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# $(\lambda, \mu)$-FUZZY STRONG $N$-INNER PRODUCT SPACE

S. Vijayabalaji[1], G. Shyamsundar[2]

[1]Department of Mathematics
University College of Engineering Panruti
(A Constituent College of Anna University Chennai)
Panruti, 607 106, Tamilnadu, INDIA

[2]Department of Mathematics
Kongunadu College of Engineering and Technology
Tiruchirappalli, Dist-621215, Tamilnadu, INDIA

**Abstract:** Inspired by the theory of fuzzy strong $n$-inner product space and from the theory of $(\lambda, \mu)$-fuzzy normal and quotient subgroups, in this paper we introduce the notion of $(\lambda, \mu)$-fuzzy $n$-inner product space. Given two fuzzy strong $n$-inner product space we define their equivalence condition and provide some results on it.

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**Key Words:** $n$-inner product, fuzzy $n$-inner product, fuzzy strong $n$-inner product, $\alpha$-$n$-inner product

## 1. Introduction

Remarkable contribution in the theory of 2-inner product space and $n$-inner product space has been effectively made by eminent researchers in [3, 4, 5, 6, 7]. Recently S.Vijayabalaji and N.Thillaigovindan have introduced the notion of fuzzy $n$-inner product space in [9]. In [11] as a natural generalization of fuzzy $n$-inner product space, the notion of fuzzy strong $n$-inner product space has been introduced.

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S. Vijayabalaji and N. Thillaigovindan raised a problem of constructing $\alpha$-$n$-inner product space in [9] and answer to this problem is provided in [11] by constructing $\alpha$-strong $n$-inner product space.

Analogue of $\alpha$-$n$-inner product space in [9] the notion of $\alpha$-strong $n$-inner product space is introduced in [11]. We have also introduced approximation theory in $\alpha$-$n$-normed linear space [10]. Following X. Yuan et al[12], in[13] B. Yao introduced the theory of $(\lambda, \mu)$-fuzzy normal and quotient subgroups as a generalization of fuzzy normal and quotient subgroups. $\lambda$ and $\mu$ are called as thresholds in $[0, 1]$ with $\lambda < \mu$.

In this paper, we first define the notion of $(\lambda, \mu)$-fuzzy strong $n$-inner product space as a generalization of fuzzy strong $n$-inner product space and then we define their equivalent conditions by providing some interesting results on it.

## 2. Preliminaries

In this section we recall some familiar concepts which will be needed in the sequel.

**Definition 2.1** [2]**.** Let $n$ be a natural number greater than 1 and X be a real linear space of dimension greater than or equal to $n$ and let $(\bullet, \bullet|\bullet, ..., \bullet)$ be a real valued function on $\underbrace{X \times ... \times X}_{n+1} = X^{n+1}$ satisfying the following conditions:

(1) $(i)(x, x|x_2, ..., x_n) \geq 0$,
$(ii)(x, x|x_2, ..., x_n) = 0$ if any only if $x, x_2, ..., x_n$ are linearly dependent,
(2) $(x, y|x_2, ..., x_n) = (y, x|x_2, ..., x_n)$,
(3) $(x, y|x_2, ..., x_n)$ is invariant under any permutation of $x_2, ..., x_n$,
(4) $(x, x|x_2, ..., x_n) = (x_2, x_2|x, x_3, ..., x_n)$,
(5) $(ax, x|x_2, ..., x_n) = a(x, x|x_2, ..., x_n)$ for every $a \in R$(real),
(6) $(x + x', y|x_2, ..., x_n) = (x, y|x_2, ..., x_n) + (x', y|x_2, ..., x_n)$.
Then $(\bullet, \bullet|\bullet, ..., \bullet)$ is called an $n$-inner product on X and $(X, (\bullet, \bullet|\bullet, ..., \bullet))$ is called an $n$-inner product space.

**Definition 2.2.**[9] Let $X$ be a linear space over a field $F$. A fuzzy subset $J : X^{n+1} \times R\,(R-$ set of real numbers) is called a fuzzy $n$-inner product on $X$ if and only if:
(1) For all $t \in R$ with $t \leq 0$, $J(x, x|x_2, ..., x_n, t) = 0$;
(2) For all $t \in R$ with $t > 0$, $J(x, x|x_2, ..., x_n, t) = 1$ if and only if $x, x_2, ..., x_n$ are linearly dependent;
(3) For all $t > 0$, $J(x, y|x_2, ..., x_n, t) = J(y, x|x_2, ..., x_n, t)$;

(4) $J(x,y|x_2,...,x_n,t)$ is invariant under any permutation of $x_2,...,x_n$;
(5) For all $t>0$, $J(x,x|x_2,...,x_n,t)=J(x_2,x_2|x,x_3,...,x_n,t)$;
(6) For all $t>0$, $J(ax,bx|x_2,...,x_n,t)=J(x,x|x_2,...,x_n,\frac{t}{|ab|})$, $a,b\in R$(real);
(7) For all $s,t\in R$,

$$J(x+x^{'},y|x_2,...,x_n,t+s)\geq\min\{J(x,y|x_2,...,x_n,t),J(x^{'},y|x_2,...,x_n,s)\};$$

(8) For all $s,t\in R$ with $s>0$, $t>0$,

$$J(x,y|x_2,...,x_n,\sqrt{ts})\geq\min\{J(x,x|x_2,...,x_n,t),J(y,y|x_2,...,x_n,s)\};$$

(9) $J(x,y|x_2,...,x_n,t)$ is a non-decreasing function of $t\in R$ and

$$\lim_{t\to\infty}\text{J}(x,y|x_2,...,x_n,t)\text{=}1.$$

Then $(X,J)$ is called a fuzzy $n$-inner product space or in short f-n-IPS.

**Definition 2.3 [11]** Let $X$ be a linear space over a field $F$. A fuzzy subset $J:X^{n+1}\times R\,(R-$set of real numbers) is called a fuzzy strong $n$-inner product on $X$ if and only if:
(1) For all $t\in R$ with $t\leq 0$, $J(x,x|x_2,...,x_n,t)=0$;
(2) For all $t\in R$ with $t>0$, $J(x,x|x_2,...,x_n,t)=1$ if and only if $x,x_2,...,x_n$

are linearly dependent;
(3) For all $t>0$, $J(x,y|x_2,...,x_n,t)=J(y,x|x_2,...,x_n,t)$;
(4) $J(x,y|x_2,...,x_n,t)$ is invariant under any permutation of $x_2,...,x_n$;
(5) For all $t>0$, $J(x,x|x_2,...,x_n,t)=J(x_2,x_2|x,x_3,...,x_n,t)$;
(6) For all $t>0$, $J(ax,bx|x_2,...,x_n,t)=J(x,x|x_2,...,x_n,\frac{t}{|ab|})$, $a,b\in R$(real);
(7) For all $s,t\in R$,

$$J(x+x^{'},y|x_2,...,x_n,t+s)=\min\{J(x,y|x_2,...,x_n,t),J(x^{'},y|x_2,...,x_n,s)\};$$

(8) For all $s,t\in R$ with $s>0$, $t>0$,

$$J(x,y|x_2,...,x_n,\sqrt{ts})=\min\{J(x,x|x_2,...,x_n,t),J(y,y|x_2,...,x_n,s)\};$$

(9) $J(x,y|x_2,...,x_n,t)$ is a non-decreasing function of $t\in R$ and

$$\lim_{t\to\infty}\text{J}(x,y|x_2,...,x_n,t)\text{=}1.$$

Then $(X,J)$ is called a fuzzy strong $n$-inner product space or in short f-ST-$n$-IPS.

**Example 2.4 [11].** Let $(X,(\bullet,\bullet|\bullet,...,\bullet))$ be an $n$-inner product space.

Define

$$J(x,y|x_2,...,x_n,t) = \begin{cases} \dfrac{t}{t+|(x,y|x_2,...,x_n)|}, & \text{when } t>0, t\in R, \\ & (x,y|x_2,...,x_n)\in X^{n+1} \\ 0, & \text{when } t\le 0. \end{cases}$$

Then $(X,J)$ is a f-ST-$n$-IPS.

**Theorem 2.5 [11].** Let (X,J) be a f-ST-$n$-IPS. Assume the condition that (10) $J(x,x|x_2,...,x_n,t)>0$ implies $x,x_2,...,x_n$ are linearly dependent.
Define $(x,x|x_2,...,x_n)_\alpha$=inf$\{t:$ J$(x,x|x_2,...,x_n,t)\ge\alpha\}$,$\alpha\in(0,1)$.
Then $\{(\bullet,\bullet|\bullet,...,\bullet)_\alpha:\alpha\in(0,1)\}$,is an ascending family of strong $n$-inner products on $X$. We call these $n$-inner products as strong $\alpha$-$n$-inner product on $X$ corresponding to the fuzzy strong $n$-inner product on $X$.

**Remark 2.6 [11].** We assume that (10) For $x_1,x_2,...,x_n$ linearly independent, $J(x,y|x_2,...,x_n,t)$ is continuous functions and strictly increasing on the subset $\{t:0<J(x,y|x_2,...,x_n,t)<1\}$ of $R$.

**Definition 2.7 [13].** Let $\lambda,\mu\in[0,1]$ and $\lambda<\mu$ and let $A$ be a fuzzy subset of $G$. Then $A$ is called a fuzzy subgroup with thresholds of $G$ if for all $x,y\in G$,

*(i)* $A(xy)\vee\lambda\ge A(x)\wedge A(y)\wedge\mu$
*(ii)* $A(x^{-1})\vee\lambda\ge A(x)\wedge\mu$.

## 3. Equivalent $(\lambda,\mu)$-Fuzzy Strong $n$-Inner Product Spaces

We now enter into our new notion of equivalent $(\lambda,\mu)$-fuzzy strong $n$-inner product space. Before proceeding we generalize the notion of fuzzy strong $n$-inner product space, by defining $(\lambda,\mu)$-f-ST-$n$-IPS as follows.

**Definition 3.1.** Let $X$ be a linear space over a field $F$. A fuzzy subset $J:X^{n+1}\times(0,\infty)$ is called a $(\lambda,\mu)$-fuzzy strong $n$-inner product on $X$ if and only if:
(1) $J(x,x|x_2,...,x_n,t)\vee(\lambda\wedge\mu)>0$;
(2) $J(x,x|x_2,...,x_n,t)\vee(\lambda\wedge\mu)=1$ if and only if $x,x_2,...,x_n$

are linearly dependent;
(3) $J(x,y|x_2,...,x_n,t)\vee(\lambda\wedge\mu)=J(y,x|x_2,...,x_n,t)\vee(\lambda\wedge\mu)$;

(4) $J(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu)$ is invariant under any permutation of $x_2, ..., x_n$;
(5) $J(x, x|x_2, ..., x_n, t) \vee (\lambda \wedge \mu) = J(x_2, x_2|x, x_3, ..., x_n, t) \vee (\lambda \wedge \mu)$;
(6) $J(ax, bx|x_2, ..., x_n, t) \vee (\lambda \wedge \mu) = J(x, x|x_2, ..., x_n, \frac{t}{|ab|}) \vee (\lambda \wedge \mu)$, $a, b \in F$;
(7) $J(x+x', y|x_2, ..., x_n, t+s) \vee (\lambda \wedge \mu) = J(x, y|x_2, ..., x_n, t) * J(x', y|x_2, ..., x_n, s) \vee (\lambda \wedge \mu)$;
(8) $J(x, y|x_2, ..., x_n, \sqrt{ts}) \vee (\lambda \wedge \mu) = J(x, x|x_2, ..., x_n, t) * J(y, y|x_2, ..., x_n, s) \vee (\lambda \wedge \mu)$;
(9) $J(x, y|x_2, ..., x_n, t)$ is a non-decreasing function of $t$ and

$$\lim_{t \to \infty} \mathrm{J}(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu) = 1.$$

Then $(X, J)$ is called a $(\lambda \wedge \mu)$-fuzzy strong $n$-inner product space or in short $(\lambda, \mu)$-f-ST-$n$-IPS. Here $\lambda$ and $\mu$ are called as thresholds in $[0, 1]$ with $\lambda < \mu$.

**Remark 3.2.** In the above definition when $n = 1$, it is called as $(\lambda, \mu)$ -fuzzy inner product space.

The following example justifies the definition 3.1.

**Example 3.3.** Let $(X, (\bullet, \bullet|\bullet, ..., \bullet))$ be an $n$-inner product space.

Define $a * b = \min\{a, b\}$ and $J(x, y|x_2, ..., x_n, t) = \dfrac{(\lambda \wedge \mu)t}{(\lambda \wedge \mu)t + |(x, y|x_2, ..., x_n)|}$, where $\lambda, \mu \in [0, 1]$ and $t \in (0, \infty)$. Then $(X, J)$ is a $(\lambda, \mu)$-f-ST-$n$-IPS.

**Remark 3.4 .** Let (X,J) be a $(\lambda, \mu)$-f-ST-$n$-IPS. Assume the condition that (10) $J(x, x|x_2, ..., x_n, t) \vee (\lambda \wedge \mu) > 0$ implies $x, x_2, ..., x_n$ are linearly dependent. Define $(x, x|x_2, ..., x_n)_\alpha = \inf\{t : \mathrm{J}(x, x|x_2, ..., x_n, t) \vee (\lambda \wedge \mu) \geq \alpha\}$,$\alpha \in (0, 1)$. Then $\{(\bullet, \bullet|\bullet, ..., \bullet)_\alpha : \alpha \in (0, 1)\}$,is an ascending family of strong $n$-inner products on $X$. We call these $n$-inner products as strong $\alpha$-$n$-inner product on $X$ corresponding to the $(\lambda, \mu)$-fuzzy strong $n$-inner product on $X$.

**Definition 3.5.** Let $A = (X, J_1)$ and $B = (X, J_2)$ be two $(\lambda, \mu)$-f-ST-$n$-IPS. Then $A$ and $B$ are said to be equivalent if there exists positive constants $a$ and $b$ such that

$$J_2(x, y|x_2, ..., ax_n, t) \vee (\lambda \wedge \mu) \leq J_1(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu) \leq J_2(x, y|x_2, ..., bx_n, t) \vee (\lambda \wedge \mu)$$

$\forall t \in R$. We denote it by $J_1 \sim J_2$.

**Example 3.6.** Let $(X, (\bullet, \bullet|\bullet, ..., \bullet))$ be an $n$-inner product space. Define

$$J_1(x, y|x_2, ..., x_n, t) = \frac{k_1(\lambda \wedge \mu)t}{k_1(\lambda \wedge \mu)t + |(x, y|x_2, ..., x_n)|},\ t > 0.$$

Then $A = (X, J_1)$ is a $(\lambda, \mu)$-f-ST-$n$-IPS.

Also define

$$J_2(x,y|x_2,...,x_n,t) = \frac{k_2(\lambda\wedge\mu)t}{k_2(\lambda\wedge\mu)t + |(x,y|x_2,...,x_n)|},\ t>0.$$

Then $B=(X,J_2)$ is a $(\lambda,\mu)$-f-ST-$n$-IPS.

Choose $k_1 < k_2$ and $a > b$, where $k_1,k_2,a,b>0$.
Then $A$ and $B$ are equivalent $(\lambda,\mu)$-f-ST-$n$-IPS.

**Theorem 3.7.** The relation $\sim$ defined above is an equivalence relation.

*Proof.* (i) The relation is reflexive, since

$J_2(x,y|x_2,...,1.x_n,t)\vee(\lambda\wedge\mu) \le J_1(x,y|x_2,...,x_n,t)\vee(\lambda\wedge\mu) \le J_2(x,y|x_2,...,1.x_n,t)\vee(\lambda\wedge\mu)$ $\forall t\in R$.

(ii) To prove symmetry, let

$J_2(x,y|x_2,...,ax_n,t)\vee(\lambda\wedge\mu) \le J_1(x,y|x_2,...,x_n,t)\vee(\lambda\wedge\mu) \le J_2(x,y|x_2,...,bx_n,t)\vee(\lambda\wedge\mu)$ $\forall t\in R$.

We have to prove that there exists positive numbers $c$ and $d$ such that

$J_1(x,y|x_2,...,cx_n,t)\vee(\lambda\wedge\mu) \le J_2(x,y|x_2,...,x_n,t)\vee(\lambda\wedge\mu) \le J_1(x,y|x_2,...,dx_n,t)\vee(\lambda\wedge\mu)$ $\forall t\in R$. We have

$$J_2(x,y|x_2,...,ax_n,t)\vee(\lambda\wedge\mu) \le J_1(x,y|x_2,...,x_n,t)\vee(\lambda\wedge\mu)$$
$$\Rightarrow J_2(x,y|x_2,...,x_n,\frac{t}{a})\vee(\lambda\wedge\mu) \le J_1(x,y|x_2,...,x_n,t)\vee(\lambda\wedge\mu).$$

Let $s=\frac{t}{a}$ then,

$$\begin{aligned} J_2(x,y|x_2,...,x_n,s)\vee(\lambda\wedge\mu) &\le J_1(x,y|x_2,...,x_n,as)\vee(\lambda\wedge\mu)\\ &= J_1(x,y|x_2,...,x_n,\frac{s}{\frac{1}{a}})\vee(\lambda\wedge\mu)\\ &= J_1(x,y|x_2,...,\frac{x_n}{a},s)\vee(\lambda\wedge\mu)\end{aligned}$$

$$J_2(x,y|x_2,...,x_n,s)\vee(\lambda\wedge\mu) \le J_1(x,y|x_2,...,\frac{x_n}{a},s)\vee(\lambda\wedge\mu) \tag{3.1}$$

On the other hand

$$\begin{aligned} J_1(x,y|x_2,...,x_n,t)\vee(\lambda\wedge\mu) &\le J_2(x,y|x_2,...,bx_n,t)\vee(\lambda\wedge\mu)\\ &= J_2(x,y|x_2,...,x_n,\frac{t}{b})\vee(\lambda\wedge\mu)\end{aligned}$$

Replacing $\frac{bt}{a}$ for $t$ we get,

$$J_1(x,y|x_2,...,x_n,\frac{bt}{a})\vee(\lambda\wedge\mu) \le J_2(x,y|x_2,...,x_n,\frac{t}{a})\vee(\lambda\wedge\mu)$$
$$\Rightarrow J_1(x,y|x_2,...,x_n,bs)\vee(\lambda\wedge\mu) \le J_2(x,y|x_2,...,x_n,s)\vee(\lambda\wedge\mu)$$
$$\Rightarrow J_1(x,y|x_2,...,\frac{x_n}{b},s)\vee(\lambda\wedge\mu) \le J_2(x,y|x_2,...,x_n,s)\vee(\lambda\wedge\mu) \tag{3.2}$$

Now by (3.1) and (3.2) we get,

$J_1(x,y|x_2,...,\frac{x_n}{b},s)\vee(\lambda\wedge\mu) \le J_2(x,y|x_2,...,x_n,s)\vee(\lambda\wedge\mu) \le J_1(x,y|x_2,...,\frac{x_n}{a},s)\vee$

$(\lambda \wedge \mu)$
$\Rightarrow J_1(x, y|x_2, ..., cx_n, s) \vee (\lambda \wedge \mu) \leq J_2(x, y|x_2, ..., x_n, s) \vee (\lambda \wedge \mu) \leq J_1(x, y|x_2, ..., dx_n, s) \vee (\lambda \wedge \mu)$ (3.3)
where $c = \dfrac{1}{b}$ and $d = \dfrac{1}{a}$
From (3.3) it follows that $\sim$ is symmetric.
(iii) To prove transitivity, let

$J_0(x, y|x_2, ..., ax_n, t) \vee (\lambda \wedge \mu) \leq J(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu) \leq J_0(x, y|x_2, ..., bx_n, t) \vee (\lambda \wedge \mu)$

$J_1(x, y|x_2, ..., cx_n, t) \vee (\lambda \wedge \mu) \leq J_0(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu) \leq J_1(x, y|x_2, ..., dx_n, t) \vee (\lambda \wedge \mu)$.

Then we show that there exist two positive numbers $e$ and $f$ such that

$J_1(x, y|x_2, ..., ex_n, t) \vee (\lambda \wedge \mu) \leq J(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu) \leq J_1(x, y|x_2, ..., fx_n, t) \vee (\lambda \wedge \mu)$.

Now $J_1(x, y|x_2, ..., cx_n, t) \vee (\lambda \wedge \mu) \leq J_0(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu)$
$\Rightarrow J_1(x, y|x_2, ..., x_n, \frac{t}{c}) \vee (\lambda \wedge \mu) \leq J_0(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu)$
$\Rightarrow J_1(x, y|x_2, ..., ax_n, \frac{t}{c}) \vee (\lambda \wedge \mu) \leq J_0(x, y|x_2, ..., ax_n, t) \vee (\lambda \wedge \mu)$
$\Rightarrow J_1(x, y|x_2, ..., acx_n, t) \vee (\lambda \wedge \mu) \leq J_0(x, y|x_2, ..., ax_n, t) \vee (\lambda \wedge \mu)$
So,

$$\begin{aligned} J_1(x, y|x_2, ..., acx_n, t) \vee (\lambda \wedge \mu) &\leq J_0(x, y|x_2, ..., ax_n, t) \vee (\lambda \wedge \mu) \\ &\leq J(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu) \\ &\leq J_0(x, y|x_2, ..., bx_n, t) \vee (\lambda \wedge \mu) \end{aligned} \quad (3.4)$$

Also $J_0(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu) \leq J_1(x, y|x_2, ..., dx_n, t) \vee (\lambda \wedge \mu)$
$\Rightarrow J_0(x, y|x_2, ..., bx_n, t) \vee (\lambda \wedge \mu) \leq J_1(x, y|x_2, ..., bdx_n, t) \vee (\lambda \wedge \mu)$ (3.5)
From (3.4) and (3.5)

$J_1(x, y|x_2, ..., acx_n, t) \vee (\lambda \wedge \mu) \leq J(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu) \leq J_1(x, y|x_2, ..., bdx_n, t) \vee (\lambda \wedge \mu)$.

Choose $ac = e$ and $bd = f$
$\Rightarrow J_1(x, y|x_2, ..., ex_n, t) \vee (\lambda \wedge \mu) \leq J(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu) \leq J_1(x, y|x_2, ..., fx_n, t) \vee (\lambda \wedge \mu)$ (3.6)
From (3.6) we see that $\sim$ is transitive. □

**Theorem 3.8.** Let $A$ and $B$ be two $(\lambda, \mu)$-f-ST-$n$-IPS satisfying (10) and (11). Then $A$ and $B$ are equivalent if and only if their corresponding strong $\alpha$-$n$-inner products are equivalent for all $\alpha \in (0, 1)$.

*Proof.* Let $A$ and $B$ be two equivalent $(\lambda, \mu)$-f-ST-$n$-IPS. Then there exists positive constants $a, b$ and $c, d$ such that

$J_2(x, y|x_2, ..., ax_n, t) \vee (\lambda \wedge \mu) \leq J_1(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu) \leq J_2(x, y|x_2, ..., bx_n, t) \vee (\lambda \wedge \mu)$ $\forall t \in R$.

Let $(\bullet, \bullet|\bullet, ..., \bullet)^1_\alpha$ and $(\bullet, \bullet|\bullet ..., \bullet)^2_\alpha$ where $\alpha \in (0, 1)$ be the corresponding strong

$\alpha$-$n$-inner products of $A$ and $B$ respectively.
First we show that
$J_2(x, y|x_2, ..., ax_n, t) \vee (\lambda \wedge \mu) \leq J_1(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu)$ for all $t \in R$
$\Leftrightarrow (x, y|x_2, ..., x_n)^1_\alpha \leq (x, y|x_2, ..., ax_n)^2_\alpha$ for all $\alpha \in (0, 1)$.
Suppose $J_2(x, y|x_2, ..., ax_n, t) \vee (\lambda \wedge \mu) \leq J_1(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu)$ holds for all $t \in R$.
Now $(x, y|x_2, ..., ax_n)^2_\alpha < t$
$\Rightarrow \inf\{s : J_2(x, y|x_2, ..., ax_n, s) \vee (\lambda \wedge \mu) \geq \alpha, \} < t$
$\Rightarrow \exists s_0 < t$ such that $J_2(x, y|x_2, ..., ax_n, s_0) \geq \alpha$
$\Rightarrow J_1(x, y|x_2, ..., x_n, s_0) \vee (\lambda \wedge \mu) \geq \alpha, \alpha \in (0, 1)$
$\Rightarrow (x, y|x_2, ..., x_n)^1_\alpha \leq s_0 < t$
$\Rightarrow (x, y|x_2, ..., x_n)^1_\alpha \leq (x, y|x_2, ..., ax_n)^2_\alpha \quad (3.7)$
Next we suppose that $(x, y|x_2, ..., x_n)^1_\alpha \leq (x, y|x_2, ..., ax_n)^2_\alpha$ for all $\alpha \in (0, 1)$.
Now $\nu < J_2(x, y|x_2, ..., ax_n, t) \vee (\lambda \wedge \mu)$
$\Rightarrow \nu < \sup\{\alpha \in (0, 1) : (x, y|x_2, ..., ax_n)^2_\alpha \leq t\}$
$\Rightarrow \exists \alpha_0 \in (0, 1)$ such that $\nu < \alpha_0$ and $(x, y|x_2, ..., ax_n)^2_{\alpha_0} \leq t$
$\Rightarrow (x, y|x_2, ..., x_n)^1_{\alpha_0} \leq t$
$\Rightarrow \nu < J_1(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu)$
$\Rightarrow J_2(x, y|x_2, ..., ax_n, t) \vee (\lambda \wedge \mu) \leq J_1(x, y|x_2, ..., x_n, t) \vee (\lambda \wedge \mu) \quad (3.8)$
From (3.7) and (3.8) it follows that
$J_2(x, y|x_2, ..., ax_n, t) \vee (\lambda \wedge \mu) \leq J_2(x, y|x_2, ..., bx_n, t) \vee (\lambda \wedge \mu)$
$\Leftrightarrow (x, y|x_2, ..., x_n)^1_\alpha \leq (x, y|x_2, ..., ax_n)^2_\alpha$ for all $\alpha \in (0, 1)$ □

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