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AN APPROACH TO SOLVE INTUITIONISTIC FUZZY LINEAR PROGRAMMING PROBLEM USING SINGLE STEP ALGORITHM

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Abstract: Obtaining the solution of Intuitionistic Fuzzy Linear Programming Problem (IFLPP) without affecting its originality is an interesting task. There are methods of converting IFLPP into Crisp linear programming problem (CLPP) and then to obtain optimum solution. In this paper we proposed a new algorithm to solve an IFLPP in its original version. We have considered IFLPP with Triangular Intuitionistic Fuzzy Numbers (TIFN). Triangular Intuitionistic Fuzzy Numbers (TIFN) express more abundant and flexible information than Triangular Fuzzy Numbers. Numerical examples are done to show the efficiency of the proposed study.

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1. Introduction

The traditional optimization models were dealt only with crisp and exact in-

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formation. Fuzzy optimization models reflect real life uncertainty. IFLPP is a useful tool for understanding complex problems. Atanassov [1] introduced the concept of Intuitionistic Fuzzy Sets (IFS), which is a generalization of the concept of fuzzy set [1]. An IFS has been applied in many areas including pattern recognition and image processing. The first method for solving Fuzzy Linear Programming was proposed by Zimmermann [21]. Numerous ranking methods have been proposed in literature to rank Intuitionistic Fuzzy numbers [3, 4, 6, 7, 10, 13, 16, 17, 18, 20]. In this paper we have adopted the ranking method using score function. There are many methods based on Triangular Intuitionistic Fuzzy Number to solve multi-attribute decision making [3, 4, 5, 6, 16, 17, 20]. Linear programming method for solving multi-attribute decision making is given by Deng-Feng-Li [5]. Basic arithmetic operations (addition, subtraction and multiplication) of TIFNs is defined by S.Mahapatra and T.K.Roy [11], by considering the six tuple number itself. The other operation division is defined by A.Nagoor gani and K.Ponnalagu [13]. The Solution of IFLPP with TIFNs is obtained by converting it into CLPP by D.Dubey and A.Mehra [7]. In this paper we have developed an algorithm to solve an Intuitionistic Fuzzy Linear Programming. Throughout this paper, the concept of IFLPP and related terminologies are adopted from the aticle [13]. Numerical examples are done to show the efficiency of the study.

2. Preliminaries

In this section, some basic definitions of IFNs and TIFNs are reviewed.

2.1. Intutionistic Fuzzy Number, see [11]

An intutionistic fuzzy number $\tilde{A}^I$ is
i) an intuitionistic fuzzy subset of the real line,
ii) normal, that is, there is some $x_0 \in R$ such that $\mu_{\tilde{A}^I}(x_0) = 1$, $\vartheta_{\tilde{A}^I}(x_0) = 0$,
iii) convex for the membership function $\mu_{\tilde{A}^I}(x)$, that is, $\mu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \geq min(\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2))$, for every $x_1, x_2 \in R$, $\lambda \in [0, 1]$,
iv) concave for the membership function $\vartheta_{\tilde{A}^I}(x)$, that is,$\vartheta_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \leq max(\vartheta_{\tilde{A}^I}(x_1), \vartheta_{\tilde{A}^I}(x_2))$, for every $x_1, x_2 \in R$, $\lambda \in [0, 1]$.

2.2. (α, β)-Cuts, see [11]

A set of (α, β)-cut generated by IFS $\tilde{A}^I$, where $\alpha, \beta \in [0,1]$ are fixed numbers such that $\alpha + \beta \leq 1$ is defined as $\tilde{A}^I_{\alpha,\beta} = \{< x, \mu_{\tilde{A}^I}(x), \vartheta_{\tilde{A}^I}(x) >: x \in X, \mu_{\tilde{A}^I}(x) \leq \alpha, \vartheta_{\tilde{A}^I}(x) \leq \beta, \alpha, \beta \in [0,1]\}$.
(α, β)-level interval or (α, β)-cut denoted by $\tilde{A}^I_{\alpha,\beta}$ is defined as the crisp set of elements of x which belong to $\tilde{A}^I$ at least to the degree α and which does belong to $\tilde{A}^I$ at most to the degree β.

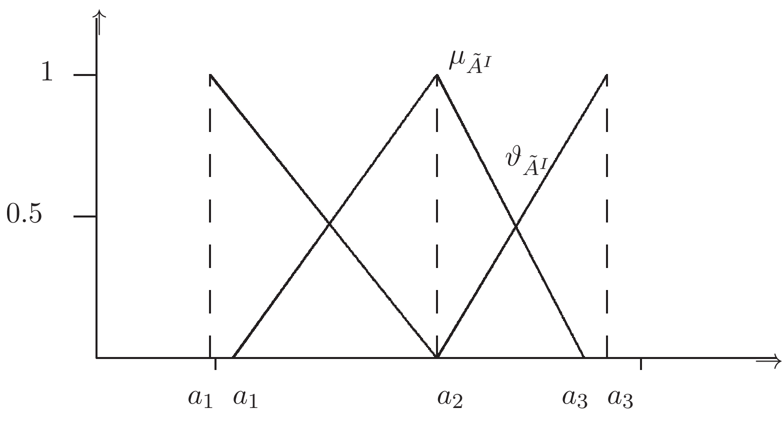
2.3. Triangular Intuitionistic Fuzzy Number, see [11]

A Triangular intuitionistic fuzzy number(TIFN) $\tilde{A}^I$ is an Intuitionistic fuzzy set in R with the following membership function $\mu_{\tilde{A}^I}(x)$ and non-membership function $\vartheta_{\tilde{A}^I}(x)$

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & if\ a_1 \leq x \leq a_2 \\ \frac{x-a_s}{a_2-a_s}, & if\ a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \quad \text{and}$$

$$\vartheta_{\tilde{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1}, & if\ a_1 \leq x \leq a_2 \\ \frac{x-a_2}{a_s-a_2}, & if\ a_2 \leq x \leq a_3 \\ 1, & \text{otherwise,} \end{cases}$$

where $a_1^{'} \leq a_1 \leq a_2 \leq a_3 \leq a_3^{'}$ and $\mu_{\tilde{A}^I}(x) + \vartheta_{\tilde{A}^I}(x) \leq 1$, or $\mu_{\tilde{A}^I}(x) = \vartheta_{\tilde{A}^I}(x)$, for all $x \in R$. This TIFN is denoted by $\tilde{A}^I = (a_1, a_2, a_3; a_1{}', a_2, a_3{}') = \{(a_1, a_2, a_3); (a_1{}', a_2, a_3{}')\}$.



Membership and non-membership functions of TIFN

2.4. Arithmetic Operations, see [11] and [13]

Arithmetic operations of Triangular Intuitionistic Fuzzy Number based on (α, β)-cuts method:
i) If $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ and $\tilde{B}^I = \{(b_1, b_2, b_3);\ (b_1',\ b_2,\ b_3')\}$ are two TIFNs , then their sum
$\tilde{A}^I + \tilde{B}^I = \{(a_1+b_1, a_2+b_2, a_3+b_3); (a_1'+b_1', a_2+b_2, a_3'+b_3')\}$ is also a TIFN.
ii) If $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ and $\tilde{B}^I = \{(b_1,\ b_2,\ b_3);\ (b_1',\ b_2,\ b_3')\}$ are two TIFNs , then
$\tilde{A}^I - \tilde{B}^I = \{(a_1 - b_3, a_2 - b_2, a_3 - b_1); (a_1' - b_3', a_2 - b_2, a_3' - b_1')\}$ is also a TIFN.
iii) If $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ and $\tilde{B}^I = \{(b_1, b_2, b_3);\ (b_1',\ b_2,\ b_3')\}$ are two TIFNs , then
$\tilde{A}^I \times \tilde{B}^I = \{(a_1 b_1, a_2 b_2, a_3 b_3); (a_1' b_1', a_2 b_2, a_3' b_3')\}$ is also a TIFN.
iv) If TIFN $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ and $y = ka$ (with $k > 0$) then
$\tilde{y}^I = k\tilde{A}^I = \{(ka_1, ka_2, ka_3); (ka_1', ka_2, ka_3')\}$ is also a TIFN.
v) If TIFN $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ and $y = ka$ (with $k < 0$) then
$\tilde{y}^I = k\tilde{A}^I = \{(ka_3, ka_2, ka_1); (ka_3', ka_2, ka_1')\}$ is also a TIFN.
vi) If $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ and $\tilde{B}^I = \{(b_1, b_2, b_3);\ (b_1',\ b_2,\ b_3')\}$ are two positive TIFNs , then
$\frac{\tilde{A}^I}{\tilde{B}^I}$ is also a TIFN, where $\frac{\tilde{A}^I}{\tilde{B}^I} = \left\{ \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1} \right); \left(\frac{a_1'}{b_3'}, \frac{a_2}{b_2}, \frac{a_3'}{b_1'} \right) \right\}$.

2.5. Intuitionistic Fuzzy Linear Programming, see [7]

Linear Programming with Triangular Intuitionistic Fuzzy Variables is defined as $(IFLP) max \tilde{Z}^I = \sum_{j=1}^{n} \tilde{c}_j^I \tilde{x}_j^I$ subject to

$$\sum_{j=1}^{n} \tilde{a}_{ij}^I \tilde{x}_j^I \le \tilde{b}_j^I$$
$$\tilde{x}_j^I \ge 0 \tag{1}$$

$i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n,$ where $\tilde{A}^I = (\tilde{a}_{ij}^I),\ \tilde{c}^I,\ \tilde{b}^I,\ \tilde{x}^I$ are $(m \times n)$, $(1 \times n)$, $(m \times 1)$, $(n \times 1)$ Intuitionistic fuzzy matrices consisting of Triangular Intuitionistic Fuzzy Numbers (TIFN).

2.6. Intuitionistic Fuzzy Optimum Feasible Solution, see [12]

Let X be the set of all intuitionistic fuzzy feasible solutions of equation (1). An intuitionistic fuzzy feasible solution $\tilde{x}_0^I \in X$ is said to be an Intuitionistic fuzzy optimum feasible solution to equation (1), if $\tilde{c}^I\tilde{x}_0^I \geq \tilde{c}^I\tilde{x}^I$ for all $\tilde{x}^I \in X$, where $\tilde{c}^I = (\tilde{c}_1^I, \tilde{c}_2^I, \ldots, \tilde{c}_n^I)$, and $\tilde{c}^I\tilde{x}^I = \tilde{c}_1^I\tilde{x}_1^I + \tilde{c}_2^I\tilde{x}_2^I + \ldots + \tilde{c}_n^I\tilde{x}_n^I$.

2.7. Score Function, see [13]

Let $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1^{'}, a_2, a_3{'})\}$ be a TIFN, then we define a Score function for membership and non-membership values respectively as $S(\tilde{A}^{I\alpha}) = \frac{a_1+2a_2+a_3}{4}$ and $S(\tilde{A}^{I\beta}) = \frac{a_1^{'}+2a_2+a_3^{'}}{4}$.

2.8. Ranking Using Score Function, see[13]

Let $\tilde{A}^I = \{(a_1, a_2, a_3);\ (a_1^{'}, a_2, a_3{'})\}$ and $\tilde{B}^I = \{(b_1, b_2, b_3);(b_1^{'},\ b_2,\ b_3{'})\}$ be two TIFNs and $S(\tilde{A}^{I\alpha})$, $S(\tilde{A}^{I\beta})$ & $S(\tilde{B}^{I\alpha})$, $S(\tilde{B}^{I\beta})$ be the scores of $\tilde{A}^I$ and $\tilde{B}^I$ respectively.
i) If $S(\tilde{A}^{I\alpha}) \leq S(\tilde{B}^{I\alpha})$ and $S(\tilde{A}^{I\beta}) \leq S(\tilde{B}^{I\beta})$, then $\tilde{A}^I < \tilde{B}^I$.
ii) i) If $S(\tilde{A}^{I\alpha}) \geq S(\tilde{B}^{I\alpha})$ and $S(\tilde{A}^{I\beta}) \geq S(\tilde{B}^{I\beta})$, then $\tilde{A}^I > \tilde{B}^I$.
iii) i) If $S(\tilde{A}^{I\alpha}) = S(\tilde{B}^{I\alpha})$ and $S(\tilde{A}^{I\beta}) = S(\tilde{B}^{I\beta})$, then $\tilde{A}^I = \tilde{B}^I$.

3. Proposed IFLPP with Minimization of Objective Function

3.1. Intuitionistic Fuzzy Linear Programming (Objective Function is to Minimize)

Linear Programming with Triangular Intuitionistic Fuzzy Variables is defined as $(IFLP) min\tilde{Z}^I = \sum_{j=1}^{n} \tilde{c}_j^I\tilde{x}_j^I$ subject to

$$\sum_{j=1}^{n} \tilde{a}_{ij}^I\tilde{x}_j^I \geq \tilde{b}_i^I$$
$$\tilde{x}_j^I \geq 0 \tag{2}$$

$i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n,$ where $\tilde{A}^I = (\tilde{a}_{ij}^I)$, $\tilde{c}^I$, $\tilde{b}^I$, $\tilde{x}^I$ are $(m \times n)$, $(1 \times n)$, $(m \times 1)$, $(n \times 1)$ Intuitionistic fuzzy matrices consisting of Triangular Intuitionistic Fuzzy Numbers (TIFN).

3.2. Standard Form

Suppose the objective function is of minimization type, then the standard form by introducing Slack variables, Surplus variables & Artificial Variables in the traditional way is given by

$$(IFLP)min\tilde{Z}^I = -max(-\tilde{Z}^I) = -\sum_{j=1}^{n} \tilde{c}_j^I \tilde{x}_j^I + \tilde{0}^I \sum_{j=n+1}^{n+m} \tilde{x}_j^I - \tilde{1}^I M \sum_{i=1}^{m} \tilde{A}_i^I \tag{3}$$

subject to

$$\begin{aligned}
\tilde{a}_{11}^I \tilde{x}_1^I + \tilde{a}_{12}^I \tilde{x}_2^I + \ldots + \tilde{a}_{1n}^I \tilde{x}_n^I - \tilde{1}^I \tilde{x}_{n+1}^I + \tilde{1}^I \tilde{A}_1^I &= \tilde{b}_1^I \\
\tilde{a}_{21}^I \tilde{x}_1^I + \tilde{a}_{22}^I \tilde{x}_2^I + \ldots + \tilde{a}_{2n}^I \tilde{x}_n^I - \tilde{1}^I \tilde{x}_{n+2}^I + \tilde{1}^I \tilde{A}_2^I &= \tilde{b}_2^I \\
&\ldots \\
\tilde{a}_{m1}^I \tilde{x}_1^I + \tilde{a}_{m2}^I \tilde{x}_2^I + \ldots + \tilde{a}_{mn}^I \tilde{x}_n^I - \tilde{1}^I \tilde{x}_{n+m}^I + \tilde{1}^I \tilde{A}_m^I &= \tilde{b}_m^I,
\end{aligned} \tag{4}$$

where

$$\tilde{x}_1^I, \tilde{x}_2^I, \ldots, \tilde{x}_n^I, \tilde{x}_{n+1}^I, \ldots, \tilde{x}_{n+m}^I \geq 0 \tag{5}$$

4. Proposed Method: A Single Step of the Simplex Method

4.1. Algorithm

For a standard IFLPP as defined in equation (1) or as in equation (2) of this paper, the following algorithm is proposed.

1. Given B, N, $X_B = B^{-1}b \geq 0$; $X_N = 0$;
2. Solve $\lambda = (B^T)^{-1}C_B$;
3. Compute $S_N = C_N - A_N^T \lambda$;
4. If $S_N \leq 0$ stop; (Optimal solution is obtained)
5. Else select $max_{S_q>0}(S_q)$, Now q is the entering index.
6. Solve $Bd = A_q$ fro d;
7. If $d < 0$; stop. (Problem is unbounded)
8. Else Calculate $min_{i/d_i>0}(\frac{X_{B_i}}{d_i})$, The variable corresponding to the minimum value ,say x_p leaves the basis.
9. Change B by adding q and removing p and vice-versa in N.

10. Perform Row operations to make d as pivot column.

11. Do the same row operations in B^{-1} to find new B^{-1}.

12. Go to step 1.

Theorem 1. *The single step method terminates at an Intuitionistic Fuzzy Optimum Feasible solution , provided the Intuitionistic Fuzzy Linear Programming Problem is non - degenerate and bounded.*

Proof. The single step method cannot visit the same Intuitionistic Fuzzy basic feasible solution X_B^* at two different iterations, because it attains a strict increase at each iteration. Since the number of possible bases is finite, and each basis defines a single Intuitionistic Fuzzy basic feasible solution, there are only a finite number of Intuitionistic Fuzzy basic feasible solutions. Hence the number of iterations is finite. Moreover, since the method is always able to take a step away from a Intuitionistic Fuzzy non-optimal basic feasible solution and since the problem is not unbounded, the method must terminate at a Intuitionistic Fuzzy Optimum feasible solution. □

Lemma 2. *If the direction $d < 0$ then the problem is unbounded.*

Proof. The condition $d < 0$ indicates that d is in the direction with a strictly increasing objective value. Hence the objective value $C^T X_B$ is unbounded above, since $C^T d > 0$ and so the problem is unbounded. □

5. Numerical Illustration

5.1 Solve the following IFLPP to maximize the objective function.
Maximize $\tilde{Z}^I = \tilde{2}^I \tilde{x}_1^I + \tilde{1}^I \tilde{x}_2^I$ subject to $\tilde{3}^I \tilde{x}_1^I + \tilde{4}^I \tilde{x}_2^I \leq \tilde{6}^I$ and $\tilde{6}^I \tilde{x}_1^I + \tilde{1}^I \tilde{x}_2^I \leq \tilde{3}^I$, $\tilde{x}_1^I, \tilde{x}_2^I \geq 0$, where
$\tilde{c}_1^I = \tilde{2}^I = \{(1.8, 2, 2.2); (1.5, 2, 2.4)\}$,
$\tilde{c}_2^I = \tilde{1}^I = \{(0.9, 1, 1.1); (0.7, 1, 1.3)\}$,
$\tilde{a}_{11}^I = \tilde{3}^I = \{(2.7, 3, 3.2); (2.5, 3, 3.3)\}$,
$\tilde{a}_{12}^I = \tilde{4}^I = \{(3.5, 4, 4.4); (3.3, 4, 4.6)\}$,
$\tilde{a}_{21}^I = \tilde{6}^I = \{(5.5, 6, 6.2); (5.2, 6, 6.5)\}$,
$\tilde{a}_{22}^I = \tilde{1}^I = \{(0.8, 1, 1.4); (0.7, 1, 1.5)\}$,
$\tilde{b}_1^I = \tilde{6}^I = \{(5.3, 6, 6.3); (5, 6, 6.5)\}$,
$\tilde{b}_2^I = \tilde{3}^I = \{(2.6, 3, 3.3); (2.4, 3, 3.7)\}$.

Solution.
we can rewrite the problem in the following standard form.
Maximize $\tilde{Z}^I = \tilde{2}^I\tilde{x}_1^I + \tilde{1}^I\tilde{x}_2^I + \tilde{0}^I\tilde{x}_3^I + \tilde{0}^I\tilde{x}_4^I$ subject to $\tilde{3}^I\tilde{x}_1^I + \tilde{4}^I\tilde{x}_2^I + \tilde{1}^I\tilde{x}_3^I = \tilde{6}^I$ and $\tilde{6}^I\tilde{x}_1^I + \tilde{1}^I\tilde{x}_2^I + \tilde{1}^I\tilde{x}_4^I = \tilde{3}^I$, $\tilde{x}_1^I, \tilde{x}_2^I, \tilde{x}_3^I, \tilde{x}_4^I \geq 0$.
Here the co-efficient of $\tilde{x}_3^I, \tilde{x}_4^I$, are given by $\tilde{1}^I = \{(1,1,1);(1,1,1)\}$ and $\tilde{0}^I = \{(0,0,0);(0,0,0)\}$. Using the arithmetic operations and Ranking based on Scoring function, we have the following iterations.

Initial Iteration:

1. Let $B = \{3,4\}$ and $N = \{1,2\}$.
   Then $B = \begin{pmatrix} \tilde{1}^I & \tilde{0}^I \\ \tilde{0}^I & \tilde{1}^I \end{pmatrix}$ and $B^{-1} = \begin{pmatrix} \tilde{1}^I & \tilde{0}^I \\ \tilde{0}^I & \tilde{1}^I \end{pmatrix}$

2. $X_B - B^{-1}b = \begin{pmatrix} \tilde{1}^I & \tilde{0}^I \\ \tilde{0}^I & \tilde{1}^I \end{pmatrix} \begin{pmatrix} \tilde{6}^I \\ \tilde{3}^I \end{pmatrix} = \begin{pmatrix} \tilde{6}^I \\ \tilde{3}^I \end{pmatrix}$

3. $\lambda = (B^T)^{-1}C_B = \begin{pmatrix} \tilde{1}^I & \tilde{0}^I \\ \tilde{0}^I & \tilde{1}^I \end{pmatrix} \begin{pmatrix} \tilde{0}^I \\ \tilde{0}^I \end{pmatrix} = \begin{pmatrix} \tilde{0}^I \\ \tilde{0}^I \end{pmatrix}$

4. Compute $S_N = C_N - A_N^T\lambda$ corresponds to the set N.
   i) $S_1 = C_1 - A_1^T\lambda = \tilde{2}^I - (\tilde{3}^I \ \ \tilde{6}^I) \begin{pmatrix} \tilde{0}^I \\ \tilde{0}^I \end{pmatrix} = \tilde{2}^I$,
   ii) $S_2 = C_2 - A_2^T\lambda = \tilde{1}^I - (\tilde{4}^I \ \ \tilde{1}^I) \begin{pmatrix} \tilde{0}^I \\ \tilde{0}^I \end{pmatrix} = \tilde{1}^I$.

5. $max_{S_q>0}(S_q) = max(\tilde{2}^I, \tilde{1}^I) = \tilde{2}^I$ (By ranking procedure).

6. Since $\tilde{2}^I$ corresponds to the index $q = 1$, $\tilde{x}_1^I$ enters the basis.

7. Solve $Bd = A_q$ for d;
   that is, $d = B^{-1}A_1 = \begin{pmatrix} \tilde{1}^I & \tilde{0}^I \\ \tilde{0}^I & \tilde{1}^I \end{pmatrix} \begin{pmatrix} \tilde{3}^I \\ \tilde{6}^I \end{pmatrix} = \begin{pmatrix} \tilde{3}^I \\ \tilde{6}^I \end{pmatrix}$

8. Calculate $min_{i/d_i>0}\left(\frac{X_{B_i}}{d_i}\right) = \begin{pmatrix} \frac{\tilde{6}^I}{\tilde{3}^I} & \frac{\tilde{3}^I}{\tilde{6}^I} \end{pmatrix} = min(\tilde{2}^I, \tilde{0.5}^I) = \tilde{0.5}^I$. Here $\frac{\tilde{6}^I}{\tilde{3}^I} = \frac{\{(5.3,6,6.3);(5,6,6.5)\}}{\{(2.7,3,3.2);(2.5,3,3.3)\}} = \{(1.66,2,2.33);(1.52,2,2.6)\} = \tilde{2}^I$, $\frac{\tilde{3}^I}{\tilde{6}^I} = \frac{\{(2.6,3,3.3);(2.4,3,3.7)\}}{\{(5.5,6,6.2);(5.2,6,6.5)\}} = \{(0.42,0.5,0.6);(0.37,0.5,0.71)\} = \tilde{1}^I$. Since the minimum value corresponds to the index $p = 4$, $\tilde{x}_4^I$ leaves the basis.

9. Reduce d to $\begin{pmatrix} \tilde{0}^I \\ \tilde{1}^I \end{pmatrix}$ by performing the following row operations.
i) $R_1 = R_1 - \frac{\tilde{3}^I}{\tilde{6}^I} R_2$ and
ii) $R_2 = \frac{R_2}{\tilde{6}^I}$.

First Iteration:

1. Let $B = \{3, 1\}$ and $N = \{4, 2\}$.
Then $B^{-1} = \begin{pmatrix} \tilde{1}^I & -0.\tilde{5}^I \\ \tilde{0}^I & 0.\tilde{17}^I \end{pmatrix}$,
where $-0.\tilde{5}^I = \{(0.44, 0.5, 0.58); (0.38, 0.5, 0.63)\}$ and
$0.\tilde{17}^I = \{(0.16, 0.17, 0.18); (0.15, 0.17, 0.18)\}$.

2. $X_B = B^{-1}b = \begin{pmatrix} \tilde{1}^I & -0.\tilde{5}^I \\ \tilde{0}^I & 0.\tilde{17}^I \end{pmatrix} \begin{pmatrix} \tilde{6}^I \\ \tilde{3}^I \end{pmatrix} = \begin{pmatrix} 4.\tilde{5}^I \\ 0.\tilde{51}^I \end{pmatrix}$,
where $4.\tilde{5}^I = \{(3.4, 4.5, 5.16); (2.67, 4.5, 5.59)\}$ and
$0.\tilde{51}^I = \{(0.42, 0.51, 0.59); (0.36, 0.51, 0.7)\}$.

3. $\lambda = (B^T)^{-1} C_B = \begin{pmatrix} \tilde{1}^I & \tilde{0}^I \\ -0.\tilde{5}^I & 0.\tilde{17}^I \end{pmatrix} \begin{pmatrix} \tilde{0}^I \\ \tilde{2}^I \end{pmatrix} = \begin{pmatrix} \tilde{0}^I \\ 0.\tilde{34}^I \end{pmatrix}$,
where $0.\tilde{34}^I = \{(0.29, 0.34, 0.4); (0.23, 0.34, 0.46)\}$.

4. Compute $S_N = C_N - A_N^T \lambda$;
i) $S_4 = C_4 - A_4^T \lambda = \tilde{0}^I - (\tilde{0}^I \ \ \tilde{1}^I) \begin{pmatrix} \tilde{0}^I \\ 0.\tilde{34}^I \end{pmatrix} = -0.\tilde{34}^I$,
ii) $S_2 = C_2 - A_2^T \lambda = \tilde{1}^I - (\tilde{4}^I \ \ \tilde{1}^I) \begin{pmatrix} \tilde{0}^I \\ 0.\tilde{34}^I \end{pmatrix} = 0.\tilde{66}^I$,
where $0.\tilde{66}^I = \{(0.6, 0.66, 0.71); (0.54, 0.66, 0.77)\}$.

5. $max_{S_q>0}(S_q) = 0.\tilde{66}^I$.

6. Since $0.\tilde{66}^I$ corresponds to the index $q = 2$, $\tilde{x}_2^I$ enters the basis.

7. Solve $Bd = A_q$ for d;
that is, $d = B^{-1}A_2 = \begin{pmatrix} \tilde{1}^I & -0.\tilde{5}^I \\ \tilde{0}^I & 0.\tilde{17}^I \end{pmatrix} \begin{pmatrix} \tilde{4}^I \\ \tilde{1}^I \end{pmatrix} = \begin{pmatrix} 3.\tilde{5}^I \\ 0.\tilde{17}^I \end{pmatrix}$,
where $3.\tilde{5}^I = \{(2.69, 3.5, 4.05); (2.35, 3.5, 4.33)\}$ and
$0.\tilde{17}^I = \{(0.13, 0.17, 0.25); (0.11, 0.17, 0.29)\}$.

8. Calculate $min_{i/d_i>0}\left(\frac{X_{B_i}}{d_i}\right) = \left(\frac{\tilde{4.5}^I}{\tilde{3.5}^I} \quad \frac{\tilde{0.51}^I}{\tilde{0.17}^I}\right)$
$= min(\tilde{1.29}^I, \tilde{3}^I) = \tilde{1.29}^I$, where
$\tilde{1.29}^I = \{(0.84, 1.29, 1.92); (0.62, 1.29, 2.38)\}$ and
$\tilde{3}^I = \{(1.68, 3, 4.54); (1.24, 3, 6, 36)\}$.
Since the minimum value corresponds to the index $p = 3$, $\tilde{x}_3^I$ leaves the basis.

9. Reduce d to $\begin{pmatrix}\tilde{1}^I\\ \tilde{0}^I\end{pmatrix}$ by performing the following row operations.
iii) $R_1 = \frac{R_1}{\tilde{3.5}^I}$ and
ii) $R_2 = R_2 - \frac{\tilde{0.17}^I}{\tilde{3.5}^I}R_1$

Second Iteration:

1. Let $B = \{2, 1\}$ and $N = \{4, 3\}$.
Then $B^{-1} = \begin{pmatrix}\tilde{0.29}^I & -\tilde{0.15}^I\\ \tilde{0.05}^I & \tilde{0.2}^I\end{pmatrix}$,
where $\tilde{0.29}^I = \{(0.25, 0.29, 0.37); (0.23, 0.29, 0.43)\}$
$\tilde{0.15}^I = \{(0.11, 0.15, 0.21); (0.09, 0.15, 0.27)\}$
$\tilde{0.05}^I = \{(0.03, 0.05, 0.09); (0.03, 0.05, 0.12)\}$ and
$\tilde{0.2}^I = \{(0.17, 0.2, 0.23); (0.16, 0.2, 0.27)\}$.

2. $X_B = B^{-1}b = \begin{pmatrix}\tilde{0.29}^I & -\tilde{0.15}^I\\ -\tilde{0.05}^I & \tilde{0.2}^I\end{pmatrix}\begin{pmatrix}\tilde{6}^I\\ \tilde{3}^I\end{pmatrix} = \begin{pmatrix}\tilde{1.29}^I\\ \tilde{0.3}^I\end{pmatrix}$,
where $\tilde{1.29}^I = \{(0.64, 1.29, 2.04); (0.15, 1, 29, 2.58)\}$ and
$\tilde{0.3}^I = \{(-0.13, 0.3, 0.6); (-0.4, 0.3, 0.85)\}$.

3. $\lambda = (B^T)^{-1}C_B = \begin{pmatrix}\tilde{0.29}^I & -\tilde{0.05}^I\\ -\tilde{0.15}^I & \tilde{0.2}^I\end{pmatrix}\begin{pmatrix}\tilde{1}^I\\ \tilde{2}^I\end{pmatrix} = \begin{pmatrix}\tilde{0.39}^I\\ \tilde{0.25}^I\end{pmatrix}$,
where $\tilde{0.39}^I = \{(0.28, 0.39, 0.6); (0.21, 0.39, 0.85)\}$.

4. Compute $S_N = C_N - A_N^T\lambda$;
i) $S_4 = C_4 - A_4^T\lambda = \tilde{0}^I - (\tilde{0}^I \ \ \tilde{1}^I)\begin{pmatrix}\tilde{0.39}^I\\ \tilde{0.25}^I\end{pmatrix} = -\tilde{0.25}^I$;
ii) $S_2 = C_2 - A_2^T\lambda = \tilde{1}^I - (\tilde{1}^I \ \ \tilde{0}^I)\begin{pmatrix}\tilde{0.39}^I\\ \tilde{0.25}^I\end{pmatrix} = -\tilde{0.39}^I$;

5. Since all $S_N \leq 0$ we teminate; that is, Optimal solution is obtained.

The objective value is

$Max\ \tilde{Z}^I = \tilde{2}^I \tilde{x}_1^I + \tilde{1}^I \tilde{x}_2^I = 1.\tilde{89}^I$ when $X_B = \begin{pmatrix} \tilde{x}_2^I \\ \tilde{x}_1^I \end{pmatrix} = \begin{pmatrix} 1.\tilde{29}^I \\ 0.\tilde{3}^I \end{pmatrix}$,

where $1.\tilde{89}^I = \{(0.35, 1.89, 3.56); (-0.49, 1.89, 5.39)\}$.
$1.\tilde{29}^I = \{(0.64, 1.29, 2.04); (0.15, 1.29, 2.58)\}$.
$0.\tilde{3}^I = \{(-0.13, 0.3, 0.6); (-0.4, 0.3, 0.85)\}$.

5.2 Solve the following IFLPP to minimize the objective function.
Minimize $\tilde{Z}^I = \tilde{1}^I \tilde{x}_1^I + \tilde{2}^I \tilde{x}_2^I$ subject to $\tilde{2}^I \tilde{x}_1^I + \tilde{5}^I \tilde{x}_2^I \geq \tilde{6}^I$ and $\tilde{1}^I \tilde{x}_1^I + \tilde{1}^I \tilde{x}_2^I \geq \tilde{2}^I$, $\tilde{x}_1^I, \tilde{x}_2^I \geq 0$, where
$\tilde{c}_1^I = \tilde{1}^I = \{(0.9, 1, 1.2); (0.7, 1, 1.3)\}$,
$\tilde{c}_2^I = \tilde{2}^I = \{(1.8, 2, 2.3); (1.5, 2, 2.4)\}$,
$\tilde{a}_{11}^I = \tilde{2}^I = \{(1.9, 2, 2.2); (1.7, 2, 2.4)\}$,
$\tilde{a}_{12}^I = \tilde{5}^I = \{(4.6, 5, 5.2); (4.5, 5, 5.4)\}$,
$\tilde{a}_{21}^I = \tilde{1}^I = \{(0.9, 1, 1.2); (0.8, 1, 1.3)\}$,
$\tilde{a}_{22}^I = \tilde{1}^I = \{(0.8, 1, 1.1); (0.6, 1, 1.2)\}$,
$\tilde{b}_1^I = \tilde{6}^I = \{(5.7, 6, 6.3); (5.5, 6, 6.4)\}$,
$\tilde{b}_2^I = \tilde{2}^I = \{(1.8, 2, 2.3); (1.6, 2, 2.4)\}$.

Solution:
Rewriting the problem in to standard form.
Minimize $\tilde{Z}^I = -maximize(-\tilde{Z}^I) = -maximize(\tilde{w}^I)$, where
$maximize(\tilde{w}^I) = \tilde{1}^I \tilde{x}_1^I + \tilde{2}^I \tilde{x}_2^I + \tilde{0}^I \tilde{x}_3^I + \tilde{0}^I \tilde{x}_4^I - M\tilde{A}_1^I - M\tilde{A}_2^I$ subject to $\tilde{2}^I \tilde{x}_1^I + \tilde{5}^I \tilde{x}_2^I - \tilde{1}^I \tilde{x}_3^I + \tilde{1}^I \tilde{A}_1^I = \tilde{6}^I$ and $\tilde{1}^I \tilde{x}_1^I + \tilde{1}^I \tilde{x}_2^I - \tilde{1}^I \tilde{x}_4^I + \tilde{1}^I \tilde{A}_2^I = \tilde{2}^I$, $\tilde{x}_1^I, \tilde{x}_2^I, \tilde{x}_3^I, \tilde{x}_4^I, \tilde{A}_1^I, \tilde{A}_2^I \geq 0$.
Here the co-efficients of $\tilde{x}_3^I, \tilde{x}_4^I, \tilde{A}_1^I, \tilde{A}_2^I$ in the constraints are given by $\tilde{1}^I = \{(1, 1, 1); (1, 1, 1)\}$ and $\tilde{0}^I = \{(0, 0, 0); (0, 0, 0)\}$.
Using the proposed algorithm, we get the optimum solution for this problem in the second Iteration.
Optimum Solution is $Minimize\ \tilde{Z}^I = 2.\tilde{64}^I$, $\tilde{x}_1^I = 1.\tilde{36}^I$, $\tilde{x}_2^I = 0.\tilde{64}^I$, where
$2.\tilde{64}^I = \{(-4.78, 2.64, 12.5); (-14.37, 2.64, 30.77)\}$
$1.\tilde{36}^I = \{(-2.19, 1.36, 5.3); (-7.84, 1.36, 12.79)\}$.
$0.\tilde{64}^I = \{(-1.56, 0.64, 2.67); (-5.92, 0.64, 5.89)\}$

5.3 Solve the following IFLPP to maximize the objective function.
Maximize $\tilde{Z}^I = \tilde{2}^I \tilde{x}_1^I + \tilde{3}^I \tilde{x}_2^I$ subject to $\tilde{1}^I \tilde{x}_2^I - \tilde{1}^I \tilde{x}_1^I \geq \tilde{0}^I$, and $\tilde{1}^I \tilde{x}_1^I \leq \tilde{4}^I$, $\tilde{x}_1^I, \tilde{x}_2^I \geq 0$, where

$\tilde{c}_1^I = \tilde{2}^I = \{(1.9, 2, 2.2); (1.8, 2, 2.4)\}$,
$\tilde{c}_2^I = \tilde{3}^I = \{(2.8, 3, 3.1); (2.7, 3, 3.3)\}$,
$\tilde{a}_{11}^I = \tilde{-1}^I = -\{(0.8, 1, 1.1); (0.7, 1, 1.2)\}$,
$\tilde{a}_{12}^I = \tilde{1}^I = \{(0.9, 1, 1.1); (0.7, 1, 1.3)\}$,
$\tilde{a}_{21}^I = \tilde{1}^I = \{(0.8, 1, 1.3); (0.7, 1, 1.4)\}$,
$\tilde{a}_{22}^I = \tilde{0}^I = \{(0, 0, 0); (0, 0, 0)\}$,
$\tilde{b}_1^I = \tilde{0}^I = \{(0, 0, 0); (0, 0, 0)\}$,
$\tilde{b}_2^I = \tilde{4}^I = \{(3.8, 4, 4.1); (3.5, 4, 4.3)\}$.

Solution:
After rewriting the first constraint as $\tilde{1}^I\tilde{x}_1^I - \tilde{1}^I\tilde{x}_2^I \leq \tilde{0}^I$ we can change the problem in to standard form as follows.
Maximize $\tilde{Z}^I = \tilde{2}^I\tilde{x}_1^I + \tilde{3}^I\tilde{x}_2^I + \tilde{0}^I\tilde{x}_3^I + \tilde{0}^I\tilde{x}_4^I$ subject to $\tilde{1}^I\tilde{x}_1^I - \tilde{1}^I\tilde{x}_2^I + \tilde{1}^I\tilde{x}_3^I = \tilde{0}^I$ and $\tilde{1}^I\tilde{x}_1^I + \tilde{0}^I\tilde{x}_2^I + \tilde{1}^I\tilde{x}_4^I = \tilde{4}^I$, $\tilde{x}_1^I, \tilde{x}_2^I, \tilde{x}_3^I, \tilde{x}_4^I \geq 0$.
Here the co-efficients of $\tilde{x}_3^I, \tilde{x}_4^I$ in the constraints are given by
$\tilde{1}^I = \{(1, 1, 1); (1, 1, 1)\}$ and $\tilde{0}^I = \{(0, 0, 0); (0, 0, 0)\}$.
Using the proposed algorithm, we get the problem has an unbounded solution in the initial iteration.

6. Conclusion

In [13], we have considered the problems of maximizing the objective function alone and also for each iteration simplex tables are constructed. The procedure discussed in this paper gives the simplest approach when compared with the traditional simplex method. Actually, we have given an algorithm to solve intutionistic fuzzy optimization problems in a different way. The algorithm has been illustrated through some examples.

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