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ON INTUITIONISTIC FUZZY SLIGHTLY PRECONTINUOUS FUNCTIONS

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Abstract: In this paper the concept of intuitionistic fuzzy slightly precontinuous functions are introduced and studied. Intuitionistic fuzzy slightly precontinuity generalize intuitionistic fuzzy precontinuity. Besides giving characterizations and basic properties of this function, preservation theorems of intuitionistic fuzzy slightly precontinuous are also obtained. We also study relationships between intuitionistic fuzzy slightly precontinuity and separation axioms. Moreover, we investigate the relationships among intuitionistic fuzzy slightly precontinuity and compactness and connectedness.

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1. Introduction

Ever since the introduction of fuzzy sets by L.A. Zadeh [16], the fuzzy concept has invaded almost all branches of mathematics. The concept of fuzzy topological spaces was introduced and developed by C.L. Chang [2], Atanassov [1] introduced the notion of intuitionistic fuzzy sets, Coker [3] introduced the

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intuitionistic fuzzy topological spaces. H. Gurcay, D. Coker and A.H. Es [7] introduced the concept of intuitionistic fuzzy precontinuity in 1997. Ekici [6] introduced fuzzy slightly precontinuous functions. In this paper we have introduced the concept of intuitionistic fuzzy slightly precontinuous functions and studied their properties. Also we have given preservation theorems of intuitionistic fuzzy slightly precontinuous functions. We also study relationships between this function and separation axioms. Moreover, we investigate the relationships among intuitionistic fuzzy slightly precontinuity and compactness and connectedness.

2. Preliminaries

Definition 2.1. [1] Let X be a nonempty fixed set and I the closed interval [0,1]. An intuitionistic fuzzy set (IFS) A is an object of the following form

$$\mathrm{A} = \{<\mathrm{x},\ \mu_A(\mathrm{x}),\ \nu_A(\mathrm{x})>;\ \mathrm{x}\in \mathrm{X}\}$$

where the mappings $\mu_A(\mathrm{x})$: X→I and $\nu_A(\mathrm{x})$: X→I denote the degree of membership(namely) $\mu_A(\mathrm{x})$ and the degree of nonmembership(namely) $\nu_A(\mathrm{x})$ for each element x∈ X to the set A respectively, and $0 \leq \mu_A(\mathrm{x}) + \nu_A(\mathrm{x}) \leq 1$ for each x∈X.

Definition 2.2. [1] Let A and B are IFSs of the form $\mathrm{A} = \{<\mathrm{x}, \mu_A(\mathrm{x}), \nu_A(\mathrm{x})>; \mathrm{x}\in\mathrm{X}\}$ and $\mathrm{B} = \{<\mathrm{x}, \mu_B(\mathrm{x}), \nu_B(\mathrm{x})>; \mathrm{x}\in\mathrm{X}\}$.Then

(i) $\mathrm{A} \subseteq \mathrm{B}$ if and only if $\mu_A(\mathrm{x}) \leq \mu_B(\mathrm{x})$ and $\nu_A(\mathrm{x}) \geq \nu_B(\mathrm{x})$;

(ii) $\bar{\mathrm{A}}$(or A^c) $= \{<\mathrm{x}, \nu_A(\mathrm{x}), \mu_A(\mathrm{x})>; \mathrm{x}\in\mathrm{X}\}$;

(iii) $\mathrm{A} \cap \mathrm{B} = \{<\mathrm{x}, \mu_A(\mathrm{x}) \wedge \mu_B(\mathrm{x}), \nu_A(\mathrm{x}) \vee \nu_B(\mathrm{x})>; \mathrm{x}\in\mathrm{X}\}$;

(iv) $\mathrm{A} \cup \mathrm{B} = \{<\mathrm{x}, \mu_A(\mathrm{x}) \vee \mu_B(\mathrm{x}), \nu_A(\mathrm{x}) \wedge \nu_B(\mathrm{x}) >; \mathrm{x}\in\mathrm{X}\}$.

We will use the notation $\mathrm{A}= \{<\mathrm{x}, \mu_A, \nu_A>; \mathrm{x}\in\mathrm{X}\}$ instead of $\mathrm{A}= \{<\mathrm{x}, \mu_A(\mathrm{x}), \nu_A(\mathrm{x})>; \mathrm{x}\in\mathrm{X}\}$

Definition 2.3. [3] $0_\sim = \{<\mathrm{x},0,1>; \mathrm{x}\in\mathrm{X}\}$ and $1_\sim = \{<\mathrm{x},1,0>; \mathrm{x}\in\mathrm{X}\}$.

Let $\alpha, \beta \in [0,1]$ such that $\alpha+\beta \leq 1$. An intuitionistic fuzzy point (IFP)$p_{(\alpha,\beta)}$ is intuitionistic fuzzy set defined by $p_{(\alpha,\beta)}(\mathrm{x}) = \begin{cases} (\alpha,\beta) & \text{if } x = p, \\ (0,1) & \text{otherwise} \end{cases}$

Definition 2.4. [3] An intuitionistic fuzzy topology (IFT) in Coker's sense on a nonempty set X is a family τ of intuitionistic fuzzy sets in X satisfying the following axioms:

(i) $0_\sim$, $1_\sim \in \tau$;

(ii) $G_1 \cap G_2 \in \tau$, for any G_1 ,$G_2 \in \tau$;

(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i$; $i \in J \} \subseteq \tau$.

In this paper by (X,τ), (Y,σ) or simply by X, Y, we will denote the intuitionistic fuzzy topological spaces (IFTS). Each IFS which belongs to τ is called an intuitionistic fuzzy open set (IFOS) in X. The complement $\bar{A}$ of an IFOS A in X is called an intuitionstic fuzzy closed set (IFCS) in X. An IFS X is called intuitionistic fuzzy clopen(IF clopen) iff it is both intuitionistic fuzzy open and intuitionistic fuzzy closed.

Let X and Y are two non-empty sets and f:(X,τ) $\rightarrow$ (Y,σ) be a function. If B = {<y, μ_B(y), ν_B(y)>;y$\in$Y} is an IFS in Y, then the pre-image of B under f is denoted and defined by $f^{-1}(B)$ = {<x, $f^{-1}(\mu_B(x))$, $f^{-1}(\nu_B(x))$>; x$\in$X } Since μ_B(x) , ν_B(x) are fuzzy sets, we explain that $f^{-1}(\mu_B(x)) = \mu_B(x)(f(x))$, $f^{-1}(\nu_B(x)) = \nu_B(x)(f(x))$

Definition 2.5. [11] Let $p_{(\alpha,\beta)}$ be an IFP in IFTS X. An IFS A in X is called an intuitionistic fuzzy neighborhood (IFN) of $p_{(\alpha,\beta)}$ if there exists an IFOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.

Definition 2.6. [3] Let (X,τ) be an IFTS and A = {<x, μ_A(x) , ν_A(x)>; x$\in$X } be an IFS in X.Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A are defined by

(i) cl(A) = $\bigcap$ {C:C is an IFCS in X and C $\supseteq$ A};

(ii) int(A) = $\bigcup$ {D:D is an IFOS in X and D $\subseteq$ A};

It can be also shown that cl(A) is an IFCS, int(A) is an IFOS in X and A is an IFCS in X if and only if cl(A) = A ; A is an IFOS in X if and only int(A) = A.

Proposition 2.1. *[3]Let (X,τ) be an IFTS and A,B be IFSs in X. Then the following properties hold:*

(i) $cl\overline{A} = \overline{(int(A))}$, $int\overline{(A)} = \overline{(cl(A))}$;

(ii) $int(A) \subseteq A \subseteq cl(A)$.

Definition 2.7. [7] An IFS A in an IFTS X is called an intuitionistic fuzzy pre open set (IFPOS) if A$\subseteq$ int(clA). The complement of an IFPOS A in IFTS X is called an intuitionistic fuzzy pre closed (IFPCS) inX.

Definition 2.8. Let f be a mapping from an IFTS X into an IFTS Y. The mapping f is called:

(i) intuitionistic fuzzy continuous if and only if $f^{-1}(B)$ is an IFOS in X, for each IFOS B in Y [7];

(ii) intuitionistic fuzzy pre continuous if and only if $f^{-1}(B)$ is an IFPOS in X, for each IFOS B in Y [7];

(iii) intuitionistic fuzzy totally continuous if and only if $f^{-1}(B)$ is an IF clopen set in X, for each IFOS B in Y [12];

Definition 2.9. [13] A function f:(X,τ)$\rightarrow$(Y,σ) from a intuitionistic fuzzy topological space (X,τ) to another intuitionistic fuzzy topological space (Y,σ) is said to be intuitionistic fuzzy pre irresolute if $f^{-1}(B)$ is an IFPOS in (X,τ) for each IFPOS B in (Y,σ).

Definition 2.10. [3, 10] Let X be an IFTS. A family of {<x, μ_{G_i}(x), ν_{G_i}(x)>;i$\in$J} intuitionistic fuzzy open sets (intuitionistic fuzzy preopen sets) in X satisfies the condition $1_\sim = \cup$ {<x, μ_{G_i}(x), ν_{G_i}(x)>;i$\in$J} is called a intuitionistic fuzzy open (intuitionistic fuzzy preopen) cover of X. A finite subfamily of a intuitionistic fuzzy open (intuitionistic fuzzy preopen) cover {<x, μ_{G_i}(x), ν_{G_i}(x)>;i$\in$J} of X which is also a intuitionistic fuzzy open (intuitionistic fuzzy preopen) cover of X is called a finite subcover of {<x, μ_{G_i}(x), ν_{G_i}(x)>;i$\in$J}.

Definition 2.11. [4] An IFTS X is called intuitionistic fuzzy compact if each intuitionistic fuzzy open cover of X has a finite subcover for X.

3. Intuitionistic Fuzzy Slightly Precontinuous Functions

Definition 3.1. A function $f:(X,\tau)\rightarrow(Y,\sigma)$ from a intuitionistic fuzzy topological space (X,τ) to another intuitionistic fuzzy topological space (Y,σ) is said to be intuitionistic fuzzy slightly precontinuous if for each intuitionistic fuzzy point $p_{(\alpha,\beta)}\in X$ and each intuitionistic fuzzy clopen set B in Y containing $f(p_{(\alpha,\beta)})$, there exists a fuzzy intuitionistic fuzzy preopen set A in X such that $f(A)\subseteq B$.

Theorem 3.1. *For a function $f : X \longrightarrow Y$, the following statements are equivalent:*

1. *f is intuitionistic fuzzy slightly precontinuous;*
2. *for every intuitionistic fuzzy clopen set B in Y, $f^{-1}(B)$ is intuitionistic fuzzy preopen;*
3. *for every intuitionistic fuzzy clopen set B in Y, $f^{-1}(B)$ is intuitionistic fuzzy preclosed;*
4. *for every intuitionistic fuzzy clopen set B in Y, $f^{-1}(B)$ is intuitionistic fuzzy preclopen.*

Proof. (1)⇒(2) Let B be IF clopen set in Y and let $p_{(\alpha,\beta)} \in f^{-1}(B)$. Since $f(p_{(\alpha,\beta)}) \in B$, by (1) there exists a IFPOS $A_{p_{(\alpha,\beta)}}$ in X containing $p_{(\alpha,\beta)}$ such that $A_{p_{(\alpha,\beta)}} \subseteq f^{-1}(B)$. We obtain that $f^{-1}(B) = \bigcup\limits_{p_{(\alpha,\beta)\in f^{-1}(B)}} A_{p_{(\alpha,\beta)}}$ Thus, $f^{-1}(B)$ is IF preopen.
(2)⇒(3) Let B be IF clopen set in Y. Then $\overline{B}$ is IF clopen. By (2), $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$ is IF preopen. Thus $f^{-1}(B)$ is IF preclosed.
(3)⇒(4) Let B be IF clopen set in Y. Then by (3) $f^{-1}(B)$ is IF preclosed. Also $\overline{B}$ is IF clopen and (3) implies $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$ is IF preclosed. Hence $f^{-1}(B)$ is IF preopen. Thus $f^{-1}(B)$ is IFpreclopen.
(4)⇒(1) Let B be IF clopen set in Y containing $f(p_{(\alpha,\beta)})$. By (4), $f^{-1}(B)$ is IF preopen. Let us take A $= f^{-1}(B)$. Then $f(A) \subseteq B$. Hence f is IF slightly precontinuous. □

Lemma 3.1. *[8] Let g:X→X×Y be a graph of a mapping f:(X,τ)→(Y,σ). If A and B are IFS's of X and Y respectivly, then $g^{-1}(1_\sim \times B) = (1_\sim \cap f^{-1}(B))$*

Lemma 3.2. *[8] Let X and Y be intuitionistic fuzzy topological spaces, then (X,τ) is product related to (Y,σ) if for any IFS C in X, D in Y whenever $\bar{A} \not\supseteq C$, $\bar{B} \not\supseteq D$ implies $\bar{A} \times 1_\sim \bigcup 1_\sim \times \bar{B} \supseteq C \times D$ there exists $A_1 \in \tau$, $B_1 \in \sigma$ such that $\overline{A_1} \supseteq C$ and $\overline{B_1} \supseteq D$ and $\overline{A_1} \times 1_\sim \bigcup 1_\sim \times \overline{B_1} = \bar{A} \times 1_\sim \bigcup 1_\sim \times \bar{B}$.*

Theorem 3.2. *Let f:X→Y be a function and assume that X is product related to Y.If the graph g:X→ X×Y of f is IF slightly precontinuous then so is f.*

Proof. Let B be IF clopen set in Y.Then by lemma 3.1, $f^{-1}(B) = 1_\sim \cap f^{-1}(B) = g^{-1}(1_\sim \times B)$. Now $1_\sim \times$ B is a IF clopen set in X×Y. Since g is IF slightly precontinuous then $g^{-1}(1_\sim \times B)$ is IF pre open in X. Hence $f^{-1}(B)$ is IF pre open in X. Thus f is IF slightly precontinuous.

□

Proposition 3.1. *Every intuitionistic fuzzy precontinuous function is intuitionistic fuzzy slightly precontinuous. But the converse need not be true, as shown by the following example.*

Example 3.1. Let $X = \{a, b, c\}$,, τ ={$0_\sim, 1_\sim, A\}$, $\sigma = \{0_\sim, 1_\sim, B, C, B \cup C, B \cap C\}$ where

A = {<x,$(\frac{a}{0.7},\frac{b}{0.6},\frac{c}{0.6})$,$(\frac{a}{0.3},\frac{b}{0.4},\frac{b}{0.4})$>; x∈X},

B = {<x,$(\frac{a}{0.1},\frac{b}{0.1},\frac{c}{0.5})$,$(\frac{a}{0.9},\frac{b}{0.9},\frac{c}{0.5})$>; x∈X},

C = {<x,$(\frac{a}{0.9},\frac{b}{0.9},\frac{c}{0.3})$,$(\frac{a}{0.1},\frac{b}{0.1},\frac{b}{0.7})$>; x∈X}

Define an intuitionistic fuzzy mapping $f : (X, \tau) \to (X, \sigma)$ by $f(a) = a, f(b) = b$, $f(c) = c$. Then f is IF slightly precontinuous. But f is not precontinuous, since $f^{-1}(B{\cap}C)$ is not IFPOS in X as $f^{-1}(B{\cap}C) \nsubseteq int(clf^{-1}(B{\cap}C) = 0_\sim$

Proposition 3.2. *Every intuitionistic fuzzy pre irresolute function is intuitionistic fuzzy slightly precontinuous. But the converse need not be true, as shown by the following example.*

Example 3.2. Let $X = \{a, b\}, Y = \{c, d\}$, τ ={$0_\sim, 1_\sim, A\}$, $\sigma = \{0_\sim, 1_\sim, B\}$ where

A = {<x,$(\frac{a}{0.3},\frac{b}{0.4})$,$(\frac{a}{0.6},\frac{b}{0.5})$>; x∈X}, B = {<y,$(\frac{c}{0.4},\frac{d}{0.5})$,$(\frac{c}{0.5},\frac{d}{0.5})$>; y∈Y}.

Define an intuitionistic fuzzy mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = d, f(b) = c$. Then f is IF slightly precontinuous. But it is not IF pre irresolute, since $f^{-1}(B) \nsubseteq intclf^{-1}(B)$.

Theorem 3.3. *Suppose that Y has a base consisting of IF clopen sets. If $f : X \to Y$ is IF slightly precontinuous, then f is IF precontinuous.*

Proof. Let $p_{(\alpha,\beta)} \in X$ and let C be IFOS inY containing $f(p_{(\alpha,\beta)})$. Since Y has a base consisting of IF clopen sets, there exists a IF clopen set B containing $f(p_{(\alpha,\beta)})$ such that $B \subseteq C$. Since F is IF slightly precontinuous, then there exists a IFPOS A in X containing $p_{(\alpha,\beta)}$ such that $f(A) \subseteq B \subseteq C$. Thus f is IF precontinuous. □

Theorem 3.4. *The following hold for functions $f : X \to Y$ and $g : Y \to Z$*
(i)If f is IF slightly precontinuous and g is IF totally coninuous then $g\circ f$ is IF precontinuous.
(ii)If f is IF pre irresolute and g is IF slightly precontinuous then $g\circ f$ is IF slightly precontinuous.

Proof. (i) Let B be an IFOS in Z. Since g is IF totally continuous, g^{-1}(B) is an IF clopen set in Y. Now $(g\circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$. Since f is IF slightly precontinuous, $f^{-1}(g^{-1}(B))$ is IFPOS in X. Hence $g\circ f$ is IF precontinuous.
(ii) Let B be IF clopen set in Z. Since g is IF slightly precontinuous, g^{-1}(B) is an IFPOS in Y. Now $(g\circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$. Since f is IF pre irresolute, $f^{-1}(g^{-1}(B))$ is IFPOS in X which implies $g\circ f$ is IF slightly precontinuous. □

4. Intuitionistic Fuzzy p-Separation Axioms

In this section, we investigate the relationships between IF slightly precontinuous functions and IF p- separation axioms.

Definition 4.1. An IFTS (X, τ) is called $p - T_1(co - T_1)$ if and only if for each pair of distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}$, $y_{(\nu,\delta)}$ in X there exits intuitionistic fuzzy preopen sets (IF clopen sets) $U, V \in X$ such that $x_{(\alpha,\beta)} \in U$, $y_{(\nu,\delta)} \notin U$ and $y_{(\nu,\delta)} \in V$, and $x_{(\alpha,\beta)} \notin V$.

Theorem 4.1. *If $f : (X, \tau) \to (Y, \sigma)$ is IF slightly precontinuous injection and Y is $co - T_1$, then X is IF $p - T_1$.*

Proof. Suppose that Y is IF $co - T_1$. For any distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}$, $y_{(\nu,\delta)}$ in X, there exists IF clopen sets A, B in Y such that $f(x_{(\alpha,\beta)}) \in A$, $f(y_{(\nu,\delta)}) \notin A, f(x_{(\alpha,\beta)}) \notin B$ and $f(y_{(\nu,\delta)}) \in B$. Since f is IF slightly precontinuous, $f^{-1}(A)$ and $f^{-1}(B)$ are IF preopen sets in X such that $x_{(\alpha,\beta)} \in f^{-1}(A)$, $y_{(\nu,\delta)} \notin f^{-1}(A)$, $x_{(\alpha,\beta)} \notin f^{-1}(B)$, $y_{(\nu,\delta)} \in f^{-1}(B)$. This shows that X is IF $p - T_1$. □

Definition 4.2. An IFTS X is said to be $p - T_2$ or p-Hausdorff($co - T_2$ or co-Hausdorff) if for all pair of distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}$, $y_{(\nu,\delta)}$ in X, there exits IF preopen sets (IF clopen sets) $U, V \in X$ such that $x_{(\alpha,\beta)} \in U$, $y_{(\nu,\delta)}$ in V and $U \cap V = 0_\sim$.

Theorem 4.2. *If $f : (X, \tau) \to (Y, \sigma)$ is IF slightly precontinuous injection and Y is $co - T_2$, then S is IF $p - T_2$.*

Proof. Suppose that Y is IF $co - T_2$. For any distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}$, $y_{(\nu,\delta)}$ in X, there exists IF clopen sets A, B in Y such that $f(x_{(\alpha,\beta)}) \in A$, and $f(y_{(\nu,\delta)}) \in B$. Since f is IF slightly precontinuous, $f^{-1}(A)$ and $f^{-1}(B)$ are IF preopen sets in X such that $x_{(\alpha,\beta)} \in f^{-1}(A)$, and $y_{(\nu,\delta)} \in f^{-1}(B)$. Also we have $f^{-1}(A) \cap f^{-1}(B) = 0_\sim$. Hence X is IF $p - T_2$. □

Definition 4.3. An IFTS X is said to be IF co-regular(respectively IF strongly p-regular) if for each IF clopen(respectively IF preclosed) set C and each IFP $x_{(\alpha,\beta)} \notin C$, there exist intuitionistic fuzzy open sets A and B such that $C \subseteq A$, $x_{(\alpha,\beta)} \in B$ and $A \cap B = 0_\sim$

Definition 4.4. An IFTS X is said to be IF co-normal(respectively IF strongly p-normal) if for each IF clopen(respectively IF preclosed) sets C_1 and C_2 in X such that $C_1 \cap C_2 = 0_\sim$, there exist intuitionistic fuzzy open sets A and B such that $C_1 \subseteq A$ and $C_2 \subseteq B$ and $A \cap B = 0_\sim$.

Theorem 4.3. *If F is IF slightly precontinuous injective IF open function from an IF strongly p-regular space X onto a IF space Y, then Y is IF co-regular.*

Proof. Let D be IF clopen set in Y and $y_{(\nu,\delta)} \notin D$. Take $y_{(\nu,\delta)} = f(x_{(\alpha,\beta)}$. Since f is IF slightly precontinuous, $f^{-1}(D)$ is a IF preclosed se in X. Let $C = f^{-1}(D)$. So $x_{(\alpha,\beta)} \notin C$. Since X is IF strongly p-regular, there exist intuitionistic fuzzy open sets A and B such that $C \subseteq A$, $x_{(\alpha,\beta)} \in B$ and $A \cap B = 0_\sim$. Hence, we have $D = f(C) \subseteq f(A)$ and $y_{(\nu,\delta)} = f(x_{(\alpha,\beta)} \in f(B)$ such that $f(A)$ and $f(B)$ disjoint IF open sets. Hence Y is IF co-regular. □

Theorem 4.4. *If f is IF slightly precontinuous, injective, IF open function from a IF strongly p-normal space X onto a IF space Y, then Y is IF co-normal.*

Proof. Let C_1 and C_2 be be disjoint IF clopen sets in Y. Since f is IF slightly precontinuous, $f^{-1}(C_1)$ and $f^{-1}(C_2)$ are IF preclosed sets in X. Let us take $C = f^{-1}(C_1)$ and $D = f^{-1}(C_2)$. We have $C \cap D = 0_\sim$. Since X is IF strongly p-normal, there exist disjoint IF open sets A and B such that $C \subseteq A$ and $D \subseteq$B. Thus $C_1 = f(C) \subseteq f(A)$ and $C_2 = f(D) \subseteq f(B)$ such that $f(A)$ and $f(B)$ disjoint IF open sets. Hence Y is IF co-normal. □

5. Intuitionistic Fuzzy Covering Properties and Intuitionistic Fuzzy p-Connectedness

In this section we investigate the relationships between IF slightly precontinuous and IF compactes and between IF slightly precontinuous and IF connectedness.

Definition 5.1. An IFTS X is said to be

(i) [10]IF precompact if every IF preopen cover of X has a finite subcover.

(ii)[10] IF countably precompact if every preopen countably cover of x has a finite subcover.

(iii)[10] IF pre-Lindelof if every cover of X by IF preopen sets has a countable subcover.

(iv) IF mildly compact if every IF clopen cover of X has a finite subcover.

(v) IF mildly countably compact if every IF clopen countably cover of X has a finite subcover.

(vi) IF mildly Lindelof if every cover of X has IF clopen sets has a countable subcover.

Theorem 5.1. *Let $f : (X, \tau) \to (Y, \sigma)$ be a IF slightly precontinuous surjection. Then the following statements hold:*
(1) if X is IF precompact, then Y is IF mildly compact.
(2) if X is IF pre-Lindelof, then Y is IF mildly Lindelof.
(3) if X is IF countably precompact, then Y is IF mildly countably compact.

Proof. (1) Let $\{A_\alpha; \alpha \in I\}$ be any IF clopen cover of Y. Since f is IF slightly precontinuous, then $\{f^{-1}(A_\alpha); \alpha \in I\}$ is IF preopen cover of X. Since X is IF precompact, there exists a finite subset I_0 of I such that $1_{\sim X} = \bigcup\{f^{-1}(A_\alpha); \alpha \in I_0\}$. Thus, we have $1_{\sim Y} = \bigcup\{A_\alpha; \alpha \in I_0\}$ and Y is IF mildly compact.
(2) Let $\{A_\alpha; \alpha \in I\}$ be any IF clopen cover of Y. Since f is IF slightly precontinuous, then $\{f^{-1}(A_\alpha); \alpha \in I\}$ is IF preopen cover of X. Since X is IF pre-Lindelof, there exists a countable subset I_0 of I such that $1_{\sim X} = \bigcup\{f^{-1}(A_\alpha); \alpha \in I_0\}$. Thus, we have $1_{\sim Y} = \bigcup\{A_\alpha; \alpha \in I_0\}$ and Y is IF mildly Lindelof.
(3) Let $\{A_\alpha; \alpha \in I\}$ be any IF clopen cover of Y. Since f is IF slightly precontinuous, then $\{f^{-1}(A_\alpha); \alpha \in I\}$ is IF preclopen cover of X. Since X is IF countably precompact, for countable preclopen cover $\{f^{-1}(A_\alpha); \alpha \in I\}$ in X, there exists a finite subset I_0 of I such that $1_{\sim X} = \bigcup\{f^{-1}(A_\alpha); \alpha \in I_0\}$. Thus, we have $1_{\sim Y} = \bigcup\{A_\alpha; \alpha \in I_0\}$ and Y is IF mildly countable compact. □

Definition 5.2. An IFTS X is said to be
(1) IF preclosed-compact if every preclosed of X has a finite subcover.
(2) IF preclosed-Lindelof if every cover of X by preclosed sets has a countable subcover.
(3) IF countably preclosed-compact if every countable cover of X by preclosed sets has a finite subcover.

Theorem 5.2. *Let $f : X \longrightarrow Y$ be a IF slightly precontinuous surjection. Then the following statements hold:*
(1) if X is IF preclosed-compact, then Y is mildly compact.
(2) if X is IF preclosed-Lindelof, then Y is mildly Lindelof.
(3) if X is IF countably preclosed-compact, then Y is mildly countably compact.

Proof. (1) Let $\{A_\alpha; \alpha \in I\}$ be any IF clopen cover of Y. Since f is IF slightly precontinuous, then $\{f^{-1}(A_\alpha); \alpha \in I\}$ is IF preclosed cover of X. Since X is TF preclosed-compact, there exists a finite subset I_0 of I such that $1_{\sim X} = \bigcup\{f^{-1}(A_\alpha); \alpha \in I_0\}$. Thus, we have $1_{\sim Y} = \bigcup\{A_\alpha; \alpha \in I_0\}$ and Y is IF mildly compact.
Similarly, we can obtained the proof for (2) and (3). □

Definition 5.3. An IFTS (X, τ) is said to be intuitionistic fuzzy p-disconnected(IF p-disconnected) if there exists IFPOS A, B in X such that $A \neq 0_\sim$, $B \neq 0_\sim$ such that $A \cup B = 1_\sim$ and $A \cap B = 0_\sim$. If X is not IF p-disconnected then it is said to be intuitionistic fuzzy p-connected(IF p-connected).

Theorem 5.3. *If Let f:(X,τ)$\rightarrow$(Y,σ) be a IF slightly precontinuous surjection, (X, τ) is an IF p-connected, then (Y,σ) is IF connected.*

Proof. Assume that (Y,σ) is not IF connected then there exists nonempty intuitionistic fuzzy open sets A and B in (Y,σ) such that A$\cup$B = $1_\sim$ and A$\cap$B = $0_\sim$. Therefore, A and B are intuitionistic fuzzy clopen sets in Y. Since f is IF slightly precontinuous, C = f^{-1}(A) $\neq 0_\sim$ D = f^{-1}(B) $\neq 0_\sim$ which are intuitionistic fuzzy preopen sets in X. And f^{-1}(A)$\cup f^{-1}$(B) = $f^{-1}(1_\sim) = 1_\sim$ which implies $C \cup D = 1_\sim$.
f^{-1}(A)$\cap f^{-1}$(B) = $f^{-1}(0_\sim) = 0_\sim$ which implies C$\cap$D = $0_\sim$. Thus X is IF p-disconnected , which is a contradiction to our hypothesis. Hence Y is IF connected. □

Definition 5.4. [5] An intuitionistic fuzzy set A in intuitionistic fuzzy topological space (X, τ) is called intuitionistic fuzzy dense if there exists no intuitionistic fuzzy closed set B in (X, τ) such that $A \subseteq B \subseteq 1_\sim$.

Definition 5.5. An IFTS X is called hyperconnected if every IF open set is dense.

Remark 5.1. The follwoing example shows that IF slightly precontinuous surjection do not necessarily preserve IF hyperconnectedness.

Example 5.1. Let $X = \{a, b, c\}$, τ =$\{0_\sim, 1_\sim, A\}$, $\sigma = \{0_\sim, 1_\sim, B, C, B \cup C, B \cap C\}$ where

A = {<x,$(\frac{a}{0.7},\frac{b}{0.6},\frac{c}{0.6}),(\frac{a}{0.3},\frac{b}{0.4},\frac{b}{0.4})$>; x∈X},

B = {<x,$(\frac{a}{0.1},\frac{b}{0.1},\frac{c}{0.5}),(\frac{a}{0.9},\frac{b}{0.9},\frac{c}{0.5})$>; x∈X},

C = {<x,$(\frac{a}{0.9},\frac{b}{0.9},\frac{c}{0.3}),(\frac{a}{0.1},\frac{b}{0.1},\frac{b}{0.7})$>; x∈X}.

Define an intuitionistic fuzzy mapping $f : (X, \tau) \to (X, \sigma)$ by $f(a) = a, f(b) = b, f(c) = c$. Then f is IF slightly precontinuous surjective. (X, τ) is hyperconnected. But (X, σ) is not hyperconnected.

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