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SIZE-OF-RISK AVERSION, TIME VARYING UTILITY AND HEALTHY BEHAVIORS

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Abstract: In the context of time varying utility of wealth, this paper analyzes risk behavior in the case of a Decision Maker who can differently act as a Buyer or a Seller in order to study the appealing concept of size-of-risk attitude. Sufficient conditions ensuring existence and regularity of the proposed definitions are presented, both in the case of temporal size-of-risk aversion both in the case of instantaneous size-of-risk aversion. Sufficient conditions ensuring such behaviors are then investigated.

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Key Words: size-of-risk attitude, temporal utility, Buyer-Seller viewpoints

1. Introduction

As recently pointed out in [1], risk averse individuals are less likely to engage in unhealthy behaviors. The authors in fact find that individuals who are risk averse are less likely to smoke, drink, be overweight or drive over the speed limit. In other words, risk averse individuals are more likely to use a seat belt. If individual risk preference is related to health-related behaviors, it is very well-known that individual choices under uncertainty may be inconsistent with the hypothesis underlying some models proposed in literature. Between them, the Expected Utility (EU) model ([2], [6], [13]) has been one of the

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main scientific results of modern economic analysis: some attempts to develop alternative models have been proposed, but despite that, very often they don't keep the simple and tractable formulation of EU. In particular, specific models have been proposed in order to cover the case of temporal preferences.

A trade-off between time changing preferences and formulation tractability is obtained by Temporal Expected Utility Theory, proposed by Nachman [10]. Starting from the shareable observation that if generally the utility for wealth is assumed to be timeless even if clearly preferences may change with time, the Author proposed a generalization of EU in order to overcome this limit and to measure aversion to risk that allows for time varying preferences. He assumes that a Decision Maker (DM) at a fixed time must compare her present wealth position with some future wealth position: more precisely, she has well-defined preferences over uncertain prospects where outcomes are date-wealth pairs. In some way, the standard static EU de Finetti-Arrow-Pratt model is extended: Nachman compared a current certain reward with a lottery whose resulting payment will be resolved on time t: in some way, the preferences of the DM may change during the time. This differs from some other proposals where Authors examined the meaning of temporal risk attitude by studying induced preferences over temporal prospects. In [7] the temporal generalization of the standard static EU framework as proposed by Nachman ([10]) has been analysed in order to study the behavior of different DMs. More precisely, the problem of measuring aversion to risk that allows for time varying preferences is considered in the particular case of a DM who acts as a Buyer. The notions of temporal risk premium, instantaneous risk premium and time preference premium are proposed and characterized in the case of a Buyer DM. With the purpose of analysing healthy or unhealthy behaviors, we refer to the concept of size-of-risk aversion (for losses and for gains), a definition due to Zeckhauser and Keeler ([14]): this is a form of risk aversion in which the DM will pay greater percentage premiums over fair actuarial value to insure against risks of greater magnitude. In their proposal, the study is presented with reference to a Seller whose risk aversion may be interpreted in some way as an attitude, the greater is the size of the potential loss; if, moreover, the DM considers the certainty equivalent of favorable simple lotteries, then there is risk aversion when the certainty equivalent is a proportionally decreasing function of the gain. Our work analyses this model and generalises it: more precisely, we are interested in the study of the distinction between agents who assume risk and agents who dispose themselves of it, in the case of time varying preferences. In this way our main goal is to study the notions of size-of-risk aversion in the case of a DM who can act as a Buyer or a Seller in the particular dynamic framework of

Nachman ([10]).

The paper is organized as follows. In Section 2 we first set the objects of preferences and the basic assumptions on them: definitions of temporal size-of-risk premium, instantaneous size-of-risk premium and time preference premium are proposed in the case of a Seller and of a Buyer DM. In order to analyse the case of a DM in a problem of choice over time, first of all it is necessary to define the notions of temporal size-of-risk attitude, by generalizing the proposal in [14]: Section 3 is devoted to this aim. In Section 4 sufficient conditions for instantaneous and temporal size-of-risk aversion are investigated. Finally, in Section 5 some conclusive observations and hints for future research are presented.

2. The Dynamic Model

We recall some basic definitions and some preparatory results in the field of Temporal Decision Theory (for more details see [7] and [10]). We suppose that a Decision Maker at a fixed time point must compare her present wealth position w with some future wealth position: for doing so, she must be assumed to have well-defined preferences over uncertain prospects, namely over prospects whose outcomes are date(t)-wealth(w) pairs (t, w). Let $\mathbb{R}^+ = [0, +\infty)$ be the range of the variable t and let $\mathbb{R}$ be the range of the wealth w. $\mathcal{D} = \mathbb{R}^+ \times \mathbb{R}$ is the set of all the possible fortunes (t, w); it is endowed with the relative topology from $\mathbb{R}^2$ if it is necessary to consider $\mathcal{D}$ as a topological space, with the σ-field of Borel sets generated by this topology in case $\mathcal{D}$ has to be considered a measurable space.

We hypothesize that the DM is identified with a preference order on fortune valued random prospects, that is on probability measures on the Borel subsets of $\mathcal{D}$. In the following, her preference ordering is assumed to admit an EU representation where the utility function $u : \mathcal{D} \to \mathbb{R}$ satisfies the following set of conditions:

U1 (*continuous differentiability*): u is continuously differentiable on $\mathcal{D}$;

U2 (*monotonicity*): u as a function of w is strictly increasing, namely

$$u(t, w_1) < u(t, w_2)$$

for each (t, w_1) and $(t, w_2) \in \mathcal{D}$ where $w_1 < w_2$;

U3 (*comparability*): for each $t_1, t_2 \in \mathbb{R}^+$ where $t_1 < t_2$ and for each $w_2 \in \mathbb{R}$ there exists $w_1 \in \mathbb{R}$ such that

$$u(t_1, w_1) = u(t_2, w_2).$$

The time section $u_t(w) = u(t, w)$ may be considered as the instantaneous (at time t) utility function of the wealth w, that is a utility function in the de Finetti-Arrow-Pratt framework. Note that assumptions $U1$ and $U2$ ensure the existence and the uniqueness of the certainty equivalent in the static Arrow-Pratt framework, i.e. when u doesn't vary with time. By $U3$ assumption it follows the existence of a wealth level w_1 which is *equivalent* (in terms of utility) to a future value of wealth w_2: this is true for every $0 \leq t_1 < t_2$. It is possible to define the wealth section $u_w(t) = u(t, w)$ with the meaning of function representing preferences on the time in which to possess the wealth level w. Note that the preferences represented by the utility function u relate to two different preference relations: we consider preferences varying with time in the sense of time varying utility of wealth, that is with reference to the instantaneous utility u_t which may change with t.

Given the utility function u and two times s and t where $s < t$, we consider the random variable X and we interpret it as a random increment to present value w to be received immediately. The real-valued random variable X is defined on some probability space $(\Omega, \mathcal{D}, P)$ and $\mathcal{Q}$ is the set of all the random variables with compact support that are non-degenerate. In the following all the expected (utility) values are assumed to be finite. Nachman ([10]) set his analysis by referring to a Seller DM while in [7] the Buyer's *temporal risk premium* has been defined and characterised.

The choice of healthy or unhealthy behaviors is strictly related to magnitude of risk: this is why we refer to the work of Zeckhauser and Keeler ([14]). They introduced the concept of *size-of-risk aversion* (for losses and for gains), a form of risk aversion in which the DM will pay greater percentage premiums over fair actuarial value to insure against risks of greater magnitude. In more detail, let w be the initial wealth position of the DM and let Φ be a simple lottery with two payoffs, one of which is zero: this lottery results in a loss λ (gain γ) with probability p, and the same wealth with probability $1 - p$. The question faced by the Authors refers to the certainty equivalent the DM would pay (accept, respectively) to avoid the lottery vary with the size of λ (γ). More precisely, let y (y^*, respectively) the insurance premium the DM is willing to pay (receive) to ensure against a loss (gain) of size λ (γ), which may occur with probability p, where $1 \leq p \leq 1$; then it is

$$u(w - y) \quad = \quad pu(w - \lambda) + (1 - p)u(w) \qquad (1)$$

$$(pu(w+\gamma)+(1-p)u(w) \quad = \quad u(w+y^*)). \tag{2}$$

The DM exhibits *size-of-risk aversion for losses* if y/λ is an increasing function of λ, for all $\lambda > 0$. Similarly her attitude is of *size-of-risk aversion for gains* if y^*/γ is a decreasing function of γ, for all $\gamma > 0$. Note that in both cases, the DM plays as a Seller, so she dresses up as a particular DM who can choose to sell a lottery. In the following we propose the temporal extension of the notion of size of risk aversion, with no assumptions on the role played by the DM, that is we set different definitions for Seller and Buyer DMs.

Definition 1. The Buyer's temporal risk premium $\tau^B_{st}(w,\Phi)$ is the solution of

$$Eu(s, w+\Phi+p\lambda+\tau^B_{st}(w,\Phi))) = u(t,w). \tag{3}$$

Definition 2. The Seller's temporal risk premium $\tau^S_{st}(w,\Phi)$ is the solution of

$$u(s, w-p\lambda-\tau^S_{st}(w,\Phi)) = Eu(t, w+\Phi). \tag{4}$$

Note that in (3) the quantity $-p\lambda-\tau^B_{st}(w,\Phi)$ is the Buyer's temporal certainty equivalent and it represents the real number that makes the DM indifferent between buying the random amount Φ at time s and possessing the amount w at time t. That is, $-p\lambda-\tau^B_{st}(w,\Phi)$ is the maximum price the DM will pay at s in order to buy Φ. Analogous observations in the case of a Seller DM.

In order to determine if part of the temporal risk premium is related to risk preferences and/or to time preferences, it is necessary to introduce the notion of *instantaneous risk premium* and to recall the notion of *time preference premium.*

Definition 3. The Buyer's instantaneous risk premium at time s, $\pi^B_s(w,\Phi)$, is the solution of

$$Eu(s, w+\Phi+p\lambda+\pi^B_s(w,\Phi))) = u(s,w). \tag{5}$$

Note that $\pi^B_s(w,\Phi)$ is the Buyer's risk premium for the instantaneous utility function u_s. The quantity $-p\lambda-\pi^B_s(w,\Phi)$ is the Buyer's instantaneous certainty equivalent of Φ at time s, given the wealth w.

Definition 4. The Seller's instantaneous risk premium at time t, $\pi^S_t(w,\Phi)$, is the solution of

$$u(t, w-p\lambda-\pi^B_t(w,\Phi))) = Eu(t, w+\Phi). \tag{6}$$

If we consider the price the DM would pay at time s for a deterministic increment z to wealth w in order to receive z at the next time t, the definition of time preference premium (see [10]) easily follows.

Definition 5. The time preference premium $\nu_{st}(w,z)$ is the solution of

$$u(s, w+z-\nu_{st}(w,z)) = u(t, w+z). \tag{7}$$

Note that $w+z-\nu_{st}(w,z)$ is the present value of $w+z$, that is $z-\nu_{st}(w,z)$ is the present value at wealth w of the future increment z. Existence and uniqueness of Buyer's instantaneous risk premium (see [12]) and of time preference premium follow by assumptions on the utility function u.

Moreover, in [10] and [7] the following decompositions of temporal risk premium have been presented

$$\tau^B_{st}(w,\Phi) = \pi^B_s(w-\nu_{st}(w,0),\Phi) - \nu_{st}(w,0) \tag{8}$$
$$\tau^S_{st}(w,\Phi) = \pi^S_t(w,\Phi) + \nu_{st}(w, E(\Phi)-\pi^S_t(w,\Phi)). \tag{9}$$

In particular with reference to (8), note that $-p\lambda - \tau^B_{st}(w,\Phi)$ i.e. the price the DM is disposed to pay at time s in order to receive Φ at the same time s, is the sum of the price $-p\lambda - \pi^B_s(w-\nu_{st}(w,0),\Phi)$ she would pay at s for Φ when $w-\nu_{st}(w,0)$ is her initial wealth, and the present value $\nu_{st}(w,0)$ (at wealth w) of a null future increment.

3. Size-of-Risk Aversion: The Dynamic Proposal

As it is well-known, in order to evaluate random prospects it is important to distinguish whether the evaluator is a Seller or a Buyer: despite the role played by a Buyer has been examined only in passing by Pratt ([13]) and Kimball ([8]) who noted that the *asking price* of a risk is different from the *bid price* of the same risk, this diversity is common and well distinguished in actuarial literature ([3], [4])) and in financial literature ([9], [11], [12], [5]). In order to analyse the case of a DM in a problem of choice over time, first of all we define the notion of instantaneous size-of-risk attitude, by generalizing the proposal in [14] with reference to the analysis proposed in [10] and [7].

Definition 6. A DM is instantaneous size-of-risk averse for losses if the ratio between the instantaneous risk premium $\pi^i_s(w,\Phi)$ and the loss λ is an increasing function of λ:

$$\frac{\pi^i_s(w,\Phi)}{\lambda_1} \le \frac{\pi^i_s(w,\Phi)}{\lambda_2} \qquad \forall (s,w)\in\mathcal{D},\ \forall\,\Phi\in\mathcal{Q},\ \forall\,0<\lambda_1<\lambda_2. \tag{10}$$

In the previous definition the instantaneous risk premium $\pi_s^i(w,\Phi)$ is related to a Buyer for $i = B$, to a Seller when $i = S$, respectively; in other words, we are referring to a general DM who is instantaneous size-of-risk averse for losses. In particular, a Buyer B is instantaneous size-of-risk averse for losses if the the ratio $\pi_s^B(w,\Phi)/\lambda$ satisfies the previous inequality, where $\pi_s^B(w,\Phi)$ is solution of the following equation

$$pu(s, w + y - \lambda) + (1 - p)u(s, w + y) = u(s, w)$$

where $\pi_s^B(w,\Phi) = y - p\lambda$. Note that the request of monotonicity of y/λ is equivalent to that of $\pi_s^B(w,\Phi)/\lambda$, given that $y/\lambda = \pi_s^B(w,\Phi)/\lambda + p$. The notion of instantaneous size-of-risk aversion for a Seller DM clearly corresponds to that of Zeckhauser and Keller.

The dynamic formulation of size of risk aversion is given in term of monotonicity of $\tau_{st}(w,\Phi)/\lambda$. In fact we assume that

Definition 7. A DM is temporal size-of-risk averse for losses if the ratio between the temporal risk premium $\tau_{st}^i(w,\Phi)$ and the loss λ is an increasing function of λ:

$$\frac{\tau_{st}^i(w,\Phi)}{\lambda_1} \le \frac{\tau_{st}^i(w,\Phi)}{\lambda_2} \tag{11}$$

for all $(t, w) \in \mathcal{D}$, for all s such that $0 \le s < t$, for all $\Phi \in \mathcal{Q}$ and all λ_1, λ_2 such that $0 < \lambda_1 < \lambda_2$.

As before, the temporal risk premium $\pi_s^i(w,\Phi)$ may be related to a Buyer for $i = B$ or to a Seller when $i = S$: in each case the behavior is that of a particular DM who is temporal size-of-risk averse for losses.

By assumptions made on utility function $u(t, x)$, existence and uniqueness of $\pi_s^S(w,\Phi)$ and $\pi_s^S(w,\Phi)$, $\tau_s^B(w,\Phi)$ and $\tau_s^S(w,\Phi)$ are ensured. Moreover, it is possible to investigate conditions for which the DM's temporal risk premium is a continuous function of wealth. We explicitly consider the case of a Buyer DM for sake of completeness, while the dual case of a Seller DM directly follows by postulates $U_1 - U_3$. In the following, u'_i $(i = 1, 2)$ will denote the first partial derivative of u with respect to i-component.

Proposition 1. *Let $U_1 - U_3$ assumptions be true, then the temporal risk premium $\tau_{st}^B(w,\Phi)$ is well-defined for each wealth w and each $\lambda \in \mathbb{R}^+$, $p \in [0, 1]$. If moreover*

$$Eu'_2(s, w + \Phi + p\lambda + \tau_{st}^B)) \neq 0$$

then τ_{st}^B is a continuously differentiable function of w and

$$\frac{\mathrm{d}\tau_{st}^B}{\mathrm{d}w} = \left[\frac{u_2'(t,w) - Eu_2'[s, w+\Phi+p\lambda+\tau])}{Eu_2'[s, w+\Phi+p\lambda+\tau])}\right]\Bigg|_{\tau=\tau_{st}^B}.$$

Proof. Let

$$\mathcal{F}(\pi) = E[u(s, w+\Phi-\pi) - u(s,w)]$$

be a real-valued function of the real variable π; in other words we assume that $\Phi \in \mathcal{Q}$, $w \in \mathbb{R}$ and $s \in \mathbb{R}^+$ are fixed. Given the probability density function of Φ, $f_\Phi(x)$, let us consider the set

$$A(\Phi) = \overline{\{x \in \Omega : f_\Phi(x) \neq 0\}}$$

where $\overline{A}$ denotes the closure of the set A. Then $\inf A(\Phi) = \min A(\Phi)$ and $\sup A(\Phi) = \max A(\Phi)$. Assumption U_2 ensures that

$$\mathcal{F}(\max A(\Phi)) \leq 0 \qquad\qquad \mathcal{F}(\min A(\Phi)) \geq 0.$$

Given that the function $\mathcal{F}$ is a continuous function of π, there exists at least one value, say π^*, such that $\mathcal{F}(\pi^*) = 0$. Let $\pi_s^B = -p\lambda - \pi^*$. By decomposition (8) it is

$$\tau_{st}^B(w,\Phi) = \pi_s^B(w - \nu_{st}(w,0), \Phi) - \nu_{st}(w,0)$$

and existence and uniqueness results on $\tau_{st}^B(w,\Phi)$ follow. Let

$$\mathcal{G}(w,\tau) = Eu[s, w+\Phi+p\lambda+\tau)] - u(t,w)$$

be a real-valued function of the real variables w and τ: by definition $\mathcal{G}$ is a continuous function of w and τ. Moreover it is

$$\frac{\partial \mathcal{G}(w,\tau)}{\partial \tau} = Eu_2'[s, w+\Phi+p\lambda+\tau)]$$

and assumptions made on $\tau = \tau_{st}^B$ ensure that it is never null. Then by implicit function theorem there exist an open set W containing w, an open set T containing τ_{st}^B, and a unique continuously differentiable function $g: W \to T$ such that

$$\{(w, g(w))\} = \{(w,\tau) : Eu[s, w+\Phi+p\lambda+\tau)] - u(t,w) = 0\} \cap (W \times T).$$

So $\tau_{st}^B = g(w)$ results to be a differentiable function of w and

$$\frac{\mathrm{d}\tau_{st}^B}{\mathrm{d}w} = g'(w) = \frac{u_2'(t,w) - Eu_2'[s, w+\Phi+p\lambda+\tau)]}{Eu_2'[s, w+\Phi+p\lambda+\tau)]}$$

where $\tau = \tau_{st}^B$. □

Note that τ_{st}^B is locally a continuously differentiable function of w, that is, as stated in the proof, there exist an open set W containing w, an open set T containing τ_{st}^B, and a unique continuously differentiable function $g : W \to T$ such that $\tau_{st}^B = g(w)$; moreover, the pair $(w, g(w)) \in W \times T$ solves the equation

$$Eu(s, w + \Phi - (E(\Phi) - g(w))) - u(t, w) = 0.$$

4. Temporal Size-of-Risk Aversion

As pointed out in [7], the behaviour of a Buyer DM is quite different from that of a Seller DM: the case of size-of-risk aversion confirms it, as the following result highlights. With reference to instantaneous size-of-risk aversion for losses it is possible to deduce that

Proposition 2. *A Buyer B exhibits instantaneous size-of-risk aversion for losses if u is concave in w for each fixed $s \in \mathbb{R}^+$.*

Proof. Let

$$g_s(z) = u(s, w + z) - u(s, w)$$

be a real valued function of z, then equation

$$pu(s, w + \pi_s^B - \lambda) + (1 - p)u(s, w + \pi_s^B) = u(s, w)$$

may be rewritten as follows

$$(1 - p)g_s(\pi_s^B) + pg_s(\pi_s^B - \lambda) = 0. \tag{12}$$

Clearly, $0 < \pi_s^B < \lambda$. The function $\pi_s^B(w, \Phi)/\lambda$ is increasing in λ ($\lambda > 0$) if

$$\left(\pi_s^B\right)' \geq \frac{\pi_s^B}{\lambda} \qquad \forall \lambda > 0.$$

By differentiability of logarithmic transformation of (12) it is

$$\frac{g_s'(\pi_s^B)}{g_s(\pi_s^B)} \frac{(\pi_s^B)'}{(\pi_s^B)' - 1} = \frac{g_s'(\pi_s^B - \lambda)}{g_s(\pi_s^B - \lambda)}$$

by which it is $\left(\pi_s^B\right)' < 1$.
The DM is size-of-risk averse if

$$\frac{\dfrac{g_s'(\pi_s^B)}{g_s(\pi_s^B)}}{\pi_s^B} \leq \frac{\dfrac{g_s'(\pi_s^B - \lambda)}{g_s(\pi_s^B - \lambda)}}{\pi_s^B - \lambda}.$$

Mean Value Theorem (applied to g_s in $[\pi_s^B - \lambda, 0]$ and $[0, \pi_s^B])$ and decreasing monotonicity of g_s' ensure that

$$\frac{g_s'(\pi_s^B)}{\frac{g_s(\pi_s^B)}{\pi_s^B}} \leq 1 \leq \frac{g_s'(\pi_s^B - \lambda)}{\frac{g_s(\pi_s^B - \lambda)}{\pi_s^B - \lambda}}. \qquad \square$$

The temporal size-of-risk aversion for losses is related to impatience and instantaneous attitude, as the following result proves.

Proposition 3. *A Buyer B exhibits temporal size-of-risk aversion for losses if u is concave in w for each fixed $s \in \mathbb{R}^+$ and u is decreasing in t for each fixed w.*

Proof. Decreasing monotonicity of the wealth section function $u_w(t) = u(t, w)$ ensures impatience attitude of the DM (see [10] and [7]). Then $\nu_{st}(w, 0)/\lambda$ is an increasing function of λ. Concavity assumption on the time section function $u_s(w) = u(s, w)$ implies instantaneous size-of-risk aversion.

Finally, decomposition (8) proved in [7] gives the result. $\square$

5. Concluding Remarks

If the parallelism/distinction between Buyer and Seller risk premia have been investigated in the static framework, nevertheless in the case of problems of choice over time some questions are still open. In fact, the role played by time in the choice of when to resolve uncertainty is basic in interpreting the meaning of temporal risk aversion. As a consequence, in this paper some attention has been devoted to the analysis of a particular notion of risk attitude, namely that of size-of-risk aversion, in order to analyse healthy or unhealthy behaviors given the magnitude of the underlying risk, when the Decision Maker is characterized by time changing preferences. This paper confirms that the role played by a Buyer DM in a dynamic context presents only some links with the role played by a Seller DM. Starting from the notions of temporal risk premium, instantaneous risk premium and time preference premium we directed the analysis towards the study of temporal size-of-risk aversion: after the basic definitions of a Buyer who exhibits temporal, instantaneous size-of-risk averse preferences, and impatient preferences, some characterisations of these notions have been proposed. Differently from the case of a Seller DM, the temporal risk premium cannot be decomposed into the sum of two components which are related only to instantaneous risk aversion and to impatience: this is why the links between the proposed attitudes may be different. The obtained results in the case of problems of choice over time suggest that further research may be addressed to the analysis of appropriate local measures of temporal size-of-risk aversion.

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