, g)$, the relation (3.2) holds.*

Next, substituting X and Y by ξ in (4.1) and using (2.9) and (3.2), we obtain

$$D(Z) = \frac{\{6\xi[\alpha^2 - \beta^2] - (3)\xi[\xi\beta]\}\eta(Z)}{F} - \frac{3Z\xi[\beta] + 3\phi Z[\xi\alpha] + \xi[r]\eta(Z)}{F} + D(\xi)\frac{\{[6(\alpha^2 - \beta^2) - 3\xi\beta - r]\eta(Z) - 3\phi Z[\alpha] - 3Z[\beta]\}}{F} - \left\{\frac{\xi[F]}{F^2}\right\}\left\{[2n(\alpha^2 - \beta^2) - (\xi\beta)]\eta(Z) - Z[\beta] - \phi Z[\alpha] - \frac{r}{3}\eta(Z)\right\} \tag{35}$$

for any vector Z,
If $gradF$ and ξ are orthogonal then by virtue of (3.2) and (3.3) we get

$$\begin{aligned} D(Z) &= \frac{\{6\xi[\alpha^2 - \beta^2] - 3\xi[\xi\beta]\}\eta(Z)}{F} \\ &\quad - \frac{3Z\xi[\beta] + 3\phi Z[\xi\alpha] + \xi[r]\eta(Z)}{F} \\ &\neq 0 \end{aligned}$$

If $gradF$ and ξ are not orthogonal then by virtue of (3.4) we get $D(Z) \neq 0$
Again setting $X = Z = \xi$ in (4.1) and proceeding in a similar manner as above we get

$$B(Y) = \frac{\{6\xi[\alpha^2 - \beta^2] - 3\xi[\xi\beta]\}\eta(Y)}{F} - \frac{3Y\xi[\beta] + 3\phi Y[\xi\alpha] + \xi[r]\eta(Y)}{F} + B(\xi)\frac{\{[6(\alpha^2 - \beta^2) - 3\xi\beta - r]\eta(Y) - 3\phi Y[\alpha] - 3Y[\beta]\}}{F} - \left\{\frac{\xi[F]}{F^2}\right\}\left\{[2(\alpha^2 - \beta^2) - (\xi\beta)]\eta(Y) - Y[\beta] - \phi Y[\alpha] - \frac{r}{3}\eta(Y)\right\} \tag{36}$$

for any vector Y,
If $gradF$ and ξ are orthogonal then by virtue of (3.2) and (3.3) we get

$$\begin{aligned} B(Y) &= \frac{\{6\xi[\alpha^2 - \beta^2] - 3\xi[\xi\beta]\}\eta(Y)}{F} \\ &\quad - \frac{3Y\xi[\beta] + 3\phi Y[\xi\alpha] + \xi[r]\eta(Y)}{F} \\ &\neq 0 \end{aligned}$$

If $gradF$ and ξ are not orthogonal then by virtue of (3.4) we get $B(Y) \neq 0$
Again putting $Y = Z = \xi$ in (4.1) and using (2.11) and (3.2), we get

$$A(X) = \frac{X(F + r)}{F} + \frac{6\alpha\{(X\alpha) - \eta(X)(\xi\alpha) - (\phi X)\beta\}}{F} + \frac{6\beta[(\phi X)[\alpha] + \{X[\beta] - \xi\beta\eta(X)\}] - X[r]}{F} + A(\xi)\frac{\{[6(\alpha^2 - \beta^2) - 3\xi\beta - r]\eta(X) - 3\phi X[\alpha] - 3X[\beta]\}}{F} - \left\{\frac{\xi[F]}{F^2}\right\}\left\{[2(\alpha^2 - \beta^2) - (\xi\beta)]\eta(X) - X[\beta] - \phi X[\alpha] - \frac{r}{3}\eta(X)\right\} \tag{37}$$

for any vector X,
If $gradF$ and ξ are orthogonal then by virtue of (3.2) and (3.3) we get

$$A(X) = \frac{X(F + r)}{F} + \frac{6\alpha\{(X\alpha) - \eta(X)(\xi\alpha) - (\phi X)\beta\}}{F}$$

$$+\frac{6\beta[(\phi X)[\alpha]+\{X[\beta]-\xi\beta\eta(X)\}]-X[r]}{F}$$
$$\neq \quad 0$$

If $gradF$ and ξ are not orthogonal then by virtue of (3.4) we get $A(X)\neq 0$ This leads the following:

Theorem 4. *There exists no weakly Concircular Ricci symmetric trans-Sasakian manifold M^3, if sum of the associated $1-$forms, D, B and A is not everywhere zero.*

References

[1] C. S. Bagewadi, N. S. Basavarajappa, D. G. Prakasha, Venkatesh, On weakly symmetric and special weakly Ricci symmetric Lorentzian $\alpha-$Sasakian manifolds, *Analele Unversitatii de Vest, Timisora, Seria Mathematica-Informatica*, **XLVI**, No. 1 (2008), 3-12.

[2] D. E. Blair, *Contact manifolds in Riemannian geometry*, Lecture Notes in Math. , **509**, Springer-Verlag (1976).

[3] D. E. Blair, J. A. Oubina, Conformal and related changes of metric on the product of two almost contact metric manifolds, *Publ. Math. Debrecen*, **34** (1990), 199-207.

[4] U. C. De, S. Bandyopadhyay, On weakly symmetric Riemannian spaces, *Publ. Math. Debrecen*, **54** (1999), 377-381.

[5] U. C. De, G. C. Ghosh, On weakly concircular Ricci symmetric manifolds, *South East Asian J. Math. and Math. Sci.* , **3**, No. 2 (2005), 9-15.

[6] U. C. De, T. Q. Binh, A. A. Shaikh, On weakly symmetric and weakly Ricci symmetric K-contact manifolds, *Acta Math. Acad. Paedag. Nyigyhaz*, **16** (2000), 65-71.

[7] U. C. De, A. A. Shaikh, S. Biswas, On weakly Ricci symmetric contact metric manifolds, *Tensor N. S.* , **28** (1994), 123-129.

[8] U. C. De, M. M. Tripathi, Ricci tensor in 3-dimensional trans-Sasakian manifolds, *Kyungpook Math. J.* , **43**, No. 2 (2003), 247-225.

[9] J. C. Marrero, The local structure of trans-Sasakian manifolds, *Ann. Mat. Pura Appl.* , **162**, No. 4 (1992), 77-86.

[10] J. S. Kim, R. Prasad, M. M. Tripathi, On generalized Ricci-recurrenttrans-Sasakian manifolds, *J. Korean Math. Soc.* , **39**, No. 6 (2002), 953-961.

[11] J. A. Oubina, New class of almost contact metric manifolds, *Publ. Math. Debrecen*, **32** (1985), 187-193.

[12] C. Özgür, On weakly symmetric Kenmotsu manifolds, *Diff. Geom. Dynam. Syst.* , **8** (2006), 204-209.

[13] A. A. Shaikh, Shyamal Kumar Hui, On weak symmetries of trans-Sasakian manifolds, *Proceedings of the Estonian Academy of Sciences*, **58**, No. 4 (2009), 213-223.

[14] A. A. Shaikh, Hui, On weakly concircular symmetric manifolds, *Ann. Sti. Ale Univ. "Al. I. Cuza"*, Din Iasi, **1** (2009), 167-186.

[15] Shyamal Kumar Hui, On weak concircular symmetries of kenmotsu manifolds, *Acta Universitatis Apulensis*, **26** (2011), 129-136.

[16] L. Tamassy, T. Q. Binh, On weakly symmetric and weakly projective symmetric Rimannian manifolds, *Coll. Math. Soc. J. Bolyai*, **56** (1989), 663-670.

[17] L. Tamassy, T. Q. Binh, On weak symmetries of Einstein and Sasakian manifolds, *Tensor N. S.* , **53** (1993), 140-148.

[18] K. Yano, Concircular geometry I, Concircular transformations, *Proc. Imp. Acad. Tokyo*, **16** (1940), 195-200.