GENERALIZED TREE OF HANGING STARS

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Abstract: In this paper, we give an algorithm that enables us to compute the neighbourhood of vertices of generalized tree obtained from a family of n stars having number of branches of those stars form an arithmetic progression (geometric progression) with common difference (ratio) j and one of the branches of each of those stars merged with one different point of a common path on n vertices successively in increasing order.

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1. Introduction

A simple undirected graph $G = (V(G), E(G))$ with p vertices and q edges. A function f is called graph labeling of graph G if $f: V \rightarrow \{0, 1, 2, \dots, q\}$ is injective and the induced function $f^*: E \rightarrow \{1, 2, 3, \dots, q\}$ defined as $f^*(e = uv) = |f(u) - f(v)|$ is bijective. All edge values are unique and distinct.

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Gallian,[3] gives extensive survey on graceful labeling. Huang, Kotzig, and Rosa [4] gives a new class of graceful trees, Sethuraman and Jesintha [10] shows a new class of graceful rooted trees, they showed generating new graceful trees[11] and Michelle Edwards and Lea Howard, given a survey of graceful trees[8]. In [5, 6] the authors found graceful labeling for trees with hanging stars which are in arithmetic progression and geometric progression respectively and they also proved that distinct and uniqueness of labeling.

Definition 1. (see [5]) Let P_n be basic path of $T_{A(n),a}^j$ tree. Let $s_1, s_2, \dots, s_n$ be such vertices, which are termed as supporting vertices of $T_{A(n),a}^j$ tree. In $T_{A(n),a}^j$ at each s_i , a star S_i with i branches having centre c_i with one of the branch vertex of S_i merged with s_i and denote the free leaves of each of the stars S_i by $(f_{(i)}^1, f_{(i)}^2, \dots, f_{(i)}^{i-1})$ for $i = 1, 2, \dots, n$. Here $(|S_1|, |S_2|, \dots, |S_n|)$ forms an arithmetic progression with common difference j and hence it has been denoted as $T_{A(n),a}^j$ tree, where $(|E(S_1)|) = a$.

For an example, a general tree $T_{A(n),a}^j$ is given below.

Figure 1: $T_{A(n),a}^j$ tree

Definition 2. (see [6]) Let P_n be basic of $T_{G(n),a}^r$ tree. Let $s_1, s_2, \dots, s_n$ be such vertices, which are, term it as supporting vertices of $T_{G(n),a}^r$ tree. In $T_{G(n),a}^r$ at each s_i , a star S_i with i branches having centre c_i with one of the branch vertex of S_i merged with s_i and denote the free leaves of each of the stars S_i by $(f_{(i)}^1, f_{(i)}^2, \dots, f_{(i)}^{i-1})$ for $i = 1, 2, \dots, n$. Here $(|S_1|, |S_2|, \dots, |S_n|)$ form geometric progression with common ratio r and hence it has been denoted as $T_{G(n),a}^r$ tree, where $(|E(S_1)|) = a$.

For example a general tree $T_{G(n),a}^r$ follows.

A topological index is a real number related to any (molecular) graph. It must be a structural invariant, i.e., it does not depend on the labeling or the pictorial representation of a graph. There are several topological indices

Figure 2: $T_{G(n),a}^r$ tree

[8] namely Randic[9]; Zagreb[9]; Szeged and Balaban [10]; Wiener[11]; PI[12]; have been defined and many of them have found applications as means to model chemical, pharmaceutical and other properties of molecules. For all these indices calculation we requires the number of adjacent vertices to i , i.e., adjacent vertices of each vertex i (neighbourhood of vertex i).

2. Neighbourhood of Vertices

In this section, we give an algorithm that enables us to compute the neighbourhood of the vertices of tree with hanging stars. For this purpose, the following algorithm is presented:

- At first, we assign to any vertex one number.
- We determine all of adjacent vertices set of the vertex (neighborhood) and this set denoted by $N[i]$.

First, we design algorithm for tree with hanging stars in arithmetic progression. It requires the number of hanging stars (m), number of stars in first star (a) and common difference (d). The complete procedure is given in Algorithm 1. Next, we design the algorithm for tree with hanging stars in geometric progression. It requires the number of hanging stars (m), number of stars in first star (a) and common ratio (r). The complete procedure is given in Algorithm 2.

Algorithm 1

```

Step 1: Read m, a, d
Step 2: N := [ ] ( array)
Step 3: n =  $\frac{m}{2}(2a+(m-1)d)+m$ 
Step 4: t1 = 1
Step 5: for i = 1 to m
  ti = a + (i-1) d
  t2 = t1 + ti
  for j = t1 to t2
    if(j = 1) then N[j] = [j+1, j+ti+1]
    else if (j = n - a - (m-1)d) then
      N[j] = [j - ti]
    else if (i=t1 and j > 1) then
      N[j] = [j - ti - 1, t2+1]
    else if (j = t1+1) then
      N[j] = [j+1, j+2, ... , t2]
    else N[j] = [t1+ 1]
  end if
end for
t1 = t2 + 1
end for
Step 6: for i = 1 to n
  Display N[i]
end for
Step 7: Stop

```

Algorithm 2

```

Step 1: Read m; r; a;
Step 2: N:= [ ] (array)
Step 3: n =  $\frac{r^m-1}{r-1}a+m$ 
Step 4: t1=1
Step 5: for i= 1 to m
  ti =  $a(r^{i-1})$ 
  t2 = t1+ti
  for j = t1 to t2
    if (j = 1) then
      N[j] = [j+1, j+ti+1]
    else if (j = n - a(ri-1)) then
      N[j] = [j -  $\frac{ti}{r}$ -1]
    else if (i=t1 and j > 1) then
      N[j] = [j -  $\frac{ti}{r}$ - 1, t2+1]
    else if (j = t1+1) then
      N[j] = [j+1, j+2, ... , t2]
    else N[j] = [t1+ 1]
  end if
end for
t1=t2+1
end for
Step 6 : for i=1 to n
  Display N[i]
end for
Step 7: stop

```

3. Conclusions

It takes a long time to compute the topological indices of a graph theoretically, especially when the graph has many vertices. An algorithm has been presented for computing the neighborhood of the vertices of generalized tree of hanging stars. We can write a program to compute these neighborhoods quickly. Based on these neighborhoods we can find degree and topological indices of this generalized tree of hanging stars.

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