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GLOBAL PROPERTIES OF THE RATIONAL DIFFERENCE EQUATIONS $x_{n+1} = \frac{\alpha x_n + \gamma y_n}{A x_n + C y_n}$ AND $y_{n+1} = \frac{\beta x_n + \delta y_n}{B x_n + D y_n}$ IN EXCEPTION HANDLING

S. Elizabeth[1], R. Jothilakshmi[2 §]

[1,2]Department of Mathematics
Auxilium College (Autonomous)
Vellore, 632 006, Tamilnadu, INDIA

Abstract: We present some results for convergence properties of a system of non linear rational difference equations subject to multi -point exception handling. The convergence of solution to this equation is investigated by introducing an exception handling techniques. We also prove the convergence properties and boundedness concepts for a more general class of rational difference equation. The obtained results are applied to the analysis of exception handling techniques which is associated with error identification and noise reduction in non linear filters like Extended Kalman Filter associated with non linear difference equations. Finally some numerical examples are showed for exception handling techniques and the same is draw it by *MATLAB*.

AMS Subject Classification: 39A10, 39A11
Key Words: rational difference equations, convergence, exceptions, boundary conditions

1. Introduction

The study of the nonlinear rational difference equations of a higher order is quite challenging and rewarding [7]. The results about these equations offer prototypes towards the development of the basic theory of the global behavior

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§Correspondence author

of nonlinear difference equations of a higher order [2,4]. Recently, many researchers have investigated the behavior of the solution of difference equations. Difference equations arise in the situations in which the discrete values of the independent variable involve. Many practical phenomena are modeled with the help of difference equations [1,8]. In engineering, difference equations arise in control engineering, digital signal processing, electrical networks, etc. In social sciences, difference equations arise to study the national income of a country and then its variation with time, Cobweb phenomenon in economics, etc.

Periodic solutions of difference equations have been investigated by many researchers, and various methods have been proposed for the existence and qualitative properties of the solution [11]. The study of rational difference equations of order greater than one is quite challenging and rewarding [5]. There has been a great interest in studying the global attractivity, the boundedness character and the periodicity nature of nonlinear difference equations [19].

Recently there has been a great deal of interest in studying the behavior of rational difference equations [13, 14, 21]. Our goal in this paper is to investigate the behavior of positive solutions of the recursive equation

$$x_{n+1} = \frac{\alpha x_n + \gamma y_n}{A x_n + C y_n} \quad \text{and} \quad y_{n+1} = \frac{\beta x_n + \delta y_n}{B x_n + D y_n}, \tag{1}$$

where $\alpha, \beta, \gamma, \delta, A, B, C$ and D are non negative parameter. We develop several approaches which allow us to extend boundary conditions of the above rational difference equations and handled the exceptions.

The paper is organized as follows: In Section II, we recall the basic concepts and definitions and results which are used throughout the paper, also we obtained the main result of the paper which is to identify and overcome (reduce) the exceptions which are going to produce the noise of any system of signals. Some numerical simulations to the equation are given to illustrate our results in Section III which are implemented in an Extended Kalman Filter. Section IV concludes the paper.

2. Main Results

Consider the following system of difference equations,

$$x_{n+1} = \frac{\alpha x_n + \gamma y_n}{A x_n + C y_n} \quad \text{and} \quad y_{n+1} = \frac{\beta x_n + \delta y_n}{B x_n + D y_n}.$$

In order to analyze this system, we consider $u_n = \frac{y_n}{x_n}$, we can always arrange at least one non zero coefficient is equal to one in either the numerator or the denominator or both. Let $x_n = 1$, then (1) becomes,

$$x_{n+1} = \frac{\alpha + \gamma u_n}{A + C u_n} \quad \text{and} \quad y_{n+1} = \frac{\beta + \delta u_n}{B + D u_n}. \tag{2}$$

From the above equations we extend u_n into u_{n+1}, then we have

$$u_{n+1} = \frac{(\beta + \delta u_n)(A + C u_n)}{(B + D u_n)(\alpha + \gamma u_n)}. \tag{3}$$

We will break up the analysis into the various cases, depending upon the coefficients of u_{n+1}. Thus the difference equation reduces to a single variable for these cases [9, 20].

Theorem 2.1. *Suppose x_n and y_n satisfying system (1). Then for any given choice of parameters, exactly one of the following must happen :*

(i) Every solution converges to a fixed point of the system;

(ii) There exists a unique positive prime period two solution and every solution converges either to the prime period two solution or to a fixed point of the system;

(iii) There exist unbounded solutions.

Proof. We will now go through the different cases depending upon which the coefficients of u_{n+1} are positive in (1).

Case (1) Assume that C, D, γ and δ are all positive. By the change of variables, we may express (1) in the form,

$$u_{n+1} = \frac{(\beta + u_n)(A + u_n)}{(B + u_n)(\alpha + u_n)}. \tag{4}$$

Lemma 2.2. *Consider the difference equation,*

$$u_{n+1} = \frac{(a + u_n)(b + u_n)}{(c + u_n)(d + u_n)},$$

with a, b, c and d are non negative constants. For all values of the parameters a, b, c and d every solution converges either to a fixed point or to a unique positive prime period two solution. Furthermore the fixed point to which the solution converges must be positive with the following exceptions.

(i) If $b = 0$, and $a < cd$ then there exist solutions so that $u_n \to 0$, as $n \to \infty$.

(ii) If $b = 0$, and $a = cd$ and $c + d \geq 1$, then there exist solutions so that $u_n \to 0$ as $n \to \infty$.

By the Lemma (2.2), every solution converges either to a fixed point of (4) or to a unique positive prime period two solution. The fixed point must be positive with certain exceptions. By excluding these exceptions the system converges either to a positive fixed point of (1) or to a unique positive prime period two solution. To identify the exception consider the following various cases.

Case 1.1 Assume that $A = 0, \beta = B\alpha$ and $B + \alpha \geq 1$..

By exception (ii) in Lemma 2.2, there exist solutions so that $u_n \to 0$. From this we get

$$x_{n+1} = \frac{\alpha + u_n}{u_n} \to \infty \quad \text{if} \quad \alpha > 0.$$

If $\alpha = 0$, then there exist solutions so that $x_{n+1} = 1$, for all $n \geq 0$ and $y_{n+1} = \frac{u_n}{B+u_n} \to 0$.

Case 1.2 Assume that $\beta = 0, A = B\alpha$ and $B + \alpha \geq 1$. By exception (ii) in Lemma 2.2, there exist solutions so that $u_n \to 0$. If $A > 0$, then there exists a solution,

$$x_{n+1} = \frac{\alpha + u_n}{A + u_n} \to \frac{\alpha}{A} \quad \text{and} \quad y_{n+1} = \frac{u_n}{B + u_n} \to 0.$$

Case 1.3 Assume that $A = 0$, and $\beta < B\alpha$.

By exception (ii) in Lemma 2.2, there exist solutions so that $u_n \to 0$. Thus $x_{n+1} = \frac{\alpha+u_n}{u_n} \to \infty$

Case 1.4 Assume that $\beta = 0$ and $A < B\alpha$. By exception (ii) in Lemma (2.2), there exist solutions so that $u_n \to 0$. If $A > 0$, then there exist a solution so that,

$$x_{n+1} = \frac{\alpha + u_n}{A + u_n} \to \frac{\alpha}{A} \quad \text{and} \quad y_{n+1} = \frac{u_n}{B + u_n} \to 0.$$

Case 2 Assume that γ, C and δ are positive and $D = 0$ in (2). By relabeling the parameter and change of variables, we can express the system in the following form,

$$x_{n+1} = \frac{\alpha x_n + y_n}{A x_n + y_n} textand \quad y_{n+1} = \frac{\beta x_n + \delta y_n}{x_n}, \tag{5}$$

where $\delta > 0$.

Case 2.1 Assume that $\alpha > 0, A > 0$ and $\beta > 0$. By the change of variables $x_n \to y_n$ and $y_n \to x_n$, the system becomes

$$x_{n+1} = \frac{\delta x_n + \beta y_n}{y_n} \quad \text{and} \quad y_{n+1} = \frac{x_n + \alpha y_n}{x_n + a y_n}.$$

After a further change of variables and relabeling the parameters, we can express the system in the form,

$$x_{n+1} = \frac{\alpha x_n + y_n}{y_n} \quad \text{and} \quad y_{n+1} = \frac{\beta x_n + y_n}{B x_n + y_n}, \tag{6}$$

with α, β and B are being positive. We apply (1) in the lemma (2.2), then we have,

$$x_{n+1} = \frac{u_n(u_n + \delta)}{(u_n + \beta)(u_n + \alpha)}. \tag{7}$$

Under the assumption of $\delta \neq \beta$ and $\alpha > 0$, now by the lemma 2.2 every solution of (1) converges to a fixed point. Furthermore, the fixed point must be positive with the exceptions to be covered. The exceptions are:

(i) Assume that $\beta < \alpha B$;
(ii) Assume that $\beta = \alpha B$ and $\alpha + B \geq 1$..

Case 2.2 Assume that $\alpha > 0, A > 0$ and $\beta = 0$. Here we have the following equations,

$$x_{n+1} = \frac{\alpha + u_n}{A + u_n}, \quad y_{n+1} = \delta u_n \quad \text{and} \quad u_{n+1} = \frac{\delta u_n(u_n + A)}{u_n + \alpha}.$$

Case 2.3 Assume that $\delta = 1$ and $A > \alpha$ or $\delta > 1$ and $A\delta \geq \alpha$.

We know that $u_{n+1} > u_n$ and it is obvious that $u_n \to 0$. Hence $y_{n+1} = \delta u_n \to \infty$ and then unbounded solution exists.

Case 2.4 Assume that $\delta < 1$ and $A\delta \leq \alpha$ or $\delta = 1$ and $A < \alpha$. We have $u_{n+1} < u_n$ then it is easy to conclude that $u_n \to 0$. If $\alpha > 0$ and $A = 0$, then $x_{n+1} \to \infty$. Otherwise

$$x_{n+1} = \frac{\alpha + u_n}{A + u_n} \to \frac{\alpha}{A}, \quad \text{and} \quad y_{n+1} = \delta u_n \to 0.$$

Case 2.5 Assume that $\delta > 1$ and $A\delta \leq \alpha$.

Note that,0 and $\bar{u} = \frac{\alpha - A\delta}{\delta - 1} > 0$ are the only fixed points of (7). Also we observe that $\lim\limits_{u\to\infty} \frac{f(u)}{u} = \delta > 1$ and $\lim\limits_{u\to\infty} \frac{f(u)}{u} = \frac{A\delta}{\alpha} < 1$. Thus $f(u) < u$ for $u \in (0, \bar{u})$ and $f(u) > u$ for $\bar{u} \in (\bar{u}, \infty)$. Since f is strictly increasing, then whenever $u_0 \in (0, \bar{u})$ then $u_n \to 0$ and whenever $u_0 \in (\bar{u}, \infty)$ then $u_n \to \infty$. Thus $y_{n+1} = \delta u_n \to \infty$ and so unbounded solution exist.

Case 2.6 Assume that $\delta < 1$ and $A\delta > \alpha$. Consider the fixed points of (7) i.e. 0 and $\bar{u} = \frac{\alpha - A\delta}{\delta - 1} > 0$. Further observing that $\lim\limits_{u\to\infty} \frac{f(u)}{u} = \delta > 1$ and

$\lim\limits_{u\to\infty} \frac{f(u)}{u} = \frac{A\delta}{\alpha} > 1$. Thus $f(u) > u$ for $u \in (0, \bar{u})$ and $f(u) < u for \in ?(\bar{u}, \infty)$, Since$f$ is strictly increasing then by analysis we get $u_n \to \bar{u}$ for all possible positive initial conditions and hence $\{x_n, y_n\}$ converges to all its initial values.

Case 2.7 Assume that $\delta \geq 1$. Here we have $u_{n+1} > u_n$ and it obvious that $u_n \to \infty$. Thus $y_{n+1} = \beta + \delta u_n$ and hence unbounded solution exist.

Case 2.7 Assume that $\delta < 1$. Obviously there is exactly one positive fixed point say $\bar{u}$. If we consider 0 is the fixed point then either $\beta = 0$ or $A = 0$. If both $\beta = 0$ and $A = 0$ then it is trivial to see that $u_n \to \bar{u}$, where $\bar{u}$ is the unique positive fixed point and hence the system converges to its unique positive fixed point. If suppose β and A are not equal to zero, then there exists a critical point and it gives an absolute minimum value. Then check that a necessary condition for here to be a prime period two solution m and M. If there are no prime period two solutions then the system converges to its unique positive fixed point.

Case 2.8 Assume that $\alpha > 0$ and $A = 0$.
In this case we have

$$u_{n+1} = u_n \frac{\delta u_n + \beta}{u_n + \alpha}.$$

Now we consider the following Theorem and Lemma about exception handling techniques of rational difference equations. □

Theorem 2.3. *Consider*

$$u_{n+1} = \frac{A(u_n + a)(u_n + b}{u_n + c}, \tag{8}$$

with a, b and c are non negative,$A > 0$ and the latest one of a, b or c equals to zero. Then every solution converges to the unique positive fixed point with the following exceptions:

(i) Suppose that $c = 0, a > 0, b > 0$ and $A \geq 1$ then every solution $u_n \to \infty$.

(ii) Suppose that $a = 0, A > 1$ and $Ab \geq c$ or that $a = 0, A = 1$ and $b < c$. Then $u_n \to \infty$.

(iii) Suppose that $a = 0, A = 1$ and $b = c$, then $u_n = u_0$ for all $n \geq 1$.

(iv) Suppose that $a = 0, A > 1$ and $Ab < c$. Then if $u_n \in (0, \bar{u})$ we have $u_n \to 0$. If $u_0 \in (\bar{u}, \infty)$ then $u_n \to \infty$.

(v) If one reverses the roles of a and b in exceptions (ii) through (iv) the same results will follow.

Now consider the following lemma,

Lemma 2.4. *Consider the system (1) and the rational difference equation (3) with $C, A, \gamma, \alpha, D, B, \delta$ and β are non negative constants. Then it follows that*

(i) $(\bar{x}, \bar{y})$ is a positive fixed point of (1) if and only if $\bar{u}$ is a fixed point of (3), where

$$x_{n+1} = \frac{\alpha + \gamma u_n}{Cu_n + A}, \quad y_{n+1} = \frac{\beta + \delta u_n}{B + Du_n} \quad \text{and} \quad \bar{u} = \frac{\bar{x}}{\bar{y}}.$$

Furthermore the system converges to $(\bar{x}, \bar{y})$ if and only if the solution for (3) converges to $\bar{u}$.

(ii) (m_1, m_2) and (M_1, M_2) are positive prime period two solutions of the system if and only if m and M is a positive prime period two solution of (3) where,

$$m = \frac{m_2}{m_1} \quad \text{and} \quad M = \frac{M_2}{M_1},$$

$$m_1 = \frac{\alpha + \gamma m}{A + Cm}, \quad m_2 = \frac{\beta + \delta m}{B + Dm}, \quad M_1 = \frac{\alpha + \gamma M}{A + CM} \quad \text{and} \quad M_2 = \frac{\beta + \delta M}{B + DM}.$$

Furthermore, the system converges to (m_1, m_2) and (M_1, M_2) if and only if the equation (3) converges to the prime period two solutions m and M. By Theorem (2.3) and Lemma (2.4), every solution of the system converges to the unique positive fixed point with the exceptions shown in Table 1.

Theorem 2.5. *Suppose*

$$u_{n+1} = \frac{1}{(a + u_n)(b + u_n)}, \tag{9}$$

where a and b are non negative constants, then for all positive initial conditions every solution converges either to a unique positive prime period two solution or to a positive fixed point with the following exception. If either $a = 0$ or $b = 0$ then whenever $u_0 \in (0, \bar{u})$ we have $u_{2n} \to 0$ and $u_{2n+1} \to \infty$ and whenever $u_0 \in (\bar{u}, \infty)$, we have $u_{2n} \to \infty$ and $u_{2n+1} \to 0$ as $n \to \infty$.

Proof. f is strictly increasing on $(0, \infty)$ and hence the proof is trivial.

Case 2.9 Assume that γ and D are positive and that C and δ are both obtain the value of zero. In this case we express our equation (2) as

$$x_{n+1} = \alpha_n + u_n, \quad y_{n+1} = \frac{1}{B + u_n} \quad \text{and} \quad u_{n+1} = \frac{1}{(\alpha + u_n)(B + u_n)}.$$

S.No	Condition	Exceptions
1	Suppose that $\delta > 1$ and $\beta \geq 1$	Then by exception (ii) of Theorem 2.3, every solution $u_n \to \infty$ and so $y_{n+1} = \beta + u_n \to \infty$.
2	Suppose that $\delta = 1$ and $\beta \geq \delta\alpha$	Then by exception (ii) of Theorem 2.3, every solution $u_n \to \infty$ and so $y_{n+1} = \beta + u_n \to \infty$.
3	Suppose that $\delta = 1$ and $\alpha = \beta$	Then by exception (ii) of Theorem 2.3, $u_n = u_0$ for all $n \geq 1$.Thus $x_{n+1} = \frac{\alpha x_0 + y_0}{y_0}$ and $y_{n+1} = \frac{\alpha x_0 + y_0}{x_0}$ for all $n \geq 0$.
4	Suppose that $\delta > 1$ and $\beta \leq \alpha$ or $\delta = 1$ and $\beta < \alpha$	Then by exception (iv) of Theorem 2.3, if $u_o \in (0, \bar{u})$, we have $u_n \to 0$, and so $x_{n+1} = \frac{\alpha + u_n}{u_n} \to \infty$.
5	Suppose that $\delta > 1$ and $\beta < \alpha$	Then by exception (v) of Theorem 2.3, if $u_o \in (0, \bar{u})$ we have $u_n \to \infty$. Thus if $y_0/x_0 \in (o, \bar{u})$ then we have $x_{n+1} = \frac{\alpha + u_n}{u_n} \to \infty$.

Table 1: Exceptions of Case 2.8

By Theorem (2.3) and Lemma (2.4) every solution of the system converges either to a unique positive prime period two solution or to a positive fixed point with the exception that either $\alpha = 0$ or $B = 0$. By Theorem (2.3) if $u_{\in}(\bar{u}, \infty)$ then $u_{2n} \to \infty$. Thus if $\frac{y_0}{x_0} > \bar{u}$, then $x_{2n+1} = \alpha + u_{2n} \to \infty$.

Case 2.10 Assume that γ, δ and D are all positive and that $\gamma = 0$.

This shows that $\alpha > 0$, otherwise x_n will becomes zero and the original form of x_{n+1} and y_{n+1} are,

$$x_{n+1} = \frac{1}{A + u_n} \quad \text{and} \quad y_{n+1} = \frac{\beta + \delta y_n}{B + u_n}.$$

where $u_{n+1} = \frac{(\delta u_n + \beta)(A + u_n)}{u_n + B}$ with $\delta > 0, \beta > 0, A > 0$ and $B > 0$.

Case 2.11 Assume that A, B and β are all positive. Now we relabeled x_n by y_n and then y_n by x_n. Now we get the system of equations in the form of,

$$x_{n+1} = \frac{\alpha' x_n + y_n}{A' x_n + y_n} \quad \text{and} \quad y_{n+1} = \frac{y_n}{B' x_n + y_n},$$

where α', A' and B' are positive. This provides us,

$$x_{n+1} = \frac{\alpha' + u_n}{A' + u_n} \quad \text{and} \quad y_{n+1} = \frac{u_n}{B' + u_n},$$

S.No	Condition	Exceptions
1	If $\beta > 0, B = 0, A > 0$ and $\delta \geq 1$	By Theorem 2.3, exception (i), every solution $u_n \to \infty$ and hence $x_{n+1} \to 0$ and $y_{n+1} \to \delta$.
2	If $\beta = 0$ and $\delta A > B$ and $\delta \geq 1$	By Theorem 2.3, exception (ii), every, every solution $u_n \to \infty$ and hence $x_{n+1} \to 0$ and $y_{n+1} \to \delta$
3	If $\beta = 0, \delta = 1$ and $A > B$	By Theorem 2.3 exception (i), every solution $u_n \to \infty$ and hence $x_{n+1} \to 0$ and $y_{n+1} \to \delta$.
4	If $\beta = 0, \delta = 1$ and $A = B$	By Theorem 2.3 exception (iii) $u_n = u_0$ for all $n \geq 1$. Thus $x_{n+1} = \frac{x_0}{\alpha x_0 + y_0}$ and $y_{n+1} = \frac{y_0}{\alpha + y_0}$ for all $n \geq 0$.
5	If $\beta = 0, \delta < 1$ and $\alpha\delta \leq B$	By Theorem 2.3 exception (iv) $u_n \to 0$. Then $x_n = \frac{1}{A}$ and $y_n \to 0$.
6	If $\beta = 0, \delta > 1$ and $\alpha\delta < B$	By Theorem 2.3 exception (v) $?u_0 \in (0, \bar{u})$ then $u_n \to 0$, and if $u_0 \in (\bar{u}, \infty)$ then $u_n \to \infty$
7	If $A > 0, \delta > 1, \beta \geq B$	By Theorem 2.3, exception (ii), for every solution $u_n \to \infty$ and hence $x_n \to 0$ and $y_n \to \delta$.
8	If $A = 0, \delta = 1, \beta = B$	By Theorem 2.3, exception (ii), for every solution $u_n = u_0$ for all $n \geq 1$. Thus $x_{n+1} = \frac{x_0}{y_0}$ and $y_{n+1} = 1$, for all $n \geq 1$.
9	If $A = 0, \delta < 1, \beta \leq B$	By Theorem 2.3 exception (iv) $u_n \to 0$. Then $x_n \to \infty$.

Table 2: Exceptions of Case 2.12

with

$$u_{n+1} = \frac{u_n(A' + u_n)}{(u_n + \alpha')(u_n + B')}. \tag{10}$$

By Lemma (2.2) every solution of (10) converges to a fixed point. Furthermore, the fixed point must be positive with a couple of exceptions. By Lemma (2.4) the system must also converge to a positive fixed point. These exceptions are

(i) Suppose $A' < \alpha' B'$ then the solutions will be $x_{n+1} \to \infty$.

(ii) Suppose $A' = \alpha' B'$ and $\alpha' + B' \geq 1$, then the solutions will be $x_n \to \infty$.

Case 2.12 Assume that at least one of A, B and β is equal to zero. By Theorem (2.3) and Lemma (2.4), every solution of the system converges to a unique positive fixed point with the exceptions shown in Table 2.

This concludes the Theorem (2.1) □

3. Simulation

In discrete time a wide variety of data filtering, time series analysis, and digital filtering systems and algorithms are described by difference equations. The extended Kalman filter [10] (EKF) is a very popular method in engineering, that allows to compute an estimate of the state of a dynamical system from several sensors measurements, possibly corrupted by measurement's noise [6]. The principle is to merge predictions from a trusted model of the dynamics of the system with measurements, in order to efficiently filter the noise and get an accurate estimate of the (unknown) internal state of the system in real time [12,18]. The method presented here deals with the measurement of the parameters of a system signal which is usually contaminated with noise and high disturbances [15,17].

It has been shown that the system of difference equations estimated by a signals and systems version of the EKF [3, 16]. Then, the filtering problem is equivalent to the unbiased minimization problem subject to assumptions on the noises and the initial conditions [6]. The solutions of the above system of difference equation problem lead to the standard EKF model. In this case, we study the solution of the following system of the difference equations

$$x_{n+1} = \frac{\alpha x_n + \gamma y_n}{A x_n + C y_n} \quad \text{and} \quad y_{n+1} = \frac{\beta x_n + \delta y_n}{B x_n + D y_n}.$$

Here $n \in N_0$ and the initial conditions are arbitrary real numbers such that $x_{-3}, x_{-2}, x_{-1}, x_0$ and $y_{-3}, y_{-2}, y_{-1}, y_0$ with non negative parameters $\alpha, \beta, \gamma, \delta, A$, B, C and D.

Example 3.1. We consider some interesting numerical example for the system of rational difference equation (1) with initial conditions, where $x_{-3} = 0.8, x_{-2} = 0.2, x_{-1} = 1.4, x_0 = 0.3, y_{-3} = 1.7, y_{-2} = 0.5, y_{-1} = 0.5$, and $y_0 = 0.3$. The values of the non negative parameters are $\alpha = 1.7, \beta = 0.7, \gamma = 0.6, \delta = 1.4, A = 1.7, B = 1.9, C = 2.4$ and $D = 0.2$ (see Figure 1)

Example 3.2. Figure 2 shows the behavior of the solution of the system of rational difference equation (1) with the initial conditions, where $x_{-3} = 0.4, x_2 = 0.5, x_{-1} = 1.6, x_0 = 0.7, y_{-3} = 2.7, y_{-2} = 0.5, y_{-1} = 0.5$, and $y_0 = 0.8$. The values of the non negative parameters are $\alpha = 1.9, \beta = 0.9, \gamma = 0.8, \delta = 1.4, A = 1.9, B = 2.1, C = 2.6$ and $D = 0.2$ (see Figure 2)

Example 3.3. Figure 3 shows the behavior of the solution of the system of rational difference equation (1) with the initial conditions, where $x_{-3} =$

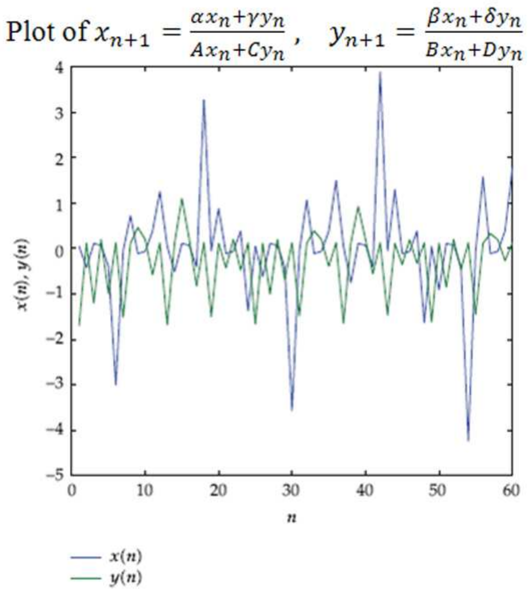

Figure 1

$1.5, x_{-2} = 0.8, x_{-1} = 1.9, x_0 = 0.9, y_{-3} = 1.8, y_{-2} = 0.5, y_{-1} = 0.5$, and $y_0 = 0.3$. The values of the non negative parameters are $\alpha = 2.4, \beta = 1.4, \gamma = 1.6, \delta = 1.3, A = 1.4, B = 0.5, C = 1.7$ and $D = 0.2$ (see Figure 3)

Example 3.4. Figure 4 shows the behavior of the solution of the system of rational difference equation (1) with the initial conditions, where $x_{-3} = 1.4, x_{-2} = 0.7, x_{-1} = 1.1, x_0 = 0.7, y_{-3} = 1.6, y_{-2} = 1, y_{-1} = 0.7$, and $y_0 = 0.2$. The values of the non negative parameters are $\alpha = 0.7, \beta = 0.7, \gamma = 0.6, \delta = 1.4, A = 1.7, B = 1.9, C = 2.4$ and $D = 0.2$ (see Figure 4)

4. Conclusion

In this paper we have analyzed boundedness properties of some rational difference equations with a new concept as exception handling techniques. We applied this technique to noise reduction and enhanced the stability of a non linear extended Kalman filters via rational difference equations. Finally Numerical examples are simulated to support our result through *MATLAB*.

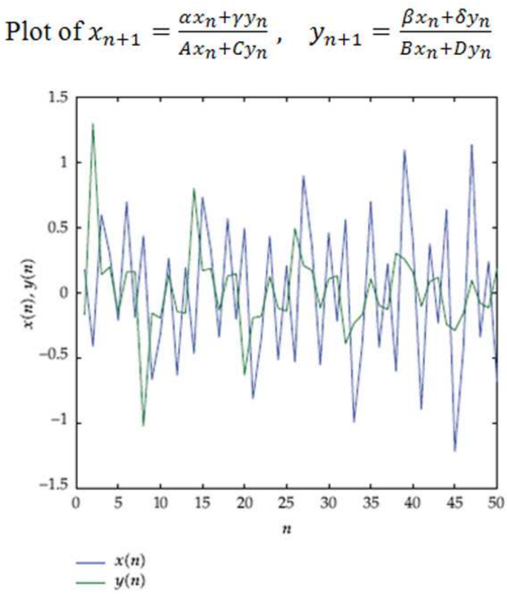

Figure 2

Acknowledgments

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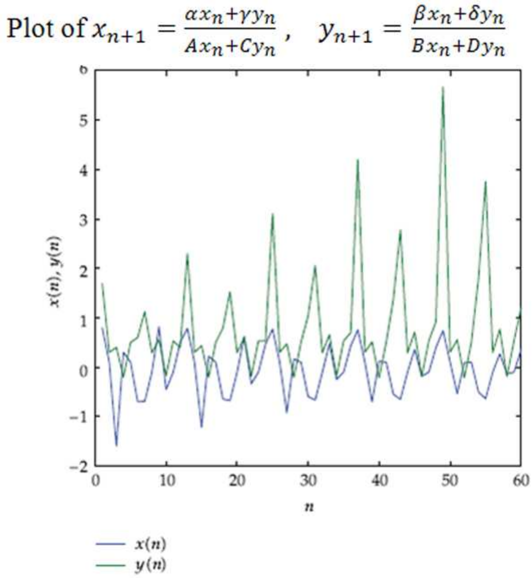

Figure 3

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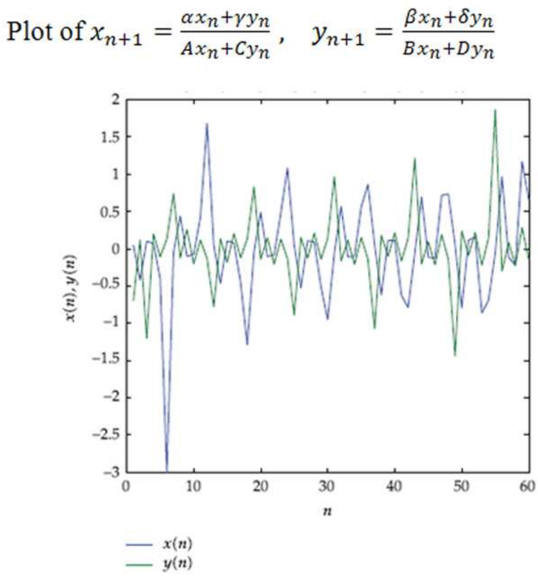

Figure 4

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