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QUALITATIVE PROPERTIES OF SELF-SIMILAR SOLUTIONS OF A NONHOMOGENEOUS EQUATION

Badia Hamri[1], Abdelilah Gmira[2], Youssef Jabri[3 §]

[1,2]Department of Mathematics
Faculty of Science
University Abdelmalek Essaâdi
B.P. 2121, Tétouan, MOROCCO

[3]Department of Mathematics
National School of Applied Sciences
University Mohammed I
B.P. 669, Oujda, MOROCCO

Abstract: In [2], we have proved the existence and uniqueness of self-similar radially symmetric solutions for the nonhomogeneous equation

$$u_t = div\left(|\nabla u|^{p-2}\nabla u\right) + |x|^l\,|u|^{q-1}\,u, \quad (t,x) \in (0,+\infty)\times\mathbb{R}^N.$$

where $p > 2$, $q \geq 1$, $N \geq 1$, $-p < l$, $-N < l$.

We have studied the asymptotic behaviour of such a solution in [3]. Our aim, in this paper, is to classify the solutions (positiveness, compact support,...) according to the parameters p, q and l.

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1. Introduction

In [2], we have investigated the existence and uniqueness of self-similar radially

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§Correspondence author

symmetric solutions of the parabolic nonhomogeneous equation

$$w_t = \Delta_p w + |x|^l |w|^{q-1} w, \quad \text{in } \mathbb{R}^{\mathbb{N}} \times (0, +\infty) \tag{1.1}$$

where $p > 2$, $q \geq 1$, $N \geq 1$, $-p < l < 0$, $-N < l < 0$.

Self-similar solutions have the form

$$w(x,t) = t^{-\alpha} U(t^{-\beta} x), \text{ in } \mathbb{R}^{\mathbb{N}} \times (0, +\infty), \tag{1.2}$$

And the profile U satisfies the nonhomogeneous elliptic equation

$$\Delta_p U + \beta x \nabla U + \alpha U + |x|^l |U|^{q-1} U = 0, \text{ for } x \in \mathbb{R}^{\mathbb{N}} \tag{1.3}$$

where the scaling factors are given by

$$\alpha = \frac{p+l}{p(q-1) + l(p-2)}, \ \beta = \frac{q-p+1}{p(q-1)+l(p-2)} \text{ and } l \neq -\frac{p(q-1)}{p-2} \tag{1.4}$$

Radially symmetric solutions satisfy

$$U(y) = u(|y|) = u(r).$$

In [3], we studied the asymptotic behaviour of the solution when α and β are both positive.

In this paper, we shall consider the qualitative properties of the solutions of (P) like positiveness, and compact support according to the values of the parameters α, β, p, q and l.

2. Qualitative Properties of the Solutions

We first start with the following result

Proposition 2.1. *Assume $p > 2$, $\alpha \geq N\beta > 0$. There is no positive solution of (P).*

Proof. Suppose by contrary that u is a positive solution of problem (P). Therefore after integration from 0 to r, we get

$$r^{N-1} |u'|^{p-2} u' = -\beta r^N u(r) - (\alpha - N\beta) \int_0^r s^{N-1} u(s) ds - \int_0^r s^{l+N-1} u^q(s) ds \tag{2.1}$$

As $\alpha \geq N\beta > 0$, the following inequality holds

$$\left|u'\right|^{p-2} u' < -\beta r u(r) \text{ for all } r \in \mathbb{R} \tag{2.2}$$

Hence

$$u^{\frac{p-2}{p-1}} < a^{\frac{p-2}{p-1}} - \frac{p-2}{p} \beta^{\frac{1}{p-1}} r^{\frac{p}{p-1}}$$

The following result ensures the existence of strictly positive solutions when $0 < \alpha < N\beta$ □

Proposition 2.2. *Assume $p > 2$, $0 < \alpha < N\beta$, $-\frac{p}{p-1} < l < 0$ and $-N < l$. Then if $q > 1 - \frac{l(p-2)}{p}$ (resp. $q < 1 - \frac{l(p-2)}{p}$) there exists a_* such that if $a \in (0, a_*)$ (resp. $a \in (a_*, \infty)$), the solution of (P) is strictly positive.*

To prove this result we write (P) as a perturbation of the following problem

$$(Q)\begin{cases} (|w'|^{p-2} w')' + \frac{N-1}{r} |w'|^{p-2} w' + \alpha w + \beta r w' = 0 \\ w(0) = 1, \ \lim_{r\to 0} r^{N-1} |w'|^{p-2} w' = 0. \end{cases} \tag{2.3}$$

For this purpose, let a be a real positive and define the function v by

$$u(r) = av(\xi), \ \xi = a^{-\frac{p-2}{p}} r \text{ for any } r > 0. \tag{2.4}$$

where u is the solution of (P). Then v satisfies

$$(Q_b)\begin{cases} (|v'|^{p-2} v')' + \frac{N-1}{\xi} |v'|^{p-2} v' + \alpha v + \beta \xi v' + b\xi^l |v|^{q-1} v = 0 \\ v(0) = 1, \ \lim_{r\to 0} \xi^{N-1} |v'|^{p-2} v' = 0. \end{cases} \tag{2.5}$$

where $b = a^{q-1+\frac{l(p-2)}{p}}$.

To emphasize the dependence of v to the parameter b, we denote it by $v(\zeta, b)$. If we set w the solution of (Q) the following result holds

Proposition 2.3. *Assume $p > 2$, $0 < \alpha < N\beta$, $-\frac{p}{p-1} < l < 0$ and $-N < l$. For every $0 < \varepsilon < \frac{\alpha}{2\beta N}$ and $R > 0$ there exists b_0 such that for $b \in (0, b_0)$ we have*

$$|v(\zeta, b) - w(\zeta)| \leq \varepsilon \quad \text{in } [0, R]. \tag{2.6}$$

Proof. First, we claim that w is strictly positive and strictly decreasing. If not, let ξ_0 the first zero of w. Then integrate (2.3) on $(0, \xi_0)$ we get

$$\xi_0^{N-1} \left|w'\right|^{p-2} w'(\xi_0) = -(\alpha - N\beta) \int_0^{r_0} s^{N-1} w(s) ds. \tag{2.7}$$

Now $0 \leq \alpha < N\beta$, the right hand side is strictly positive as the left hand side is negative which is a contradiction. Consequently $0 < w(\xi) \leq 1$ and $\lim_{\xi\to+\infty} w(\xi) = 0$. The rest of the proof will devided into two steps

Step 1. $|v(\xi,b) - w(\xi)| \leq \varepsilon$ in $[0, R_0]$, where $R_0 = w^{-1}(1 - \frac{\alpha}{2\beta N})$

Take b small enough. As v and w are continuous fonctions and satisfying $v(0,b) = w(0)$, then there exists $R(b)$ such that for any $\xi \in [0, R(b)]$

$$|v(\xi,b) - w(\xi)| \leq \varepsilon. \tag{2.8}$$

If $R(b) \geqslant R_0$, we have the desired result.

Assume in the sequel $R_0 > R(b)$. As w is decreasing and satisfies $0 < w(\xi) \leq w(\xi) = 1$; then without loss of generality we can assume that

$$\varepsilon \leq w(R_0) < w(R(b)) < w(\xi) \leq 1 \text{ in } [0, R(b)]\,. \tag{2.9}$$

Then (2.8) implies that for any $\xi \in [0, R(b)]$

$$0 < w(R_0) - \varepsilon < w(\xi) - \varepsilon < v(\xi,b) < \varepsilon + w(\xi) \tag{2.10}$$

Now recall that w and v are given by the following implicit formula

$$w(\xi) = 1 - \int_0^\xi |f(s)|^{\frac{2-p}{p-1}} f(s)ds \quad \text{for } \xi \in [0, R(b)] \tag{2.11}$$

and

$$v(\xi,b) = 1 - \int_0^\xi |g(s)|^{\frac{2-p}{p-1}} g(s)ds \quad \text{for } \xi \in [0, R(b)] \tag{2.12}$$

where

$$f(s) = \beta s w(s) + (\alpha - N\beta)s^{1-N} \int_0^s \sigma^{N-1} w(\sigma)d\sigma \tag{2.13}$$

and

$$\begin{aligned} g(s) &= \beta s v(s,b) \\ &\quad + (\alpha - N\beta)s^{1-N} \int_0^s \sigma^{N-1} v(\sigma,b)d\sigma + b s^{1-N} \int_0^s \sigma^{l+N-1} v^q(\sigma,b)d\sigma \end{aligned} \tag{2.14}$$

From the choice of R_0, we get

$$w(\xi) \geqslant w(R_0) \geqslant 1 - \frac{\alpha}{2\beta N} \text{ for any } \xi \in [0, R(b)]\,. \tag{2.15}$$

and consequently f satifsfies

$$f(s) \geqslant \left[\beta w(R_0) + \frac{\alpha}{N} - \beta\right] s \quad \text{for any} \quad s \in [0, R(b)] \,, \tag{2.16}$$

On the other hand, combining (2.10), (2.14) and (2.15) and since $0 < v \leq 1$ in $[0, R(b)]$, we get

$$g(s) \geqslant C(R_0)s \text{ for any } s \in [0, R(b)] \,, \tag{2.17}$$

where

$$C(R_0) = \beta w(R_0) + \frac{\alpha}{N} - \beta - \beta\varepsilon + \frac{b_0}{N+l} R_0^l \, (w(R_0) - \varepsilon)^q.$$

Now choose b such that

$$b \leq \beta\varepsilon(N+l)R_0^{-l} \, (w(R_0) - \varepsilon)^{-q}. \tag{2.18}$$

So

$$C(R_0) < \beta w(R_0) + \frac{\alpha}{N} - \beta \tag{2.19}$$

Consequently if we let

$$h(s) = \inf\left(|f(s)|, |g(s)|\right) \quad s \in [0, R(b)] \tag{2.20}$$

we get from (2.11)–(2.12) and (2.18) the following inequality

$$|v(\xi, b) - w(\xi)| \leq c^{\frac{2-p}{p-1}}(R_0) \int_0^\xi |f(s) - g(s)| \, s^{\frac{2-p}{p-1}} ds \tag{2.21}$$

for any $\xi \in [0, R(b)]$.

Now using (2.13) and (2.14) we get

$$\begin{aligned} \int_0^\xi |f(s) - g(s)| \, s^{\frac{2-p}{p-1}} ds &\leq +\beta \int_0^\xi s^{\frac{1}{p-1}} |v(s, b_0) - w(s)| \, ds \\ &+ |\alpha - N\beta| \int_0^\xi s^{\frac{1}{p-1} - N} \int_0^s \sigma^{N-1} |v(\sigma, b_0) - w(s)| \, d\sigma \, ds \\ &+ b \int_0^\xi s^{\frac{1}{p-1} - N} \int_0^s \sigma^{N+l-1} v^q(\sigma, b) d\sigma. \end{aligned} \tag{2.22}$$

Using again $0 < v \leq 1$, the following estimate holds in $[0, R(b)]$.

$$|v(\xi, b_0) - w(\xi)| \leq Kbc^{\frac{2-p}{p-1}}(R_0) R_0^{l + \frac{p}{p-1}}$$

$$+ Kc^{\frac{2-p}{p-1}}(R_0)\int_0^{\xi} s^{\frac{1}{p-1}}\left|v(s,b)-w(s)\right| ds, \quad (2.23)$$

where we denote by K different constants.

Applying now Gronwall's inequality, we get

$$\left|v(\xi,b)-w(\xi)\right| \le bKc^{\frac{2-p}{p-1}}(R_0)R_0^{l+\frac{p}{p-1}}\ e^{Kc^{\frac{2-p}{p-1}}(R_0)} \quad \text{in } [0,R(b)]\,. \qquad (2.24)$$

It follows that by choosing b small enough so that the right hand side of (2.24) is less than ε, we can take $R(b) = R_0$.

Step 2. The following estimate holds

$$\left|v(\xi,b)-w(\xi)\right| \le \varepsilon \text{ for any } \xi \in [R_0,R] \qquad (2.25)$$

As to prove the global existence, we introduce the energy function associated to problem (Q_b)

$$E_b(\xi) = \frac{p}{p-1}\left|v'\right|^p + \frac{\alpha}{2}v^2 + b\xi^l\frac{|v|^{q+1}}{q+1} \qquad (2.26)$$

It is easy to see that E_b is decreasing in $[R_0,R]$. So, the following inequality holds

$$\frac{\alpha}{2}v^2(\xi,b) \le E_b(R_0) = \frac{p-1}{p}\left|v'(R_0,b)\right|^p + \frac{\alpha}{2}v^2(R_0,b) + R_0^l\frac{|v(R_0,b)|^{q+1}}{q+1} \quad (2.27)$$

for any $\xi \in [R_0,R]$ and any $b \in [0,b]$.

First, Step 1 implies that $v(.,b)$ is bounded in $[0,R_0]$ for any $b \in [0,b_0]$ (with small b_0). On the hand, integrating (2.5), we obtain

$$\left|v'(R_0,b)\right|^{p-1} \le |\alpha - N\beta|\, R_0^{1-N}\int_0^{R_0} s^{N-1}\left|v(s,b)\right| ds$$
$$+ R_0^l\int_0^{R_0} |v|^q(s,b)ds + \beta\left|R_0\right| v(R_0,b). \quad (2.28)$$

and then $v'(R_0,b)$ is bounded. Thereby (2.27) gives the boundedeness in $[R_0,R]$ of $v(\xi,b)$. Finally integrating equation (2.5) we deduce that $v'(\xi,b)$ is uniformly bounded in $[R_0,R]$ for any $b \in [0,b_0]$. By using Arzela-Ascoli theorem, there exists a function $\widetilde{w}$: $[R_0,R] \to \mathbb{R}^+$ such that $v(.,b)$ converges uniformly in $C^{1,\alpha}([R_0,R])$ to $\widetilde{w}$ as b goes to 0. Hence, if we integrate equation (2.5) on $[R_0,\xi]$ (where $0 < R_0 < \xi \le R$) and letting b go to 0, we obtain

$$\xi^{N-1}\left|\widetilde{w}'\right|^{p-2}\widetilde{w}'(\xi)-R_0^{N-1}\left|\widetilde{w}'\right|^{p-2}\widetilde{w}'(R_0)=-\beta\xi^N\widetilde{w}(\xi)+\beta R_0^N\widetilde{w}(R_0)$$
$$-(\alpha-N\beta)\xi^{1-N}\int_{R_0}^{\xi}s^{N-1}\widetilde{w}(s)ds. \quad (2.29)$$

Moreover, by letting b go to 0 in the equation:

$$\begin{aligned}R_0^{N-1}\left|v'\right|^{p-2}v'(R_0)= \quad & -\beta R_0^N v(R_0)-(\alpha-N\beta)\int_0^{R_0}s^{N-1}v(s)ds\\ & -b\int_0^{R_0}s^{N+l-1}\left|v\right|^{q-1}v(s)ds\end{aligned} \quad (2.30)$$

and using step1, we obtain

$$R_0^{N-1}\left|\widetilde{w}'\right|^{p-2}\widetilde{w}'(R_0)=-\beta R_0^N w(R_0)-(\alpha-N\beta)\int_0^{R_0}s^{N-1}w(s)ds \quad (2.31)$$

Recall (2.3), we deduce $\widetilde{w}'(R_0)=w'(R_0)$. Consequently (2.29) implies that $\widetilde{w}(\xi)=w(\xi)$ for any $\xi\in[R_0,R]$, and thereby (2.25) follows.

Combining step1 and step2, we get (2.6), which completes the proof. ☐

Now Proposition 1.2, is just a consequence of Proposition 1.3.

In the sequel we introduce the following transformation

$$u(r)=av(\xi), \qquad \xi=a^{\frac{q+1-p}{p+l}}r \quad (2.32)$$

then v satisfies

$$(S_b)\begin{cases}\left(|v'|^{p-2}v'\right)+\dfrac{N-1}{\xi}\left|v'\right|^{p-2}v'+\xi^l\left|v\right|^{q-1}\xi+b(\alpha v+\beta\xi v')=0\\ v(0)=1,\ \ \lim_{\xi\to0}\xi^{N-1}\left|v'\right|^{p-2}v'(\xi)=0\end{cases} \quad (2.33)$$

where $b=a^{\gamma}$ with $\gamma=-\dfrac{l(p-2)+p(q-1)}{p+l}$.

Let w the solution following problem

$$(S)\ \begin{cases}\left(|w'|^{p-2}w'\right)'+\dfrac{N-1}{\xi}\left|w'\right|^{p-2}w'+\xi^l\left|w\right|^{q-1}w=0\\ w(0)=1,\ \ \lim_{\xi\to0}\xi^{N-1}\left|w'\right|^{p-2}w'(\xi)=0\end{cases} \quad (2.34)$$

Using [3], w is unique and nonpositive decreasing solution. Then there exists $0<R_0<R_*$ such that

$$\begin{cases}w(r)>0 \quad \text{for } r\in[0,R_0[\\ w(R_0)=0 \text{ and } w(r)<0 \quad \text{for } r\in[R_0,R_*]\end{cases} \quad (2.35)$$

Note that (S_b) is an approximate of (S) when b goes to 0. In fact the following result holds

Proposition 2.4. *Assume $p > 2$, $0 < \alpha < N\beta$, $-\dfrac{p}{p-1} < l < 0$, $-N < l$ and $q \neq 1 - \frac{l(p-2)}{p}$ such that $q < \frac{p(l+N)}{N-p} - 1$. Let R_* given in (2.35), then for every small positive ε there exists an b_* such that for any $b \in (0, b_*)$ we have*

$$|v(\xi, b) - w(\xi, b)| \leq \xi \qquad \text{in } [0, R_*] \tag{2.36}$$

The argument to show this result is essentially the same as in the previous proposition.

As a consequence, we get

Corollary 2.1. *Assume $p > 2$, $q < \frac{p(l+N)}{N-p} - 1$, $0 < \alpha < N\beta$, $-\frac{p}{p-1} < l < 0$ and $-N < l$. Then if $q > 1 - \frac{l(p-2)}{p}$ (resp. $q < 1 - \frac{l(p-2)}{p}$) there exists a^* such that if $a \in (0, a^*)$ (resp. $a \in (a^*, \infty)$), the solution $u(., a)$ of (P) is nonpositive.*

In what remains we are interested to the solutions with a compact support.

Proposition 2.5. *Assume $p > 2$, $0 < \alpha < N\beta$, $-\dfrac{p}{p-1} < l < 0$ and $-N < l < 0$ and $q \neq 1 - \frac{l(p-2)}{p}$. Then, there exists a constant $a_0 > 0$ such that $u(., a_0)$ has a finite support.*

Proof. First we claim that the set

$$A_+ = \{a > 0 : \ u(r, a) > 0, \ \text{for any } r \geqslant 0\}$$

is open. In fact, in view of Proposition 1.2, the set A_+ is not empty. In order to prove that A_+ is an open set, take $a_0 \in A_+$. There exists a large r_0 such that $J(r_0, a_0) > 0$ where the function J is given by

$$J(r) = J(r, a) = \left[u(r, a) + \frac{1}{\beta r} |u'|^{p-2} u'(r, a) \right] r^N$$

and also we have $J'(r_0, a_0) > 0$. By local continuous dependence of the solutions on the initial value, there is a neighbourhood $O(a_0)$ of a_0 such that the following holds for any $a \in O(a_0)$

$$\begin{cases} u(r, a) > 0 & \text{in } [0, r_0] \\ J(r_0, a) > 0 & \text{and } J'(r_0, a) > 0 \end{cases} \tag{2.37}$$

We assert that $J(r, a)$ and $J'(r, a)$ are strictly positive for any $r \geqslant r_0$ and $a \in O(a_0)$. Otherwise, there exists $r_1 > r_0$ such that $J'(r_1, a) = 0$ (r_1 is the first zero of $J'(r_1, a)$). Then, $J'(r, a) > 0$ for all $r \in [r_0, r_1[$. Since

$$J'(r) = \frac{1}{\beta} r^{N-1} \left[N\beta - \alpha - r^l u^{q-1} \right] u.$$

the function g given by

$$g(r) = N\beta - \alpha - r^l u^{q-1} \tag{2.38}$$

is strictly positive in $[r_0, r_1[$. Moreover, as $g'(r,a)$ is given by

$$g'(r,a) = r^{l-1}\left[-l\, u(r,a) - (q-1) r u'(r,a)\right] u^{q-2}(r,a). \tag{2.39}$$

We deduce that $g'(r,a) > 0$ for all $r \in [r_0, r_1[$. On the other hand $J'(r_1,a) = 0$ is equivalent to $\frac{1}{\beta} r_1^{N-1} g(r_1,a) u(r_1,a) = 0$. Then, two cases come; either $u(r_1,a) = 0$ or $g(r_1,a) = 0$. If $u(r_1,a) = 0$, then

$$J(r_1,a) = \frac{r^{N-1}}{\beta} \left|u'\right|^{p-2} u'(r_1,a) \leq 0.$$

But on the left of r_1, the function J is strictly positive and strictly increasing, which is impossible. In second case where $g(r_1,a) = 0$, we have also a contradiction, because $g(r,a) > 0$ and $g'(r,a) > 0$ for all r in $[r_0, r_1[$. Then,

$$J(r,a) > 0 \text{ and } J'(r,a) > 0 \tag{2.40}$$

for all $r \in [r_0, +\infty[$ and all $a \in O(a_0)$. In particular $u(r,a) > 0$ in $[r_0, +\infty[$. Combining this, with (2.37) we deduce that A_+ is open.

To finish the proof, we set $A_- = \{a > 0 : \ u(r,a) \text{ is nonpositive }\}$.

Note that A_- is obviously open and nonempty (from Corollary 2.1).

Since A_+ and A_- are nonempty and open, the complementary of $A_+ \cup A_-$ is nonempty. This implies that there exists an initial value $a > 0$ such that $u(r,a)$ has a compact support.

The proof is complete. □

We can easily show the following behavior of the solutions which have compact support

Proposition 2.6. *Assume $p > 2$, $0 < \alpha < N\beta$, $-\frac{p}{p-1} < l < 0$ and $-N < l$. Let u be a solution with a compact support $[0,R]$. Then*

$$\lim_{r \to R} \frac{|u'|^{p-1}}{u}(r) = \beta R \tag{2.41}$$

We finish this section with the following result

Proposition 2.7. *Assume $0 < \alpha \leq N\beta$, $-p < l < 0$ and $-N < l$. Then for each $a > 0$ the solution $u(r,a)$ has the same sign for large r.*

Proof. Let $a > 0$ and assume that $u(r,a)$ changes its sign for large r. Let r_0 and r_1 be two consecutive large zeroes. Integrating the equation

$$(P) \quad \begin{cases} (|u'|^{p-2} u')' + \frac{N-1}{r} |u'|^{p-2} u' + \alpha u + \beta r u' + r^l |u|^{q-1} u = 0 \\ u(0) = a \end{cases}$$

on (r_0, r_1), we get

$$r_1^{N-1} \left|u'\right|^{p-2} u'(r_1) - r_0^{N-1} \left|u'\right|^{p-2} u'(r_0) = -\int_{r_0}^{r_1} r^{N-1} u(r) \left[\alpha - N\beta + r^l |u|^{q-1} u(r)\right] ds \quad (2.42)$$

Assume that u is strictly positive on (r_0, r_1) (the other case is proved in the same way). Then, the left hand side of (2.41) is strictly negative. On the other hand, invoking Proposition 3.1 and using $0 < \alpha \leq N\beta$, we deduce that the right hand side is positive. This is a contradiction, and the proof is complete. □

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