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HOMOMORPHISM OF ANTI FUZZY M-SEMIGROUP

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Abstract: The theory of semigroups motivated many researchers to engage themselves to work in that field. M-semigroup theory is another remarkable concept in the theory of semigroup. Inspired by the theory of anti fuzzy M-semigroup in this paper we introduce the notion of homomorphism between two anti fuzzy M-semigroups. Anti cartesian product of anti fuzzy M-semigroups also introduced and we provide some results on it.

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1. Introduction

In 1965, Zadeh [14] introduced the notion of fuzzy sets. Kuroki [5, 6, 7] has contributed much more results in fuzzy semigroups. The theory of M-semigroup was introduced by Lakshmanan in [8]. AL.Narayanan and A.R.Meenakshi [12] have introduced the notion of fuzzy M-subsemigroup as a generalization of M-semigroup. The concept of intuitionistic fuzzy set was introduced by K.T. Atanassov [2], as a generalization of the notion of fuzzy set. MA Xue-ling and ZHAN Jian-ming [11] introduced the notion of intuitionistic fuzzy M-semigroup

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as a generalization of fuzzy M-semigroup. They also described about the properties of intuitionistic (S,T) direct product of an M-semigroup. G. Deschrijver and E. Kerre [4] introduced the cartesian product of intuitionistic fuzzy sets. F.P. Choudhury et al [3] defined fuzzy subgroups and fuzzy homomorphism of fuzzy subgroups. Recently S. Vijayabalaji and S. Sivaramakrishnan [13] is to introduced the notion of anti fuzzy M-semigroup.

The purpose of this paper is introduce the notion of homomorphism of anti fuzzy M-semigroup. Anti cartesian product of anti fuzzy M-semigroups also introduced and we provide some results on it.

2. Preliminaries

In this section, we recall some notations and basic definitions used in this paper.

Definition 2.1. (see [8]) A semigroup M is called an M-semigroup if the following conditions are satisfied:

(i) there exists at least one left identity $e \in M$ such that $ex = x$, for all $x \in M$.

(ii) for every $x \in M$, there is a unique left identity, say e_x such that $xe_x = x$, that is e_x is a two-sided identity for x.

Definition 2.2. (see [13]) Let M be an M-semigroup. Let $\gamma : M \to [0,1]$ be a fuzzy set. Then (M, γ) is called an anti fuzzy M-semigroup if:

(i) $\gamma(xy) \leq \max\{\gamma(x), \gamma(y)\}$, for every $x, y \in M$,

(ii) $\gamma(e) = 0$, for every left identity e in M.

Example 2.3. (see [13]) Let $M = \{e, f, a, b\}$ be an M-semigroup with the following operation.

.	e	f	a	b
e	e	f	a	b
f	e	f	a	b
a	a	b	e	f
b	a	b	e	f

Define the fuzzy set $\gamma : M \to [0,1]$ by

$$\gamma(x) = \begin{cases} 0, & \text{if } x = e, f, \\ \alpha, & \text{otherwise, } 0 < \alpha \leq 1. \end{cases}$$

Then (M, γ) is an anti fuzzy M-semigroup.

Definition 2.4. (see [14]) A function μ from a non-empty set X to the interval $[0, 1]$ is called a fuzzy set of X and the complement of μ is denoted by μ^c is a fuzzy set of X defined by $\mu^c(x) = 1 - \mu(x)$ for all $x \in X$.

Definition 2.5. (see [15]) A mapping $f : M \to M'$ of M-semigroups is called a homomorphism if:

(*i*) $f(xy) = f(x)f(y)$, $\forall x, y \in M$;

(*ii*) $f(e) = e'$ for every left identity $e \in M$, where e' is a left identity of M'.

Let $f : M \to M'$ be a mapping of M-semigroups. For a fuzzy set μ in M', the inverse image of μ under f,denoted by $f^{-1}(\mu)$, is defined by $f^{-1}(\mu)(x) = \mu(f(x))$ for all $x \in M$.

Theorem 2.6. (see [15]) *Let T be a continuous t-norm and $f : M \to M'$ an onto homomorphism of M-semigroups. If μ is a T-fuzzy M-subsemigroup of M, then $f(\mu)$ is a T-fuzzy M-subsemigroup of M'.*

Definition 2.7. (see [1]) Let M_1 and M_2 be any sets and let $f : M_1 \to M_2$ be any function. A fuzzy subset μ of M_1 is called f-invariant if $f(x) = f(y)$ implies $\mu(x) = \mu(y)$, $x, y \in M_1$.

Definition 2.8. (see [7]) Let μ and σ be two fuzzy subsets of M. The product $\mu \circ \sigma$ is defined by

$$(\mu \circ \sigma)(x) = \begin{cases} \sup\limits_{x=yz} \min\{\mu(y), \sigma(z)\}, & \text{if } x \text{ is expressible as } x = yz, \\ 0, & \text{otherwise.} \end{cases}$$

Definition 2.9. (see [9]) Let μ and σ be two fuzzy set of a set X. The Cartesian product of μ and σ denoted by $\mu \times \sigma$ is fuzzy set defined by $(\mu \times \sigma)(x, y) = \min\{\mu(x), \sigma(y)\}$, for $x, y \in X$.

Definition 2.10. (see [12]) Let X_1 and X_2 be semigroups. Then we define the direct product of X_1 and X_2 as the set $X_1 \times X_2$ of all pairs (x, y) of elements x of X_1 and y of X_2 with the coordinate multiplication $(x, y)(x_1, y_1) = (xx_1, yy_1)$, for $x, x_1 \in X_1$ and $y, y_1 \in X_2$.

Definition 2.11. (see [10]) Let $(X, *)$ and $(Y, \cdot)$ be semigroups. A function α from X into Y is called a homomorphism if $\forall a, b \in X$, $\alpha(a * b) = \alpha(a) \cdot \alpha(b)$. Let α be a homomorphism of X into Y. If α is one-one, then α is called monomorphism. If α is onto Y, then α is called an epimorphism.

3. Homomorphism of Anti Fuzzy M-Semigroup

We now introduce the notion of homomorphism of anti fuzzy M-semigorup in the following theorems.

Definition 3.1. Let $f : M_1 \to M_2$ of M-semigorups. If γ is an anti fuzzy M-semigroup in M_2, then the inverse image of γ under f, denoted by $f^{-1}(\gamma)$, is an anti fuzzy M-semigroup in M_1, defined by $f^{-1}(\gamma)(x) = \gamma(f(x))$ for all $x \in M_1$.

Theorem 3.2. *Let $f : M_1 \to M_2$ be homomorphism of M-semigorups. If γ is an anti fuzzy M-semigroup of M_2, then the inverse image $f^{-1}(\gamma)$ of γ under f is an anti fuzzy M-semigorup of M_1.*

Proof. Assume that γ is an anti fuzzy M-semigroup of M_2 and $x, y \in M_1$. Then we have

$$(i)\ f^{-1}(\gamma)(xy) = \gamma(f(xy)) = \gamma(f(x)f(y))(\text{since } f \text{ is homomorphism}) \leq \max\{\gamma(f(x)), \gamma(f(y))\} = \max\{f^{-1}(\gamma(x)), f^{-1}(\gamma(y))\} \Rightarrow\ f^{-1}(\gamma)(xy) \leq \max\{f^{-1}(\gamma(x)), f^{-1}(\gamma(y))\}.$$

(ii) $f^{-1}(\gamma)(e) = \gamma(f(e)) = \gamma(e^{'}) = 0$, where $e^{'}$ is a left identity of M_2.
Therefore $f^{-1}(\gamma)$ is an anti fuzzy M-semigroup of M_1. □

Theorem 3.3. *Let γ be an anti fuzzy M-semigroup of M and let $f : M \to M$ be an onto homomorphism. Then the mapping $\gamma^f : M \to [0, 1]$, defined by $\gamma^f(x) = \gamma(f(x))$ for all $x \in M$, is an anti fuzzy M-semigroup of M.*

Proof. (i) For any $x, y \in M$,

$$\gamma^f(xy) = \gamma(f(xy)) = \gamma(f(x)f(y))\ (\text{since } f \text{ is homomorphism}) \leq \max\{\gamma(f(x)), \gamma(f(y))\} = \max\{\gamma^f(x), \gamma^f(y)\}.$$

$$\Rightarrow \gamma^f(xy) \leq \max\{\gamma^f(x), \gamma^f(y)\}.$$

(ii) $\gamma^f(e) = \gamma(f(e)) = \gamma(e^{'}) = 0$. □

Theorem 3.4. *Let $f : M_1 \to M_2$ be an epimorhism of M-semigroups. Let γ be a f-invariant anti fuzzy M-semigroup of M_1. Then $f(\gamma)$ is an anti fuzzy M-semigroup of M_2.*

Proof. (i) Let $x^{'}, y^{'} \in M_2$. Then there exist $x, y \in M_1$ such that $f(x) = x^{'}$ and $f(y) = y^{'}$, then $x^{'}y^{'} = f(xy)$ and let $e^{'} \in M_2$. Then there exists $e \in M_1$ such that $f(e) = e^{'}$,where e and $e^{'}$ are the left identity of M_1 and M_2.

Since γ is f-invariant, $f(\gamma)(xy) = \gamma(x'y') \leq \max\{\gamma(x'), \gamma(y')\}$
$= \max\{f(\gamma)(x), f(\gamma)(y)\}$
$\Rightarrow f(\gamma)(xy) \leq \max\{f(\gamma)(x), f(\gamma)(y)\}$

(ii) $f(\gamma)(e) = \gamma(e') = 0$.
Therefore $f(\gamma)$ is an anti fuzzy M-semigroup of M_2. □

Theorem 3.5. *Let M be an M-semigroup and γ be a fuzzy set in M. Then γ is an anti fuzzy M-semigroup in M iff γ^c is an fuzzy M-semigroup in M.*

Proof. Let γ be an anti fuzzy M-semigroup in M. We have for all $x, y \in M$

$$\gamma^c(xy) = 1 - \gamma(xy) \geq 1 - \max\{\gamma(x), \gamma(y)\} = \min\{1 - \gamma(x), 1 - \gamma(y)\} = \min\{\gamma^c(x), \gamma^c(y)\},$$

$$\Rightarrow \gamma^c(xy) \geq \min\{\gamma^c(x), \gamma^c(y)\},$$

and $\gamma^c(e) = 1 - \gamma(e) = 1 - 0, \Rightarrow \gamma^c(e) = 1$.

Hence γ^c is also an fuzzy M-semigroup in M.

Conversely, let γ^c be a fuzzy M-semigroup in M. To prove γ is an anti fuzzy M-semigroup in M. We have

$$\gamma(xy) = 1 - \gamma^c(xy) \leq 1 - \min\{\gamma^c(x), \gamma^c(y)\} = \max\{1 - \gamma^c(x), 1 - \gamma^c(y)\} = \max\{\gamma(x), \gamma(y)\}$$

$$\Rightarrow \gamma(xy) \leq \max\{\gamma(x), \gamma(y)\},$$

and $\gamma(e) = 1 - \gamma^c(e) = 1 - 1 = 0$.

$$\Rightarrow \gamma(e) = 0.$$

Hence γ is an anti fuzzy M-semigroup in M. □

Definition 3.6. Let γ_1, γ_2 be two anti fuzzy M-semigroups. Then $(\gamma_1 \times \gamma_2)(x, y) = \max\{\gamma_1(x), \gamma_2(y)\}$ for all $x, y \in M$ is called an anti cartesian product of γ_1 and γ_2.

Theorem 3.7. *If γ_1 and γ_2 are anti fuzzy M-semigroups of M_1 and M_2 respectively, then $\gamma_1 \times \gamma_2$ is an anti fuzzy M-semigroup of $M_1 \times M_2$.*

Proof. Let $(a, b), (c, d) \in M_1 \times M_2$
(i) $(\gamma_1 \times \gamma_2)((a, b), (c, d)) = (\gamma_1 \times \gamma_2)(ac, bd) = \max\{\gamma_1(ac), \gamma_2(bd)\}$
$\leq \max\{\max(\gamma_1(a), \gamma_1(c)), \max(\gamma_2(b), \gamma_2(d))\}$

$\leq \max\{\max(\gamma_1(a), \gamma_2(b)), \max(\gamma_1(c), \gamma_2(d))\}$
$= \max\{(\gamma_1 \times \gamma_2)(a, b), (\gamma_1 \times \gamma_2)(c, d)\}$

(ii) $(\gamma_1 \times \gamma_2)(e, e^{'})$, where e and $e^{'}$ are left identity identity in M_1 and M_2
$= \max\{\gamma_1(e), \gamma_2(e^{'})\}$
$= \max\{0, 0\}$
$= 0$
Therefore $\gamma_1 \times \gamma_2$ is an anti fuzzy M-semigroup of $M_1 \times M_2$. □

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