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POISSON APPROXIMATION FOR THE NUMBER OF ISOLATED TREES IN A RANDOM INTERSECTION GRAPH

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Abstract: Let $W_{n,k}$ be the number of isolated trees of order k in a random intersection graph $\mathbb{G}(n, m, p)$. In this paper, we give the bound on Poisson approximation of $W_{n,k}$ by using the Stein-Chen method.

Key Words: random intersection graph, isolated trees and Stein-Chen and Coupling Method

1. Introduction

Given a set V with n vertices and another universal set U with m elements, define a bipartite graph $B(n, m, p)$ with independent vertex sets V and U and edges between $v \in V$ and $u \in U$ existing independently with probability p. The random intersection graph $\mathbb{G}(n, m, p)$, derived from $B(n, m, p)$, is defined on the vertex set V with vertices $v_1, v_2 \in V$ adjacent if and only if there exist some $u \in U$ such that both v_1 and v_2 are adjacent to u in $B(n, m, p)$. Also define S_i be a random subset of U such that each element of S_i is adjacent to $i \in V$, in which case two vertices $i, j \in V$ are adjacent if and only if $S_i \cap S_j \neq \phi$, and edge set $\mathrm{E}(\mathbb{G})$ is define as

$$\mathrm{E}(\mathbb{G}) = \{\{i, j\} : i, j \in V, S_i \cap S_j \neq \phi\}.$$

The properties of $\mathbb{G}(n, m, p)$ were studied in [2,3] contrasted with the well

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known random graph model $\mathbb{G}(n,p)$, in which vertices are made adjacent to each other independently and with probability p, and showed that for a fixed $\alpha > 0$, the number of elements m is taken to be $m = \lfloor n^{\alpha} \rfloor$. In 1999, Karonski, Scheinerman and Singer-Cohen[2] showed that the total variation distance between the distribution of $\mathbb{G}(n,m,p)$ and $\mathbb{G}(n,p)$ converges to 0 when $\alpha > 6$ and p is defined appropriately. Without loss of generality we consider the independent set V. For $i = 1, 2, 3, ..., n$, let

$$X_i = \begin{cases} 1 & \text{if vertex } i \text{ is an isolated in } \mathbb{G}(n,m,p) \\ 0 & \text{otherwise,} \end{cases}$$

and

$$X = \sum_{i \in \Gamma} X_i.$$

Clearly X is the number of isolated vertices in $\mathbb{G}(n,m,p)$.

In 2011, Yilum Shang (see [6]) proved that the distribution function of X can be approximated by Poisson distribution with parameter

$$\lambda := \mathrm{E}W = nP(X_i = 1) = n \sum_{s=0}^{m} \binom{m}{s} p^s (1-p)^{m-s} (1-p)^{(n-1)s}$$
$$= n[1 - p + p(1-p)^{(n-1)}]^m.$$

In 2013, M. Donganont[9] showed the another proof of Poisson approximation for the number of isolated vertices in $\mathbb{G}(n,m,p)$ by Stein-Chen and coupling method. The results as the following,

Theorem 1.1 (9). *Let W be the number of isolated vertices in a random intersection graph $\mathbb{G}(n,m,p)$.*

For $A \subseteq \{0, 1, 2, ...n\}$ and $m = \lfloor n^{\alpha} \rfloor$ for some $\alpha > 0$, we have

$$|P(W \in A) - Poi_{\lambda}(A)| \leq \frac{nC_{\lambda,A}}{e^{pm(1-(1-p)^{n-2})}}$$

where $C_{\lambda,A} = \min\left\{1, \lambda, \frac{\triangle(\lambda)}{M_A+1}\right\}$,

$$\triangle(\lambda) = \begin{cases} e^{\lambda} + \lambda - 1 & \text{if } \lambda^{-1}(e^{\lambda} - 1) \leq M_A, \\ 2(e^{\lambda} - 1) & \text{if } \lambda^{-1}(e^{\lambda} - 1) > M_A, \end{cases}$$

and

$$M_A = \begin{cases} \max\{w \mid C_w \subseteq A\} & \text{if } 0 \in A, \\ \min\{w \mid w \in A\} & \text{if } 0 \notin A. \end{cases}$$

When $C_w = \{0, 1, ..., w\}$

Corollary 1.1. *[9] Let W be the number of isolated vertices in a random intersection graph $\mathbb{G}(n,m,p)$. Let $A \subseteq \{0,1,2,...n\}$, $m = \lfloor n^\alpha \rfloor$ for some $\alpha > 0$, $q = 1-p$, and $p = \dfrac{1}{n^\gamma}$ for any $\gamma \in \mathbb{R}^+ \setminus \{1\}$,then*

$$|P(W \in A) - Poi_\lambda(A)| \leq \frac{C(\lambda, A)}{n^{2(\alpha-\gamma)(1-(q)^{n-2})-1}},$$

where $C(\lambda, A) = \min\left\{1, \lambda, \frac{\triangle(\lambda)}{M_A+1}\right\}$.

Let

$$\Gamma_{n,k} = \{i =: \{i_1, i_2, \ldots, i_k\} \mid 1 \leq i_1 < \ldots < i_k \leq n\}$$

be the set of all possible combinations of k vertices. we note that T_k is a tree of order k in $\mathbb{G}(n,m,p)$ and say that T_k is isolated in $\mathbb{G}(n,m,p)$ if there is no edge in $\mathbb{G}(n,m,p)$ between a vertex in the tree and the other outside of the tree.

For each $i \in \Gamma_{n,k}$, we define the indicator random variable

$$X_i = \begin{cases} 1 & \text{if there is an isolated tree in } \mathbb{G}(n,m,p) \text{ that spans the vertices} \\ & i = (i_1, \ldots, i_k), \\ 0 & \text{otherwise,} \end{cases}$$

and set

$$W_{n,k} = \sum_{i \in \Gamma_{n,k}} X_i.$$

Then $W_{n,k}$ is the number of isolated trees in $\mathbb{G}(n,m,p)$.

In this work, we study isolated trees of order k in $\mathbb{G}(n,m,p)$ with $m = \lfloor n^\alpha \rfloor$, $\alpha > 0$ and show that $W_{n,k}$ can be approximated by Poisson approximation with parameter

$$\lambda := \mathbb{E}(W_{n,k}) = \binom{n}{k} P(X_i = 1) = \binom{n}{k} \sum_{s=0}^{m} \binom{m}{s} p^{s+k-1}(1-p)^{m-s}(1-p)^{(n-k)s}$$
$$= \binom{n}{k} p^{k-1} \left[1 - p + p(1-p)^{(n-k)}\right]^m. \quad (1)$$

By using Stein-Chen and Coupling Method. The following theorem is our main result.

Theorem 1.2. *Let W be the number of isolated trees in a random intersection graph $\mathbb{G}(n,m,p)$.*

For $A \subseteq \{0,1,2,...n\}$ and $m = \lfloor n^\alpha \rfloor$ for some $\alpha > 0$, we have

$$|P(W \in A) - Poi_\lambda(A)| \leq \frac{C_{(\lambda,A)}(1 + \frac{n^k}{k!} pk^2) p^{k-1}}{e^{mp(1-(1-p)^{n-2k})}}$$

where $C_{(\lambda,A)} = \min\left\{1, \lambda, \frac{\triangle(\lambda)}{M_A+1}\right\}$,

$$\triangle(\lambda) = \begin{cases} e^{\lambda} + \lambda - 1 & \text{if } \lambda^{-1}(e^{\lambda} - 1) \leq M_A, \\ 2(e^{\lambda} - 1) & \text{if } \lambda^{-1}(e^{\lambda} - 1) > M_A, \end{cases}$$

and

$$M_A = \begin{cases} \max\{w \mid C_w \subseteq A\} & \text{if } 0 \in A, \\ \min\{w \mid w \in A\} & \text{if } 0 \notin A. \end{cases}$$

When $C_w = \{0, 1, ..., w\}$

Corollary 1.2. *Let W be the number of isolated trees in a random intersection graph $\mathbb{G}(n, m, p)$. Let $A \subseteq \{0, 1, 2, ..., n\}$, $m = \lfloor n^{\alpha} \rfloor$ for some $\alpha > 0$, and $p = \dfrac{1}{n^{\gamma}}$ for any $\gamma \in \mathbb{R}^{+}\{1\}$*

1. *If $\gamma > k > 0$, $q = 1 - p$, then*

$$|P(W \in A) - Poi_{\lambda}(A)| \leq \frac{C(\lambda, A)}{n^{2(\alpha-\gamma)(1-(q)^{n-2k})+(k\gamma-\gamma)}},$$

where $C(\lambda, A) = 2\min\left\{1, \lambda, \frac{\triangle(\lambda)}{M_A+1}\right\}$.

2. *If $k > \gamma > 0$, $q = 1 - p$, then*

$$|P(W \in A) - Poi_{\lambda}(A)| \leq \frac{C(\lambda, A)}{n^{2(\alpha-\gamma)(1-(q)^{n-2k})+(k\gamma-k)}},$$

where $C(\lambda, A) = 2\min\left\{1, \lambda, \frac{\triangle(\lambda)}{M_A+1}\right\}$.

This paper is organized as follows. In section 2, we introduce the Stein-Chen method for Poisson approximation and Coupling approach which use in the proof of main result in section 3.

2. Stein-Chen and Coupling Method

In 1972, Stein [1] gave a new technique to find a bound in the normal approximation to a distribution of a sum of dependent random variables. His technique was relied instead on the elementary differential equation. In 1975, Chen [4, 5] applied Stein's idea to the Poisson case. The central idea of the Stein-Chen method is the difference equation

$$I_A(j) - \mathcal{P}_{\lambda}(A) = \lambda g_{\lambda,A}(j+1) - j g_{\lambda,A}(j), \quad j \in \mathbb{N} \cup \{0\}, \tag{2}$$

where $\lambda > 0$ and $A \subseteq \mathbb{N} \cup \{0\}$ and $I_A : \mathbb{N} \cup \{0\} \to \mathbb{R}$ is defined by

$$I_A(w) = \begin{cases} 1, & w \in A, \\ 0, & w \notin A. \end{cases}$$

The equation (2) is called Stein's equation for Poisson distribution function and its solution is

$$g_{\lambda,A}(w) = \begin{cases} (w-1)!\lambda^{-w}e^{\lambda}[\mathcal{P}_\lambda(I_{A\cap C_{w-1}}) - \mathcal{P}_\lambda(I_A)\mathcal{P}_\lambda(I_{C_{w-1}})], & w \geq 1, \\ 0, & w = 0, \end{cases}$$

where

$C_{w-1} = \{0, 1, \ldots, w-1\}$ and $\mathcal{P}_\lambda(I_A) = e^{-\lambda} \sum\limits_{l=0}^{\infty} I_A(l)\frac{\lambda^l}{l!}$. [7]

By substituting j and λ in (2) by any integer-valued random variable W and $\lambda = \mathrm{E}(W)$, we have

$$P(W \in A) - Poi_\lambda(A) = \mathrm{E}(\lambda g_{\lambda,A}(W+1)) - \mathrm{E}(W g_{\lambda,A}(W)). \tag{3}$$

So far W could be $\sum\limits_{i\in\Gamma} X_i$ and $\lambda = \mathbb{E}(W) = \sum\limits_{i\in\Gamma} p_i$ where $p_i = \mathbb{E}(X_i) = P(X_i = 1)$.

In 1992, Barbour, Holst and Janson[7] constructed coupling random variable W_i and used Stein-Chen method to find the bound in Poisson approximation of W. They assumed that for each i the distribution $L(W_i)$ of W_i equals to the conditional distribution $L(W - X_i | X_i = 1)$ and gave the fundamental theorem as follows:

Theorem 2.1. *If W and W_i are defined as above, then*

$$|P(W \in A) - Poi_\lambda(A)| \leq \| g_{\lambda,A} \| \sum_{i\in\Gamma} p_i \mathbb{E}(|W - W_i|) \tag{4}$$

where $\| g_{\lambda,A} \| := \sup\limits_w [g_{\lambda,A}(w+1) - g_{\lambda,A}(w)]$.

In 2006, Santiwipanont and Teerapabolarn [8] proved that for any subset A of $\{0, 1, \ldots, n\}$,

$$\| g_{\lambda,A} \| \leq \lambda^{-1} \min\left\{1, \lambda, \frac{\triangle(\lambda)}{M_A + 1}\right\} \tag{5}$$

where

$$\triangle(\lambda) = \begin{cases} e^\lambda + \lambda - 1 & \text{if } \lambda^{-1}(e^\lambda - 1) \leq M_A, \\ 2(e^\lambda - 1) & \text{if } \lambda^{-1}(e^\lambda - 1) > M_A, \end{cases}$$

and

$$M_A = \begin{cases} \max\{w \mid C_w \subseteq A\} & \text{if } 0 \in A, \\ \min\{w \mid w \in A\} & \text{if } 0 \notin A. \end{cases}$$

In next section, we will use Theorem 2.1 and (5) to prove our main result by constructing the random variable W_i.

3. Proof of Main Result

Let $A \subseteq \mathbb{N}$. By (4), it suffice to bound $\mathbb{E}|W - W_i|$ for $i \in \Gamma_{n,k}$ where the distribution of W_i equals to the conditional distribution of $W - X_i$ given $X_i = 1$.

Let W_i be the number of isolated trees of order k in a random intersection graph $\mathbb{G}(n,m,p) - i$, $\mathbb{G}(n,m,p) - i$ obtained from $\mathbb{G}(n,m,p)$ by dropping the set $i \subseteq V$ and all the edges containing any of these vertices. Then for $w_0 \in \{0, 1, \ldots, \lfloor \frac{n-k}{k} \rfloor\}$, we have

$$\begin{aligned} P(W_i = w_0) &= \binom{\binom{n-k}{k}}{w_0} \left[\sum_{s=0}^{m} \binom{m}{s} p^{s+k-1} (1-p)^{m-s} (1-p)^{(n-2k)s} \right]^{w_0} \\ &= \binom{\binom{n-k}{k}}{w_0} p^{(k-1)w_0} \left[1 - p + p(1-p)^{(n-2k)} \right]^{mw_0} \end{aligned} \tag{6}$$

and

$$\begin{aligned} &P(W - X_i = w_0 \mid X_i = 1) = \frac{P(W - X_i = w_0, X_i = 1)}{P(X_i = 1)} \\ &= \frac{P(W = w_0 + 1, X_i = 1)}{P(X_i = 1)} \\ &= \frac{\binom{\binom{n-k}{k}}{w_0} \left[\sum_{s=0}^{m} \binom{m}{s} p^{s+k-1}(1-p)^{m-s}(1-p)^{(n-2k)s}\right]^{w_0} \sum_{s=0}^{m} \binom{m}{s} p^{s+k-1}(1-p)^{m-s}(1-p)^{(n-2k)s}}{\sum_{s=0}^{m} \binom{m}{s} p^{s+k-1}(1-p)^{m-s}(1-p)^{(n-2k)s}} \\ &= \binom{\binom{n-k}{k}}{w_0} \left[\sum_{s=0}^{m} \binom{m}{s} p^{s+k-1}(1-p)^{m-s}(1-p)^{(n-2k)s} \right]^{w_0} \\ &= \binom{\binom{n-k}{k}}{w_0} p^{(k-1)w_0} \left[1 - p + p(1-p)^{(n-2k)} \right]^{mw_0}. \end{aligned} \tag{7}$$

From (6) and (7), the distribution of W_i equals to the conditional distribution of $(W - X_i | X_i = 1)$.

For $i, j \in \Gamma_{n,k}$ such that $i \neq j$, we define the indicator random variable $X_j^{(i)}$ and E_{ij}, as follow

$$X_j^{(i)} = \begin{cases} 1 & \text{if here is an isolated tree in } \mathbb{G}(n,m,p) - i \text{ that spans the vertices} \\ & i = (i_1, \ldots, i_k), \\ 0 & \text{otherwise,} \end{cases}$$

and

$$E_{ij} = \begin{cases} 1 & \text{if there exists adjacent between } i_k \in i \text{ and } j_l \in j, \\ 0 & \text{otherwise.} \end{cases}$$

We observe that in case $X_i = 1$, that is we have an isolated tree in $\mathbb{G}(n, m, p)$ that span vertices $i = \{i_1, \ldots, i_k\}$. Thus the number of isolated trees in a random intersection graph $\mathbb{G}(n, m, p) - i$ equals to the number of isolated trees in a random intersection graph $\mathbb{G}(n, m, p)$ minus 1, that is

$$W_i = W_{n,k} - 1. \tag{8}$$

In case $X_i = 0$. For $j \in \Gamma_{n,k}$ such that $j \cap i = \emptyset$ and $j \neq i$, then

$$W_i = W_{n,k} + \sum_{j \in \Gamma_{n,k}, j \cap i = \emptyset} E_{ij} X_j^{(i)} \tag{9}$$

that is the number of isolated trees in $\mathbb{G}(n, m, p) - i$ equals to the sum of the number of isolated trees in $\mathbb{G}(n, m, p)$ and the number of isolated trees in $\mathbb{G}(n, m, p) - i$ which are connected to i.

We know that

$$|W_{n,k} - W_i| = (W_{n,k} - W_i)^+ + (W_{n,k} - W_i)^-$$

where $(W_{n,k} - W_i)^+ = \max\{W_{n,k} - W_i, 0\}$ and $(W_{n,k} - W_i)^- = -\min\{W_{n,k} - W_i, 0\}$. Since $-\min\{W_{n,k} - W_i, 0\} = \max\{W_i - W_{n,k}, 0\} = (W_i - W_{n,k})^+$, we have

$$\mathbb{E}|W_{n,k} - W_i| = \mathrm{E}(W_{n,k} - W_i)^+ + \mathrm{E}(W_i - W_{n,k})^+.$$

Form (8) and (9), we have

$$\begin{aligned} (W_{n,k} - W_i)^+ &\leq X_i \\ (W_i - W_{n,k})^+ &\leq \sum_{j \in \Gamma_{n,k}, j \cap i = \emptyset} E_{ij} X_j^{(i)}. \end{aligned}$$

We note that,

$$\begin{aligned} \mathbb{E}(X_i) &= \frac{\mathbb{E}(W_{n,k})}{\binom{n}{k}} \\ &= \frac{\binom{n}{k} \sum_{s=0}^{m} \binom{m}{s} p^{s+k-1} (1-p)^{m-s} (1-p)^{(n-k)s}}{\binom{n}{k}} \end{aligned}$$

$$= \sum_{s=0}^{m} \binom{m}{s} p^{s+k-1}(1-p)^{m-s}(1-p)^{(n-k)s}$$

$$= p^{k-1}\left[1-p+p(1-p)^{(n-k)}\right]^{m}. \tag{10}$$

and

$$\begin{aligned}\sum_{j\in\Gamma_{n,k}, j\cap i=\emptyset} \mathrm{E}[E_{ij}X_j^{(i)}] &= \sum_{j\in D, j\cap i=\emptyset} P(E_{ij}=1, X_j^{(i)}=1)\\ &= \binom{n-k}{k}(1-q^{k^2})\sum_{s=0}^{m}\binom{m}{s}p^{s+k-1}(1-p)^{m-s}\\ &\times (1-p)^{(n-2k)s}\\ &\le \frac{n^k}{k!}p^k k^2\left[1-p+p(1-p)^{(n-2k)}\right]^{m}. \end{aligned} \tag{11}$$

From (10), (11) and use the fact that $1-p \le \frac{1}{e^p}$, we have

$$\begin{aligned}\mathbb{E}(|W-W_i|) &= \mathbb{E}(W-W_i)^+ + \mathbb{E}(W_i-W)^+\\ &\le \mathbb{E}(X_i) + \sum_{j\in\Gamma_{n,k}, j\cap i=\emptyset}\mathbb{E}(E_{ij}X_j^{(i)})\\ &\le p^{k-1}\left[1-p+p(1-p)^{(n-k)}\right]^m + \frac{n^k}{k!}p^k k^2\left[1-p+p(1-p)^{(n-2k)}\right]^m\\ &\le \left[1-p+p(1-p)^{(n-2k)}\right]^m(1+\frac{n^k}{k!}pk^2)p^{k-1}\\ &= \left[1-p(1-(1-p)^{(n-2k)}\right]^m(1+\frac{n^k}{k!}pk^2)p^{k-1}\\ &\le \frac{(1+\frac{n^k}{k!}pk^2)p^{k-1}}{e^{mp(1-(1-p)^{n-2k})}} \end{aligned} \tag{12}$$

Hence, by (4), (5) and (12), we have

$$|P(W\in A) - Poi_\lambda(A)| \le \frac{C_{(\lambda,A)}(1+\frac{n^k}{k!}pk^2)p^{k-1}}{e^{mp(1-(1-p)^{n-2k})}}$$

where $C_{(\lambda,A)} = \min\left\{1, \lambda, \frac{\triangle(\lambda)}{M_A+1}\right\}$.

This complete the proof of Theorem 1.2.

4. Proof of Corollary 1.1

From (12), we have

$$\begin{aligned}\mathbb{E}(|W-W_i|) &\leq \frac{(1+\frac{n^k}{k!}pk^2)p^{k-1}}{e^{mp(1-(1-p)^{n-2k})}}\\ &= \frac{p^{k-1}}{e^{mp(1-(1-p)^{n-2k})}} + \frac{k(np)^k}{(k-1)!e^{mp(1-(1-p)^{n-2k})}} \qquad (13)\end{aligned}$$

Suppose that $p = \dfrac{1}{n^\gamma}$ for any $\gamma \in \mathbb{R}^+ \setminus \{1\}$.

1. If $\gamma > k > 0$, $q = 1-p$ and $m = \lfloor n^\alpha \rfloor$ for some $\alpha > 0$, then we observe that

$$\begin{aligned}\mathbb{E}(|W-W_i|) &\leq \frac{1}{n^{(k-1)\gamma}e^{mp(1-(1-p)^{n-2k})}} + \frac{k}{(k-1)!n^{(\gamma-1)k}e^{mp(1-(1-p)^{n-2k})}}\\ &\leq \frac{n^\gamma(k-1)!}{(k-1)!n^{k\gamma}e^{mp(1-(1-p)^{n-2k})}} + \frac{n^k k}{(k-1)!n^{k\gamma}e^{mp(1-(1-p)^{n-2k})}}\\ &\leq \frac{(n^\gamma+n^k)}{n^{k\gamma}e^{mp(1-(1-p)^{n-2k})}}\\ &\leq \frac{2}{n^{(k\gamma-\gamma)}e^{mp(1-(1-p)^{n-2k})}}\\ &\leq \frac{2}{n^{(k\gamma-\gamma)}e^{n^{\alpha-\gamma}(1-(1-p)^{n-2k})}}\\ &\leq \frac{2}{n^{(k\gamma-\gamma)}n^{2(\alpha-\gamma)(1-(q)^{n-2k})}}\\ &= \frac{2}{n^{2(\alpha-\gamma)(1-(q)^{n-2k})+(k\gamma-\gamma)}} \qquad (14)\end{aligned}$$

By (4), (5) and (14), we have

$$|P(W\in A) - Poi_\lambda(A)| \leq \frac{C(\lambda, A)}{n^{2(\alpha-\gamma)(1-(q)^{n-2k})+(k\gamma-\gamma)}},$$

where $C(\lambda, A) = 2\min\left\{1, \lambda, \frac{\triangle(\lambda)}{M_A+1}\right\}$.

2. If $k > \gamma > 0$, $q = 1-p$ and $m = \lfloor n^\alpha \rfloor$ for some $\alpha > 0$, then we observe that

$$\mathbb{E}(|W-W_i|) \leq \frac{1}{n^{(k-1)\gamma}e^{mp(1-(1-p)^{n-2k})}} + \frac{k}{(k-1)!n^{(\gamma-1)k}e^{mp(1-(1-p)^{n-2k})}}$$

$$\leq \frac{n^{\gamma}(k-1)!}{(k-1)!n^{k\gamma}e^{mp(1-(1-p)^{n-2k})}} + \frac{n^k k}{(k-1)!n^{k\gamma}e^{mp(1-(1-p)^{n-2k})}}$$
$$\leq \frac{(n^{\gamma}+n^k)}{n^{k\gamma}e^{mp(1-(1-p)^{n-2k})}}$$
$$\leq \frac{2}{n^{(k\gamma-k)}e^{mp(1-(1-p)^{n-2k})}}$$
$$\leq \frac{2}{n^{(k\gamma-k)}e^{n^{\alpha-\gamma}(1-(1-p)^{n-2k})}}$$
$$\leq \frac{2}{n^{(k\gamma-k)}n^{2(\alpha-\gamma)(1-(q)^{n-2k})}}$$
$$= \frac{2}{n^{2(\alpha-\gamma)(1-(q)^{n-2k})+(k\gamma-k)}} \tag{15}$$

By (4), (5) and (15), we have

$$|P(W\in A) - Poi_{\lambda}(A)| \leq \frac{C(\lambda, A, k)}{n^{2(\alpha-\gamma)(1-(q)^{n-2k})+(k\gamma-k)}},$$

where $C(\lambda, A, k) = 2\min\left\{1, \lambda, \frac{\triangle(\lambda)}{M_A+1}\right\}$.

This complete the proof of Corollary 1.1.

Acknowledgments

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References

[1] A.D. Barbour, Louis H.Y. Chen, *An Introduction to Stein's Method*, Singapore university Press and world scientific Publishing Co. Pte. Ltd (2005).

[2] M. Karonski, E.R. Scheinerman, K.B. Singer-Cohen, On random intersection graphs: The subgraph problem, Combinatorics, *Probability and Computing*, **8** (1999), 131-159.

[3] J. Fill, E. Scheinerman, K. Singer-Cohen, Random intersection graphs when $m = w(n)$: An equivalence theorem relating the evolution of the $G(n, m, p)$ and $G(n, p)$ models, *Random Structures Algorithms*, **16** (2000), 156-176; *Lecture Note Ser.*, Vol. 276, Cambridge University Press, Cambridge (1999), 239-298.

[4] L.H.Y. Chen, Poisson approximation for dependent trials, *Annals of Probability*, 3 (1975), 534-545.

[5] P. Erdős, A. Rényi, On the evolution of random graphs, *Publ. Math. Inst. Hungar. Acad. Sci.*, **5** (1960), 17-61.

[6] Y. Shang, On the isolated vertices and connectivity in random intersection graphs, *Hindawi Publishing Corporation International Journal of Combinatorics*, Volume 2011, Article ID 872703 (2011).

[7] A.D. Barbour, L. Holst, S. Janson, *Poisson Approximation*, Oxford Studies in Probability **2**, Clarendon Press, Oxford (1992).

[8] T. Santiwipanont, K. Teerapabolarn, Two formulas of non-uniform bounds on Poisson approximation for dependent indicators, *Thai Journal of Mathematics* (2006).

[9] M. Donganont, The bounds on Poisson Approximation of the number of isolated vertices in a random intersection graph, *International Journal of Pure and Applied Mathematics*, **87**, No. 2 (2013), 323-332.