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MIXED SAMPLING PLANS WHEN THE FRACTION DEFECTIVE IS A FUNCTION OF TIME

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Abstract: A mixed acceptance sampling scheme actually consists of two stages. The first stage sampling is concerned with variable criteria and the second stage sampling is considered with attribute criteria. In this paper, the variable criteria when the fraction defective is not a constant is taken into consideration and a new design procedure of product control for variable non conformities using a system of equations is presented. An iterative procedure of finding the parameters of the sampling plans are obtained by using new algorithm presented in this paper. Tables are constructed for selecting the parameters which will facilitate the shaft floor engineers.

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Key Words: probability of acceptance, variable fraction defective

1. Introduction

When a quality characteristic is measurable, the randomness of occurrence of variations in the measurements, in the production process may be due to environmental effects or any other factor. This variation with respect to time, variable fraction defective is taken into account and the new sampling plan is formulated by solving a stochastic differential equation. The main objective in

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any production process is to control and maintain the quality of the manufactured product so that it confirms to specified quality standards.

2. Literature Review

Hamaker (1979) has given a procedure of finding the parameters for unknown sigma variables sampling plans from known sigma variables sampling plans. Schilling (1982) has written an exclusive book on acceptance sampling which also deals conventional variables sampling plans. Bernt Oksendal (1945) has written Stochastic differential equations an introduction with applications. An introduction to Stochastic differential equations was given by Lawerence C. Evans. DevaArul (1996 - 2004) has developed several such mixed sampling plans by combining process and product control procedures following a proposal of Schilling (1967). In 2009, DevaArul investigated mixed sampling system with tightened inspection in the second stage. Suresh and DevaArul (2002),(2003) developed application oriented mixed sampling plans to suit industrial need.

3. Formulation of the Mixed Sampling Plan

The design of a mixed sampling plan in case of a one-sided upper specification u assuming that the standard deviation σ of the considered process characteristic is known, is specified by four parameters (n_1, n_2, k, c).

The parameters have the following meaning:

n_1 is the sample size of the sample used for process control;

n_2 is the sample size used for lot control, if the process has not been accepted in the first step sampling;

k is the standardized upper control limit for process control;

c is the acceptance number.

4. Operating Procedure of the Plan

Independent Mixed Sampling Plan (n_1, n_2, k, c) :

Step 1: Take a random sample of size n_1 from the lot (assumed to be large).

Step 2: The n_1 units in the sample are measured and the values $x_1(t)$, $x_2(t)$, $x_3(t)$,...,$x_n(t)$ are obtained at any time t. The mean $\overline{X}(t)$ is calculated.

Step 3: If $(\overline{X}(t) + k\sigma) \leq U$ then accept the lot.

Step 4: If $(\overline{X}(t) + k\sigma) > U$ then take a second sample of size n_2.

Step 5: If the number of non conforming items in the second sample is less than or equal to c then accept the lot, otherwise reject the lot.

Theorem 1. (Independent Plan) *The probability of acceptance is given by*

$$P_a(p(t)) = P_{n_1}(\overline{X}(t) \leq A) + P_{n_1}(\overline{X}(t) > A) \sum_{j=0}^{c} \frac{e^{-n_2 p(t)}(n_2 p(t)^j)}{j!}.$$

Proof. For mixed plans in which the two stages are kept independent the probability of acceptance is given by the complement of the product of the two probabilities of rejection for a given percent defective.

$$\begin{aligned} P_a(p(t)) =& 1 - P_{n_1}(\overline{X}(t) > A) \sum_{j=c+1}^{n_2} P_{n_2}(j; n_2) \\ =& P_{n_1}(\overline{X}(t) \leq A) + P_{n_1}(\overline{X}(t) > A) - P_{n_1}(\overline{X}(t) > A) \sum_{j=c+1}^{n_2} P_{n_2}(j; n_2) \\ =& P_{n_1}(\overline{X}(t) \leq A) + P_{n_1}(\overline{X}(t) > A) \sum_{j=0}^{c} P_{n_2}(j; n_2) \\ =& P_{n_1}(\overline{X}(t) \leq A) + P_{n_1}(\overline{X}(t) > A) \sum_{j=0}^{c} \frac{e^{-n_2 p(t)}(n_2 p(t)^j)}{j!}. \end{aligned}$$ □

Theorem 2. *Let $p(t)$ denote variable fraction defective at time t. $\frac{dp}{p}$, the relative change of variable fraction defective given by the Stochastic differential equation*

$$\frac{dp}{p} = \mu dt + \sigma dw$$

where $w(t)$ is $N(0, t)$ and $p(0) = p_0$ has the solution

$$p(t) = p_0 e^{w(t) + \left(\mu - \frac{\sigma^2}{t}\right)t}.$$

Proof. Consider the SDE $dp = \mu pdt + \sigma pdw$.
Using ITO's formula, taking $f(p) = logp$, we get

$$\begin{aligned} d(logp) &= \frac{1}{p}dp + \frac{1}{2p^2}\sigma^2 p^2 dw^2 \\ &= \frac{1}{p}(\mu pdt + \sigma pdw) + \frac{\sigma^2}{2}dt \\ logp &= \mu t + \sigma w(t) - \frac{\sigma^2}{2}t + logp_0 \\ p(t) &= p_0 e^{w(t)+\left(\mu - \frac{\sigma^2}{t}\right)t}. \end{aligned}$$

□

5. Designing and Selection of the Plan Indexed Through AQL and RQL

This section provides the procedure for designing the plan indexed through AQL and RQL. Two points on the OC curve can be fixed such that the probability of acceptance of fraction defective $P_1(t)$ is β_1 and probability of acceptance of fraction defective $P_2(t)$ is β_2.

5.1. Procedure

1. Assume that the mixed plan is independent. .Split the probability of acceptance that will be assigned to the first stage. Let it be β_1' and β_2' respectively, such that $\beta_1 \geq \beta_1'$ and $\beta_2 \geq \beta_2'$.
2. Using the standard variable procedure, determine the first size n_1 as,
$$n_1 = \left[\frac{Z(\beta_2') - Z(\beta_1')}{Z(p_1(t)) - Z(p_2(t))}\right]^2.$$
3. Calculate the acceptance limit as $A = U - \left[Z(p_1(t)) + \frac{Z(\beta_1')}{\sqrt{n_1}}\right]\sigma$.
4. Now determine β_1'' and β_2'' the probability of acceptance assigned to the attributes plan associated with second stage sample as
$$\beta_1'' = \frac{\beta_1 - \beta_1'}{1 - \beta_1'} \text{ and } \beta_2'' = \frac{\beta_2 - \beta_2'}{1 - \beta_2'}.$$

5. Determine the appropriate second stage sample of size n_2 and acceptance number from:
$$\sum_{j=0}^{c} \frac{e^{-n_2 p_1(t)}(n_2 p_1(t)^j)}{j!} = \beta_1'' \text{ for fraction defective } p_1(t),$$
$$\sum_{j=0}^{c} \frac{e^{-n_2 p_2(t)}(n_2 p_2(t)^j)}{j!} = \beta_2'' \text{ for fraction defective } p_2(t).$$

$w(t) - \frac{t}{2}$	p_1	p_2	$p_1(t)$	$p_2(t)$	n_1	n_2	k	c
.1	.005	.015	.0055	.017	29	180	2.6105	2
.2	.004	.016	.0048	.019	19	170	2.67717	2
.3	.004	.019	.0053	.025	14	100	2.6615	0
.4	.004	.01	.0059	.015	42	160	2.5786	0
.5	.002	.02	.0032	.03	7	60	2.8736	0
.6	.0025	.004	.0045	.0073	200	250	2.6287	2
.7	.0025	.005	.00503	.01	76	260	2.6235	2
.8	.0025	.006	.0056	.0134	50	150	2.5937	0
.9	.005	.01	.0123	.0245	61	80	2.2986	0

Table 1: Values of n_1, n_2, k, c for given $\beta_1 = .95$, $\beta_2 = .05$ and variation factor at $t = 1hr.$ are given below:

Example 1. Determine the mixed sampling plan if the variation factor is 0.1, $AQL = .5\%$, $\beta = .05$, $LQL = 1.5\%$ at $t = 1hr.$

Solution. From table 1, the parameters are $n_1 = 29$, $n_2 = 180$, $k = 2.6105$, $c = 2$.

6. Conclusion

There are many situations in industry where the quality of a product can be actually measured. The variations in measurement with respect to time is taken into account and this new sampling plan by solving stochastic differential equation is presented in this paper. The designing procedure is given in detail. The new OC is derived using variable fraction defective. The required sample size for inspection and corresponding acceptance values, which provides the desired levels of protection for both producers and consumers are given in the table.

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