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DIFFERENTIAL RECURRENCE RELATION OF GENERALIZED K-WRIGHT FUNCTION

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Abstract: The principal aim of this paper is to establish differential recurrence relation and Integral representation of generalized K-Wright function ${}_p\Psi_q^k(z)$ introduced by Gehlot and Prajapati [2]. In the end some interesting special cases have also been discussed.

AMS Subject Classification: generalized K-Wright function, K-gamma function, Mittag-Leffler funtion
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1. Introduction

Generalized K-Gamma function $\Gamma_k(x)$ defined as (see Diaz and Pariguan [1])

$$\Gamma_k\left(x\right) = \lim_{n\to\infty} \frac{n!\ k^n (nk)^{\frac{x}{k}\ -\ 1}}{(x)_{n,k}}\ ,\ k>0,\ x\in\mathbb{C}\backslash k\mathbb{Z}^-, \tag{1}$$

where $(x)_{n,k}$ is the k-Pochhammer symbol and is given by

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$$(x)_{n,k} = x(x+k)(x+2k)\ldots\ldots\ldots\ldots(x+(n-1)k), \tag{2}$$

where $x \in \mathbb{C}$, $k \in \mathbb{R}$, $n \in \mathbb{N}^+$. For $Re(x) > 0$, $\Gamma_k(x)$ is K-Gamma function defined as the integral

$$\Gamma_k(x) = \int_0^\infty t^{x-1} e^{\frac{-t^k}{k}}\, dt \tag{3}$$

It is easy to prove following results

$$\Gamma_k(x) = k^{\frac{x}{k}-1}\Gamma\left(\frac{x}{k}\right), \tag{4}$$

$$\Gamma_k(x+k) = x\Gamma_k(x), \tag{5}$$

$$(\gamma)_{r,k} = (k)^r \left(\frac{\gamma}{k}\right)_r, \tag{6}$$

$$(\gamma)_{r,k} = \frac{\Gamma_k(\gamma+rk)}{\Gamma_k(\gamma)}, \tag{7}$$

$$rk(x)_{r-1,k} = (x)_{r,k} - (x-k)_{r,k}, \tag{8}$$

$$(\delta)_{n+j,k} = (\delta)_{j,k}(\delta+jk)_{n,k}. \tag{9}$$

2. Generalized K-Wright Function

Concept of the generalized K-Wright function introduced by Gehlot and Prajapati [2] as:

Definition. The generalized K-Wright function is defined as ${}_p\Psi_q^k(z)$ for $k \in \mathbb{R}^+; z \in \mathbb{C}, a_i, b_j \in \mathbb{C}, \alpha_i, \beta_j \in \mathbb{R}(\alpha_i, \beta_j \neq 0; i = 1, 2, ..., p; j = 1, 2, ..., q)$ and $(a_i + \alpha_i n), (b_j + \beta_j n) \in \mathbb{C}\backslash k\mathbb{Z}^-$,

$${}_p\Psi_q^k(z) = {}_p\Psi_q^k\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\end{array}\middle|\, z\right] = \sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)z^n}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_i n)n!}. \tag{10}$$

For convergence, we use the following notations

$$\Delta = \sum_{j=1}^{q}\left(\frac{\beta_j}{k}\right) - \sum_{i=1}^{p}\left(\frac{\alpha_i}{k}\right), \quad \delta = \prod_{i=1}^{p}\left|\frac{\alpha_i}{k}\right|^{\frac{-\alpha_i}{k}}\prod_{j=1}^{q}\left|\frac{\beta_j}{k}\right|^{\frac{\beta_j}{k}},$$

$$\mu = \sum_{j=1}^{q}\left(\frac{b_j}{k}\right) - \sum_{i=1}^{p}\left(\frac{a_i}{k}\right) + \frac{p-q}{2}.$$

Theorem 2.1. *For $k \in \mathbb{R}^+$; $z \in \mathbb{C}$, $a_i, b_j \in \mathbb{C}$, $\alpha_i, \beta_j \in \mathbb{R}(\alpha_i, \beta_j \neq 0; i = 1,2,...,p; j = 1,2,...,q)$ and $(a_i + \alpha_i n), (b_j + \beta_j n) \in \mathbb{C}\backslash k\mathbb{Z}^-$,*

(a) *If $\Delta > -1$ then series (10) is absolutely convergent for all $z \in \mathbb{C}$ and generalized K-Wright function ${}_p\Psi_q^k(z)$ is an entire function of z.*

(b) *If $\Delta = -1$ then series (10) is absolutely convergent for all $|z| < \delta$ and of $|z| = \delta$, $Re(\mu) > \frac{1}{2}$.*

3. Differential Recurrence Relation

In this section we evaluate the recurrence relation of generalized K-Wright function, ${}_p\Psi_q^k(z)$.

Theorem 3.1. *For $k \in \mathbb{R}^+; z \in \mathbb{C}; a_i, b_j, \beta + s \in \mathbb{C}; Re(\beta + s) > 0$ and $\alpha + \gamma, \alpha_i, \beta_j \in \mathbb{R}$ $(\alpha_i, \beta_j \neq 0; i = 1,2,\ldots,p; j = 1,2,\ldots,q)$, then*

$$\begin{aligned}
&{}_p\Psi_{q+1}^k\left[\begin{matrix}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q},(\beta+s+k,\alpha+r)\end{matrix}\middle| z\right]\\
&\qquad - k\,{}_p\Psi_{q+1}^k\left[\begin{matrix}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q},(\beta+s+2k,\alpha+r)\end{matrix}\middle| z\right]\\
&\qquad = z^2(\alpha+\gamma)^2\,{}_p\ddot{\Psi}_{q+1}^k\left[\begin{matrix}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q},(\beta+s+3k,\alpha+r)\end{matrix}\middle| z\right]\\
&+ z\{(\alpha+\gamma)^2 + 2(\alpha+\gamma)(\beta+s+k)\}\,{}_p\dot{\Psi}_{q+1}^k\left[\begin{matrix}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q},(\beta+s+3k,\alpha+r)\end{matrix}\middle| z\right]\\
&\qquad + (\beta+s)(\beta+s+2k)\,{}_p\Psi_{q+1}^k\left[\begin{matrix}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q},(\beta+s+3k,\alpha+r)\end{matrix}\middle| z\right]. \qquad (11)
\end{aligned}$$

Proof. From definition (10) and (5), we have

$$\begin{aligned}
&{}_p\Psi_{q+1}^k\left[\begin{matrix}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q},(\beta+s+k,\alpha+\gamma)\end{matrix}\middle| z\right]\\
&= \sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)}\frac{z^n}{n!}\frac{1}{(\beta+s+(\alpha+\gamma)n)\Gamma_k(\beta+s+(\alpha+\gamma)n)}. \qquad (12)
\end{aligned}$$

So

$$ {}_p\Psi^k_{q+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\ ,(\beta+s+2k,\alpha+\gamma)\end{matrix}\middle|z\right]=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)}\frac{z^n}{n!} $$

$$ \times\frac{1}{(\beta+s+k+(\alpha+\gamma)n)(\beta+s+(\alpha+\gamma)n)\Gamma_k(\beta+s+(\alpha+\gamma)n)}, \quad (13) $$

this can be reduces to

$$ =\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)}\frac{z^n}{n!}\frac{1}{\Gamma_k(\beta+s+(\alpha+\gamma)n)} $$

$$ \times\frac{1}{k}\Big\{\frac{1}{(\beta+s+(\alpha+\gamma)n)}-\frac{1}{(\beta+s+k+(\alpha+\gamma)n)}\Big\}. $$

Using (12), we can write

$$ S={}_p\Psi^k_{q+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\ ,(\beta+s+k,\alpha+\gamma)\end{matrix}\middle|z\right] $$

$$ -k\ {}_p\Psi^k_{q+1}\left[\begin{matrix}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\ ,(\beta+s+2k,\alpha+\gamma)\end{matrix}\middle|z\right], \quad (14) $$

where

$$ S=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)}\frac{z^n}{n!}\frac{1}{(\beta+s+k+(\alpha+\gamma)n)\Gamma_k(\beta+s+(\alpha+\gamma)n)}. \quad (15) $$

Now, applying a simple identity $\frac{1}{u}=\frac{k}{u(u+k)}+\frac{1}{u+k}$; for $u=\beta+s+k+n(\alpha+p)$ to (12), we obtain

$$ S=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)}\frac{z^n}{n!}\frac{1}{\Gamma_k(\beta+s+(\alpha+\gamma)n)} $$

$$ \times\Big\{\frac{k}{(\beta+s+k+(\alpha+\gamma)n)((\beta+s+2k+(\alpha+\gamma)n))}+\frac{1}{(\beta+s+2k+(\alpha+\gamma)n)}\Big\}. $$

This leads to

$$S=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)}\frac{z^n}{n!}\times$$

$$\Big\{\frac{k(\beta+s+(\alpha+\gamma)n)}{(\beta+s+2k+(\alpha+\gamma)n)(\beta+s+k+(\alpha+\gamma)n)(\beta+s+(\alpha+\gamma)n)\Gamma_k((\beta+s+(\alpha+\gamma)n))}$$
$$+\frac{(\beta+s+(\alpha+\gamma)n)(\beta+s+k+(\alpha+\gamma)n)}{(\beta+s+2k+(\alpha+\gamma)n)(\beta+s+k+(\alpha+\gamma)n)(\beta+s+(\alpha+\gamma)n)\Gamma_k((\beta+s+(\alpha+\gamma)n))}\Big\}.$$

Using (5), we have

$$S=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)\Gamma_k(\beta+s+3k+(\alpha+\gamma)n)}\frac{z^n}{n!}$$

$$\times\{n^2(\alpha+\gamma)^2+2n(\alpha+\gamma)(\beta+s+k)+(\beta+s)+(\beta+s+2k)\} \tag{16}$$

each summation in the right side of (16) can be expressed as follows:

$$\frac{d}{dz}\left\{z\ {}_p\Psi_{q+1}^{k}\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\ ,(\beta+s+3k,\alpha+r)\end{array}\middle|z\right]\right\}$$
$$=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)(n+1)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)\Gamma_k(\beta+s+3k+(\alpha+\gamma)n)}\frac{z^n}{n!},$$

$$z\ {}_p\dot{\Psi}_{q+1}^{k}\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\ ,(\beta+s+3k,\alpha+r)\end{array}\middle|z\right]$$
$$+{}_p\Psi_{q+1}^{k}\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\ ,(\beta+s+3k,\alpha+r)\end{array}\middle|z\right]$$
$$=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)\Gamma_k(\beta+s+3k+(\alpha+\gamma)n)}\frac{(n+1)z^n}{n!},$$

$$z\ {}_p\dot{\Psi}^k_{q+1}\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\ ,(\beta+s+3k,\alpha+r)\end{array}\middle|z\right]$$
$$=\sum_{n=0}^{\infty}\frac{n\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)\Gamma_k(\beta+s+3k+(\alpha+\gamma)n)}\frac{z^n}{n!}. \quad (17)$$

Again

$$\frac{d^2}{dz^2}\left\{z^2{}_p\Psi^k_{q+1}\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\ ,(\beta+s+3k,\alpha+r)\end{array}\middle|z\right]\right\}$$
$$=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)(n+2)(n+1)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)\Gamma_k(\beta+s+3k+(\alpha+\gamma)n)}\frac{z^n}{n!},$$

Therefore

$$z^2\ {}_p\ddot{\Psi}^k_{q+1}\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\ ,(\beta+s+3k,\alpha+r)\end{array}\middle|z\right]$$
$$+4z\ {}_p\dot{\Psi}^k_{q+1}\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\ ,(\beta+s+3k,\alpha+r)\end{array}\middle|z\right]$$
$$+2\ {}_p\Psi^k_{q+1}\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\ ,(\beta+s+3k,\alpha+r)\end{array}\middle|z\right]$$
$$=\sum_{n=0}^{\infty}\frac{(n^2+3n+2)\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)\Gamma_k(\beta+s+3k+(\alpha+\gamma)n)}\frac{z^n}{n!}.$$

Using (17), above equation leads to

$$\sum_{n=0}^{\infty}\frac{n^2\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)\Gamma_k(\beta+s+3k+(\alpha+\gamma)n)}\frac{z^n}{n!}$$
$$=z^2\ {}_p\ddot{\Psi}^k_{q+1}\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\ ,(\beta+s+3k,\alpha+r)\end{array}\middle|z\right]$$
$$+z_p\dot{\Psi}^k_{q+1}\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\ ,(\beta+s+3k,\alpha+r)\end{array}\middle|z\right]. \quad (18)$$

Substituting values of (17) and (18) in (16), yields

$$S = z^2(\alpha+\gamma)^2 \ {}_p\ddot{\Psi}^k_{q+1}\left[\begin{matrix} (a_i,\alpha_i)_{1,p} \\ (b_j,\beta_j)_{1,q}, (\beta+s+3k,\alpha+r) \end{matrix}\middle| z\right]$$
$$+ z\{(\alpha+\gamma)^2 + 2(\alpha+\gamma)(\beta+s+k)\}_p\dot{\Psi}^k_{q+1}\left[\begin{matrix} (a_i,\alpha_i)_{1,p} \\ (b_j,\beta_j)_{1,q}, (\beta+s+3k,\alpha+r) \end{matrix}\middle| z\right]$$
$$+ (\beta+s)(\beta+s+2k)_p\Psi^k_{q+1}\left[\begin{matrix} (a_i,\alpha_i)_{1,p} \\ (b_j,\beta_j)_{1,q}, (\beta+s+3k,\alpha+r) \end{matrix}\middle| z\right].$$

4. Integral Representation

In this section, authors established an intersting Integral representation of generalised K-Wright function ${}_p\Psi^k_q(z)$.

Theorem 4.1. *For $k \in \mathbb{R}^+; z \in \mathbb{C}; a_i, b_j, \beta+s \in \mathbb{C}; Re(\beta+s) > 0$ and $\alpha+\gamma, \alpha_i, \beta_j \in \mathbb{R}$ $(\alpha_i, \beta_j \neq 0; i = 1,2,\ldots,p; j = 1,2,\ldots,q)$, then*

$$\int_0^1 t^{\beta+s+k-1}{}_p\Psi^k_{q+1}\left[\begin{matrix} (a_i,\alpha_i)_{1,p} \\ (b_j,\beta_j)_{1,q}, (\beta+s,\alpha+r) \end{matrix}\middle| t^{\alpha+\gamma}\right] dt$$
$$= {}_p\Psi^k_{q+1}\left[\begin{matrix} (a_i,\alpha_i)_{1,p} \\ (b_j,\beta_j)_{1,q}, (\beta+s+k,\alpha+r) \end{matrix}\middle| 1\right]$$
$$- k \ {}_p\Psi^k_{q+1}\left[\begin{matrix} (a_i,\alpha_i)_{1,p} \\ (b_j,\beta_j)_{1,q}, (\beta+s+2k,\alpha+r) \end{matrix}\middle| 1\right]. \quad (19)$$

Proof. Put $z = 1$, in (14) and (15), we have

$$S = {}_p\Psi^k_{q+1}\left[\begin{matrix} (a_i,\alpha_i)_{1,p} \\ (b_j,\beta_j)_{1,q}, (\beta+s+k,\alpha+\gamma) \end{matrix}\middle| 1\right]$$
$$- k \ {}_p\Psi^k_{q+1}\left[\begin{matrix} (a_i,\alpha_i)_{1,p} \\ (b_j,\beta_j)_{1,q}, (\beta+s+2k,\alpha+\gamma) \end{matrix}\middle| 1\right]$$
$$= \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)} \frac{1}{n!} \frac{1}{(\beta+s+k+(\alpha+\gamma)n)\Gamma_k(\beta+s+(\alpha+\gamma)n)}. \quad (20)$$

Consider following integral

$$A \equiv \int_0^1 t^{\beta+s+k-1}{}_p\Psi^k_{q+1}\left[\begin{matrix} (a_i,\alpha_i)_{1,p} \\ (b_j,\beta_j)_{1,q}, (\beta+s,\alpha+r) \end{matrix}\middle| t^{\alpha+\gamma}\right] dt.$$

This can be written as

$$A \equiv \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} \Gamma_k(a_i + \alpha_i n)}{\prod_{j=1}^{q} \Gamma_k(b_j + \beta_j n)} \frac{1}{n!} \frac{1}{\Gamma_k(\beta + s + (\alpha + \gamma)n)} \int_0^1 t^{(\alpha+\gamma)n+\beta+s+k-1} dt.$$

This immediately leads to

$$A \equiv \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} \Gamma_k(a_i + \alpha_i n)}{\prod_{j=1}^{q} \Gamma_k(b_j + \beta_j n)} \frac{1}{n!} \frac{1}{(\beta + s + k + (\alpha + \gamma)n)\Gamma_k(\beta + s + (\alpha + \gamma)n)},$$

and from (20), we derive

$$A = {}_p\Psi_{q+1}^k \left[\begin{array}{c} (a_i, \alpha_i)_{1,p} \\ (b_j, \beta_j)_{1,q}, (\beta + s + k, \alpha + \gamma) \end{array} \middle| 1 \right] - k \, {}_p\Psi_q^k \left[\begin{array}{c} (a_i, \alpha_i)_{1,p} \\ (b_j, \beta_j)_{1,q}, (\beta + s + 2k, \alpha + \gamma) \end{array} \middle| 1 \right].$$

5. Particular Case

Taking $p = 1$, $q = 2$; $a_1 = \gamma$, $\alpha_1 = r$, $k = 1$; $b_1 = \beta$, $\beta_1 = \alpha$; $b_2 = \gamma$, $\beta_2 = 0$, the results obtained by Shukla and Prajapati [4] also for suitable values of parameters the results obtained by Gupta and Debnath [3] are particular cases of results of this paper.

The function defined in equation (10) is an extension of generalized Mittag-Leffler function defined by Shukla and Prajapati (see [5], [6], [7]), and Shukla et al [8].

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