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0-SIMPLE AND SOFT M-SEMIGROUPS

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Abstract: Motivated by the theory of 0- simple semigroup and M- semigroup, in this paper we introduce the notion of 0- simple M- semigroup. We also focus our research towards soft set theory and introduce the notion of soft M-semigroup and provide some results on it. In a 0- simple M- semigroup, we prove that 0- minimal ideal is a 0- simple M- semigroup and in soft setting, we prove that the union of the two soft M- semigroups is also a soft M- semigroup with the condition that the intersection of the subsets are empty. Also we justify that the Cartesian product of the two soft M- semigroups is also a soft M- semigroup.

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1. Introduction

T. Tamura, R.B. Markel, and J.F. Latimer [6] discussed M -groupoid as the direct product of a right singular semigroup and a groupoid with two sided identity. Warner [7] proved that an M- semigroup is isomorphic to direct product of a right zero semigroup and a semigroup with two sided identity. Lakshmanan [4] also studied about the structure of the set of idempotent elements of an M-semigroup.

The concept of 0- simple semigroup was introduced by J.M.Howie [3]. The

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word simple as used in semigroup theory does not have the same import as in group theory or ring theory, where it implies the total absence of non- trivial homomorphic images. A simple semigroup can be made into a 0- simple semigroup by merely adjoining a zero element. Not all 0- simple semigroups arise from simple semigroups.

It is known that many problems in different disciplines such as economics, engineering science, social sciences, etc., we cannot successfully use classical methods because of various types of uncertainties present in these problems. In dealing with uncertainties, many theories have been recently developed, including the theory of Probability, theory of Fuzzy sets, theory of intuitionistic Fuzzy sets, and theory of Rough sets and so on. But all these theories have their inherent difficulties as the inadequency of the parametrization. To overcome these problems Molodtsov [5] developed the theory of soft sets involving enough parameters so that many difficulties we are facing become easier by applying soft sets.

2. Preliminaries

In this section, we shall give some definitions and theorems required in the sequel.

Definition 2.1. [3] A semigroup S without zero is called simple if it has no proper ideals. A semigroup S with zero is called 0 -simple if (i) 0 and S are the only ideals and (ii) $S^2 \neq 0$

Definition 2.2. [6] An M -semigroup M is a semigroup that satisfy the following conditions (i) there is atleast one left identity $e \in M$ such that $ex = x$ for all $xinM$. (ii) for each $x \in M$, there is a unique left identity e_x such that $xe_x = x$.

Definition 2.3. [5] Let U be an initial universal set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subset E$. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

Definition 2.4. [2] A soft set (F, A) over U is said to null soft set denoted by $\widetilde{\phi}$, if $\epsilon \in A$, $F(\epsilon) =$ null set ϕ.

Definition 2.5. [2] A soft set (F, A) over a semigroup S is called a soft semigroup if $\widetilde{N_S} \neq (F, A) \neq \widetilde{\phi_S}$ and $(F, A)\ \hat{0}\ (F, A) \subseteq (F, A)$.

Example 2.6. [4] The extended union of (F, A) and (G, B) denoted by $(F, A) \cup_{\varepsilon} (G, B)$ is defined as the soft set (H, C) where $C = A \cup B$ and $\forall\ e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if} A \cap B \end{cases}$$

Definition 2.7. [3] A 0- minimal ideal M in a semigroup S with zero is an ideal minimal in the set of non-zero ideals.

Theorem 2.8. [1] Let (F, A) and (H, B) be two soft groups over G and K respectively. Then the product $(F, A) \times (H, B)$ is a soft group over $G \times K$.

Theorem 2.9. [3] If J is a 0-minimal ideal of S then either $J^2 = 0$ or J is a 0-simple semigroup.

3. 0- Simple M- Semigroup

In this section we enter into our new notion of 0-simple M- semigroup as follows:

Definition 3.1. An M- Semigroup M is 0- simple if it satisfies the conditions (i) $M^2 \neq 0$ and (ii) M and 0 are the only ideals.

We now provide the necessary and sufficient condition for an M- semigroup to be 0- simple.

Theorem 3.2. *An M- semigroup M is* 0- *simple if and only if $MaM = M$ for every a in $M\setminus \{0\}$.*

Proof. Suppose that M is 0- simple. We have to prove $MaM = M$ for every a in $M\setminus \{0\}$ Since M^2 is an ideal of $M \Rightarrow M^2 \neq 0$. We must have $M^2 = M$. Hence $M^3 = M^2.M = M.M = M$. Now for any a in $M\setminus \{0\}$ the subset MaM of M is an ideal. Hence either $MaM = M$ or $MaM = 0$. It is easily seen to be $MaM = M$ because note the fact that $M^3 = M$. Conversely, if $MaM = M$ for every a in $M\setminus \{0\}$ then certainly $M^2 \neq 0$. Also if A be a non-zero ideal of M, containing a non-zero element a(say), then $A \supseteq MAM \supseteq MaM = M \Rightarrow A = M$. □

Theorem 3.3. *If I be a* 0-*minimal ideal of M then either $I^2 = 0$ or I is a* 0-*simple M- semigroup.*

*Proof.*Given I be a 0-minimal ideal of M. Since an ideal minimal is the set of non-zero ideals. I^2 is an ideal of M contained in I and so either $I^2 = 0$ or $I^2 = I$. It is easily seen to be either $I^2 = 0$ or I is a 0-simple M- semigroup.□

4. Soft M- Semigroup

We now introduce a new algebraic structure in soft set theory namely restricted product of soft sets in M- semigroup and soft M- semigroup as follows:

Definition 4.1. Let (F, A) be a soft set over a M- semigroup M. The restricted product of (F, A) and (F, A) denoted by (F, A) $\widehat{0}$ (F, A) and is defined as the soft set (G, B), where $B = A \cap A$ and $G(b) = F(b).F(b)$ for all $b \in B$.

Definition 4.2. Let M be an M-semigroup. A soft set (F, A) over a M-semigroup M is called a soft M- semigroup if $\widetilde{N}_M \neq (F, A) \neq \widetilde{\phi}_M$ and (F, A) $\widehat{0}$ $(F, A) \subseteq (F, A)$, where $\widetilde{N}_M$ be a null soft set over M and $\widetilde{\phi}_M$ be a empty soft set over M.

Example 4.3. Consider the M- semigroup $\mathbf{M} = \{\mathbf{0}, \mathbf{e}, \mathbf{f}, \mathbf{a}, \mathbf{b}\}$,

where

$$\mathbf{0} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ \mathbf{e} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ \mathbf{f} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{a} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

and the operation is matrix multiplication. The cayley table of M is

.	0	e	f	a	b
0	0	0	0	0	0
e	0	e	0	a	0
f	0	0	f	0	b
a	0	0	a	0	e
b	0	b	0	f	0

Let (F, A) be a soft set over M such that $A = \{\mathbf{e}, \mathbf{f}, \mathbf{a}, \mathbf{b}\}$ and $F(\mathbf{e}) = \{\mathbf{0}, \mathbf{e}, \mathbf{b}\}$, $F(\mathbf{f}) = \{\mathbf{0}, \mathbf{a}, \mathbf{f}\}$, $F(\mathbf{a}) = \{\mathbf{0}, \mathbf{a}, \mathbf{f}\}$, $F(\mathbf{b}) = \{\mathbf{0}, \mathbf{b}, \mathbf{e}\}$. Then it can be easily checked that $\widetilde{N}_M \neq (F, A) \neq \widetilde{\phi}_M$ and $(F, A)\ \widehat{0}\ (F, A) \subseteq (F, A)$.

Theorem 4.4. *Let (F, A) and (G, B) be two soft M- semigroups over M. If $A \bigcap B = \emptyset$ then $(F, A) \bigcup_E (G, B)$ is a soft M- semigroup over M.*

Proof. By Definition 2.6 we can write $(F, A) \bigcup_E (G, B) = (H, C)$ where $C = A \bigcup B$. Hence either $e \in A - B$ or $e \in B - A$ for all $e \in C$. If $e \in A - B$ then $H(e) = F(e)$ and if $e \in B - A$ then $H(e) = G(e)$. Thus $\widetilde{N}_M \neq (H, C) \neq \widetilde{\phi}_M$. $\Rightarrow (F, A) \bigcup_E (G, B)$ is a soft M- semigroup over M. □

Remark 4.5. If (F, A) is a soft set over M then (F, A) is a soft M-semigroup over M if and only if $\forall a \in A$, $F(a)$ is a sub M- semigroup of M whenever $F(a) \neq \emptyset$.

Theorem 4.6. *Let (F, A) and (G, B) be two soft M- semigroups over M and N respectively. Then $(F, A) \times (G, B)$ is also a soft M- semigroup over $M \times N$ whenever $\widetilde{N}_{(M \times N, A \times B)} \neq (F, A) \times (G, B)$.*

Proof. The proof is straight forward. □

References

[1] H.Aktas, N.Cagman, *Soft sets and Soft groups* , Information Sciences 177(2007)2726-2735.

[2] M.I.Ali, M.Shabir, and K.P.Shum, *On soft ideals over semigroups,* Southeast Asian Bulletin of Mathematics 34(2010) 595-610.

[3] J.M.Howie, *An Introduction to Semigroup Theory,* Academic press Inc.(London) Ltd (1976).

[4] L.Lakshmanan, *On an M- semigroup and its set of idempotent elements,* Indian J. pure appl.Math.,21(1990) 423-433.

[5] P.K.Maji,R.Bismas, A.R.Roy,*Soft Set Theory,* Comput. Math. Appl. 37(1999) 19-31.

[6] D.Molodsov, *Soft set theory first results,* Computers and Math. with Applications 37 (1999) 19-31.

[7] T.Tamura, R.S.Markel and J.F.Latimar, *Correction and supplement to the direct product of right singular semigroups and certain groupoid,* Proc. Am. Math. Soc., 16(1965) 841.

[8] R.J.Warner, *The direct product of right zero semigroup and certain groupoid,* Am. Math. Month. 74(1967) 160-164.