International Journal of Pure and Applied Mathematics
Volume 87 No. 6 2013, 757-762
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v87i6.4
ijpam.eu

LOCALLY AND WEAKLY PGPR-CLOSED SETS

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Abstract: In the year 2005, the authors introduced and studied the concept of *pgpr*-closed sets in topological spaces. The purpose of this paper is to introduce locally and weakly *pgpr*-closed sets and investigate their basic properties.

AMS Subject Classification: 54A05, 54D30
Key Words: *pgpr*-closed, *pgpr*-open, locally *pgpr*-closed and weakly *pgpr*-closed

1. Introduction

The notion of locally closed sets was introduced by Bourbaki. Several mathematicians generalized this notion by replacing open sets with nearly open sets and generalized open sets and/or by replacing closed sets with nearly closed sets and generalized closed sets. In this paper we introduce locally pgpr-closed sets and we study their relations with other locally closed sets. Throughout this chapter, X and Y represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space X, $Cl(A)$ and $Int(A)$ denote the closure of A and the interior of A in X

Received: September 6, 2013

respectively; $X - A$ denotes the complement of A in X. In this paper, A and B denote the subsets of X unless otherwise specified. The pre-closure of a subset A of X is the intersection of all pre-closed sets containing A and is denoted by $pCl(A)$. The pre-interior of a subset A of X is the union of all pre-open sets contained in A and is denoted by $pInt(A)$.

2. Preliminaries

Definition 2.1. [9, 7]

(1) A is regular open if $A = Int(Cl(A))$ and regular closed if $A = Cl(Int(A))$.

(2) A is pre-open if $A \subseteq Int(Cl(A))$ and pre-closed if $Cl(Int(A)) \subseteq A$.

Definition 2.2. [6, 8]

(1) A is generalized closed(briefly g-closed) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(2) B is regular generalized closed(briefly rg-closed) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

Definition 2.3. [6] A space X is $T_{1/2}$ if every g-closed is closed.

Definition 2.4. [5, 4] A subset A of (X, τ) is called a

(1) locally closed set if $A = U \cap F$ where $U \in \tau$ and F is closed in (X, τ).

(2) generalized locally closed set(briefly glc-set) if $A = G \cap F$ where G is g-open in (X, τ) and F is g-closed in (X, τ).

(3) glc^*-set if there exists a g-open set G and a closed set F of (X, τ) such that $A = G \cap F$.

(4) glc^{**}-set if there exist an open set U and a g-closed set F of (X, τ) such that $A = U \cap F$.

Definition 2.5. [1] A subset B of a space X is called pre-generalized pre-regular-closed(briefly $pgpr$-closed) if $pCl(B) \subseteq U$ whenever $B \subseteq U$ and U is rg-open.

Definition 2.6. [1] For a subset A of (X, τ), $pgpr$-$Cl(A) = \cap\{F : A \subseteq F$ and F is $pgpr$-closed in $X\}$ is called the $pgpr$-closure of A

Definition 2.7. Let A be a subset of (X, τ). Then A is called locally *pgpr*-closed if there exist an open set U and a *pgpr*-closed set F of X such that $A = U \cap F$. The collection of all locally *pgpr*-closed sets is denoted by $LPGPRC(X, \tau)$. It is easy to prove that the class of all locally closed sets is contained in the class of all locally *pgpr*-closed sets.

Theorem 2.8. *For a topological space* (X, τ),

$$LC(X, \tau) \subseteq LPGPRC(X, \tau).$$

Theorem 2.9. *For a submaximal space* (X, τ),

$$LPGPRC(X, \tau) \subseteq GLC(X, \tau).$$

Proof. Let $A \in LPGPRC(X, \tau)$. Then there exist an open set U and a *pgpr*-closed set F of X such that $A = U \cap F$. In a submaximal space X, every *pgpr*-closed set is g-closed. Therefore F is g-closed. Since every open set is g-open, it follows that A is an intersection of g-open set U and a g-closed set F of X. Therefore, $A \in GLC(X, \tau)$. This proves the theorem. □

Theorem 2.10. *If X is $T_{1/2}$, then*

$$LC(X, \tau) = GLC^{**}(X, \tau) = GLC(X, \tau) = GLC^{*}(X, \tau) \subseteq LPGPRC(X, \tau).$$

Proof. Let X be $T_{1/2}$. Since every g-closed set is closed in a $T_{1/2}$ space, $LC(X, \tau) = GLC^{**}(X, \tau) = GLC(X, \tau) = GLC^{*}(X, \tau)$. Also, every closed set is *pgpr*-closed, we have $GLC^{*}(X, \tau) \subseteq LPGPRC(X, \tau)$. Therefore, $LC(X, \tau) = GLC^{**}(X, \tau) = GLC(X, \tau) = GLC^{*}(X, \tau) \subseteq LPGPRC(X, \tau)$. □

Definition 2.11. A space is said to have the *pgpr*-closure preserving property if *pgpr*-$Cl(A)$ is always *pgpr*-closed.

Theorem 2.12. *Suppose X has the pgpr-closure preserving property and let A be a subset of (X, τ). Then $A \in LPGPRC(X, \tau)$ if and only if $A = U \cap pgpr - Cl(A)$ for some open set U.*

Proof. Let $A \in LPGPRC(X, \tau)$. Then $A = U \cap F$ where U is open and F is *pgpr*-closed. By Definition 2.11, $pgpr - Cl(A)$ is *pgpr*-closed in X, $A \subseteq F$ implies $pgpr - Cl(A) \subseteq F$. Now, $A = A \cap pgpr - Cl(A) = U \cap F \cap pgpr - Cl(A) = U \cap pgpr - Cl(A)$. Therefore, $A = U \cap pgpr - Cl(A)$ for some open set U. Conversely, assume that $A = U \cap pgpr - Cl(A)$ for some open set U. By Definition 2.11, $pgpr - Cl(A)$ is *pgpr*-closed in X. Therefore, $A \in LPGPRC(X, \tau)$. This proves the theorem. □

Definition 2.13. Let A be a subset of (X, τ). Then A is called *pgpr*-locally closed if there exist a *pgpr*-open set U and a *pgpr*-closed set F of X such that $A = U \cap F$.

Definition 2.14. Let A be a subset of (X, τ). Then A is called *pgpr**-locally closed if there exist a *pgpr*-open set U and a closed set F of X such that $A = U \cap F$. The collection of all *pgpr*-locally closed sets $PGPRLC$ sets of (X, τ) will be denoted by $PGPRLC(X, \tau)$. Similarly the collection of all *pgpr**-locally closed sets of (X, τ) will be denoted by $PGPRLC^*(X, \tau)$.

Proposition 2.15. *For a topological space* (X, τ),

$$LC(X, \tau) \subseteq PGPRLC^*(X, \tau) \subseteq PGPRLC(X, \tau).$$

Proof. Follows from the fact that every open set is *pgpr*-open and every closed set is *pgpr*-closed. $PGPRLC^*(X, \tau)$ and $LPGPRC(X, \tau)$ are independent of each other as seen in the next example. □

Example 2.16. Let $X = \{a, b, c\}$ and $\tau_2 = \{\phi, \{a\}, \{a, b\}, X\}$. Then $\{a, c\}$ is *pgpr**-locally closed set but not a locally *pgpr*-closed set. Let $X = \{a, b, c, d\}$ and $\tau_8 = \{\phi, \{a\}, \{a, b, c\}, X\}$. Then $\{b, c\}$ is a locally *pgpr*-closed set but not a *pgpr**-locally closed set.

Theorem 2.17. *For a sub maximal space* (X, τ),
$PGPRLC^*(X, \tau) \subseteq PGPRLC(X, \tau) \subseteq GLC(X, \tau)$.

Proof. Follows from the fact that in a sub maximal space every *pgpr*-closed set is g-closed. □

Theorem 2.18. *If* X *is* $T_{1/2}$, *then*

(1) $GLC^{**}(X, \tau) = GLC(X, \tau) = GLC^*(X, \tau) \subseteq PGPRLC(X, \tau)$.

(2) $GLC^{**}(X, \tau) = GLC(X, \tau) = GLC^*(X, \tau) \subseteq PGPRLC^*(X, \tau)$.

Proof. Let X be $T_{1/2}$. Since every g-closed set is closed in a $T_{1/2}$ space, $LC(X, \tau) = GLC^{**}(X, \tau) = GLC(X, \tau) = GLC^*(X, \tau)$. Since every closed set is *pgpr*-closed, we have $GLC^*(X, \tau) \subseteq LPGPRC(X, \tau)$. Therefore, $LC(X, \tau) = GLC^{**}(X, \tau) = GLC(X, \tau) = GLC^*(X, \tau) \subseteq PGPRLC(X, \tau)$. This proves (1). The proof for (2) is similar. □

Theorem 2.19. *Suppose a space* X *has the pgpr-closure preserving property. Then the following are equivalent.*

(1) $A \in PGPRLC(X, \tau)$

(2) $A = U \cap pgpr - Cl(A)$ *for some pgpr-open set* U.

Proof. (1) $\Rightarrow$ (2): Let $A \in PGPRLC(X, \tau)$. Then there exist a *pgpr*-open subset U and a *pgpr*-closed subset F such that $A = U \cap F$. Since $A \subseteq U$ and since $A \subseteq pgpr - Cl(A)$, $A \subseteq U \cap pgpr - Cl(A)$. By Definition 2.11, $pgpr - Cl(A)$ is *pgpr*-closed, $pgpr - Cl(A) \subseteq F$ and hence $U \cap pgpr - Cl(A) \subseteq U \cap F = A$. Therefore, $A = U \cap pgpr - Cl(A)$. This proves (2).

(2)$\Rightarrow$ (1): By Definition 2.11, $pgpr - Cl(A)$ is *pgpr*-closed. Therefore, $A = U \cap pgpr - Cl(A) \in PGPRLC(X, \tau)$. □

Theorem 2.20. *For a subset* A *of* (X, τ), *then the following are equivalent*

(1) $A \in PGLC^*(X, \tau)$

(2) $A = U \cap Cl(A)$ *for some pgpr-open set* U.

(3) $A \cup (X\text{-}Cl(A))$ *is pgpr-open.*

(4) $Cl(A)\text{-}A$ *is pgpr-closed.*

Proof. (1) $\Rightarrow$ (2) : Let $A \in PGPRLC^*(X, \tau)$. Then there exists a *pgpr*-open set U and a closed set F such that $A = U \cap F$. Since $A \subseteq U$ and $A \subseteq Cl(A)$, we have $A \subseteq U \cap Cl(A)$. Since $Cl(A) \subseteq F$, $U \cap Cl(A) \subseteq U \cap F = A$ which implies $A = U \cap Cl(A)$.

(2) $\Rightarrow$ (1) : Since U is *pgpr*-open and $Cl(A)$ is closed, $U \cap Cl(A) \in PGPRLC^*(X, \tau)$.

(2) $\Rightarrow$ (3) : Let $A = U \cap Cl(A)$ for some *pgpr*-open set U. Then we prove that $A \cup (X\text{-}Cl(A))$ is *pgpr*-open. $A \cup (X - Cl(A)) = U \cap Cl(A) \cup X - Cl(A) = U \cap Cl(A) \cup (X - Cl(A)) = U \cap X = U$ which is *pgpr*-open. Thus, $A \cup (X\text{-}Cl(A))$ is *pgpr*-open.

(3) $\Rightarrow$ (4) : Let $U = A \cup (X - Cl(A))$. Then U is *pgpr*-open. This implies that $X - U$ is *pgpr*-closed and $X - U = X - (A \cup (X - Cl(A)) = Cl(A) \cap (X - A) = Cl(A) - A$. Thus, $Cl(A) - A$ is *pgpr*-closed.

(4) $\Rightarrow$ (2) : Let $F = Cl(A) - A$. Then F is *pgpr*-closed by the assumption and $X - F = X \cup (X - (Cl(A) - A) = A \cup (X - Cl(A))$. But $X - F$ is *pgpr*-open, this shows that $A \cup X - Cl(A)$ is *pgpr*-open. □

Definition 2.21. A subset A of X is called weakly *pgpr*-closed if

$$pCl(pInt(A)) \subset G \text{ whenever } A \subset G \text{ and } G \text{ is } rg\text{-open in } X.$$

Every *pgpr*-closed set is weakly *pgpr*-closed. Examples can be constructed to show that the converse is not true.

Theorem 2.22. *If a subset A of X is weakly $pgpr$-closed then $pCl(pInt(A)) - A$ contains no non-empty rg-closed set.*

Proof. Suppose A is weakly $pgpr$-closed in X. Let F be a rg-closed set such that $F \subseteq pCl(pInt(A)) - A$. Since $X - F$ is rg-open and $A \subseteq X - F$, from the Definition 2.11, $pCl(pInt(A)) \subseteq X - F$. That is, $F \subseteq X - (pCl(pInt(A)))$ that implies $F \subseteq pCl(pInt(A) \cap (X - (pCl(pInt(A))))) = \phi$. This proves the theorem. □

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