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COMMON FIXED POINT THEOREMS OF WEAKLY COMPATIBLE MAPS SATISFYING (E.A.) AND (CLR) PROPERTY

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Abstract: We prove common fixed-point theorems in a metric space, which generalizes the result of Jay G. Mehta and M.L. Joshi, using (E.A)-property and Common Limit Range Property (CLR-property).

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Key Words: common fixed point, contractive modulus, weakly compatible maps, (E.A.) property, (CLR) property

1. Introduction

The first important result on fixed-point for contractive-type mappings was the well-known Banach fixed point theorem, published for the first time in 1922. Gerald Jungck [2] initiated the concept of compatibility in 1986.

In 1998, Jungck and Rhoades [4] introduced the notion of weakly compatible mappings and showed that compatible maps are weakly compatible but not conversely.

Branciari [1] obtained a fixed point result for a single mapping satisfying an analogue of Banach's contraction principle for an integral-type inequality. The second author [3] proved a fixed point theorem for a pair of maps satisfying a general contractive condition of integral type.

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In 2002, Aamri and Moutawakil [7] intoduced the notion of (E.A.)-property.

In 2011, Sintunavarat and S. Kumam [8] introduced the notion of (CLR)-property.

The main purpose of this paper is to present fixed point results for two pair of maps satisfying a new contractive condition of integral type by using the concept of weakly compatible maps with (E.A.) and (CLR) property in a metric space.

Definition 1.1. [4] Let f and g be two self-maps defined on a set, then f and g are said to be weakly compatible if they commute at coincidence points. That is, if $fu = gu$ for some $u \in X$, then $fgu = gfu$.

Definition 1.2. A function $\psi : [0, \infty) \to [0, \infty)$ is said to be a contractive modules if $\psi : [0, \infty) \to [0, \infty)$ and $\psi(s) < s$ for $s > 0$.

Definition 1.3. [7] Let f and g be two self-mappings defined on a metric space (X, d). We say that f and g satisfy the (E.A.) property if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = u, \quad \text{for some } u \in X.$$

Definition 1.4. [8] Let f and g be two self-mappings defined on a metric space (X, d). We say that f and g satisfy the (CLR_f)-property, if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = fx.$$

Lemma 1.5. *[9] Let $\psi : [0, \infty) \to [0, \infty)$ be a right continuous function such that $\psi(s) < s$ for every $s > 0$, then $\lim_{n\to\infty} \psi^n(s) = 0$, where ψ^n denotes the n-times repeated composition of ψ with itself.*

2. Main Result

Theorem 2.1. *Let f, g, h and k be self-maps defined on a metric space (X, d) satisfying the following conditions:*

(i) $f(X) \subseteq h(X)$, $g(X) \subseteq k(X)$,

(ii) *For all $x, y \in X$, there exists a right continuous function $\psi : [0, \infty) \to [0, \infty), \psi(0) = 0$ and $\psi(s) < s$ for $s > 0$ such that*

$$\int_0^{d(fx,gy)} \phi(t)dt \le \psi\Big(\int_0^{M(x,y)} \phi(t)dt\Big), \tag{2.1}$$

where $\phi : R^+ \to R^+$ is a lebesgue integrable mapping which is summable, nonnegative and such that:

$$\int_0^\varepsilon \phi(t)dt > 0 \quad \text{for each } \varepsilon > 0 \tag{2.2}$$

$$M(x,y) = \max\Big\{d(kx,hy), d(kx,fx), d(gy,hy), \frac{1}{2}[d(kx,gy)+d(hy,fx)]\Big\} \tag{2.3}$$

(1) *The pairs (f,k) or (g,h) satisfy (E.A) property.*

(2) *The pairs (f,k) and (g,h) are weakly compatible.*

If$k(X)$ is closed. Then f, g, h and k have a unique common fixed point.

Proof. Suppose that the pair (f,k) satisfy (E.A) property, so there exists a sequence $\{x_n\}$ in X such that:

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} kx_n = u, \quad \text{for some } u \in X. \tag{2.4}$$

Since $f(X) \subseteq h(X)$, so there exists a sequence $\{y_n\}$ in X such that $fx_n = hy_n$. Hence

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} hy_n = u\,. \tag{2.5}$$

Now, we claim that $\lim_{n\to\infty} gy_n = u$. For this, put $x = x_n$, $y = y_n$ in (ii), we have

$$\int_0^{d(fx_n,gy_n)} \phi(t)dt \le \psi\Big(\int_0^{M(x_n,y_n)} \phi(t)dt\Big)$$

where

$$M(x_n,y_n) = \max\Big\{d(kx_n,hy_n), d(kx_n,fx_n), d(gy_n,hy_n), \frac{1}{2}[d(kx_n,gy_n)+d(hy_n,fx_n)]\Big\}$$

Taking limit as $n \to \infty$, we get

$$\lim_{n\to\infty} \int_0^{d(fx_n,gy_n)} \phi(t)dt \le \lim_{n\to\infty} \psi\Big(\int_0^{M(x_n,y_n)} \phi(t)dt\Big).$$

Since

$$\lim_{n\to\infty} d(fx_n, gy_n) = d(u, gy_n) = \lim_{n\to\infty} d(gy_n, hy_n) = \lim_{n\to\infty} (kx_n, gy_n),$$
$$\lim_{n\to\infty} d(kx_n, hy_n) = d(u,u) = 0 = \lim_{n\to\infty} d(kx_n, fx_n) = \lim_{n\to\infty} (hy_n, fx_n)$$

We may conclude that

$$\int_0^{d(u,gy_\mathsf{n})} \phi(t)dt \le \psi \int_0^{d(u,gy_\mathsf{n})} \phi(t)dt$$

If $d(u, gy_n) \neq 0$, then.

$$\int_0^{d(u,gy_\mathsf{n})} \phi(t)dt \le \psi\Big(\int_0^{d(u,gy_\mathsf{n})} \phi(t)dt\Big) < \int_0^{d(u,gy_\mathsf{n})} \phi(t)dt, \text{ a contradiction.}$$

Therefore

$$\lim_{n\to\infty} gy_n = u\,.$$

Hence

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} kx_n = \lim_{n\to\infty} gy_n = \lim_{n\to\infty} hy_n = u. \tag{2.6}$$

Suppose that $k(X)$ is closed, so there exists $v \in X$ such that $kv = u$.

Now, we claim that $fv = u$. Put $x = v$, $y = y_n$ in (ii)

$$\int_0^{d(fv,gy_\mathsf{n})} \phi(t)dt \le \psi\Big(\int_0^{M(v,y_\mathsf{n})} \phi(t)dt\Big)$$

where

$$M(v, y_n) = \max\Big\{d(kv, hy_n), d(kv, fv), d(gy_n, hy_n), \frac{1}{2}[d(kv, gy_n) + d(hy_n, fv)]\Big\},$$
$$\lim_{n\to\infty} d(fv, gy_n) = d(fv, u) = \lim_{n\to\infty} d(kv, fv) = \lim_{n\to\infty} d(hy_n, fv),$$
$$\lim_{n\to\infty} d(kv, hy_n) = 0 = \lim_{n\to\infty} d(gy_n, hy_n) = \lim_{n\to\infty} d(kv, gy_n).$$

We may conclude that

$$\int_0^{d(fv,u)} \phi(t)dt \le \psi\Big(\int_0^{d(fv,u)} \phi(t)dt\Big)$$

If $d(fv,u) \neq 0$, then

$$\begin{aligned}\int_0^{d(fv,u)} \phi(t)dt &\le \psi\Big(\int_0^{d(fv,u)} \phi(t)dt\Big)\\ &< \int_0^{d(fv,u)} \phi(t)dt, \text{ a contradiction}\end{aligned}$$

Therefore

$$d(fv,u) = 0 \quad \Rightarrow \quad fv = u\,. \tag{2.7}$$

Therefore

$$fv = kv = u.$$

Hence v is a coincidence point of (f,k).
Since $f(X) \subseteq h(X)$, so there exists $w \in X$, such that

$$fv = hw = u. \tag{2.8}$$

Now, we claim that $gw = u$. Put $x = v$, $y = w$ in (ii)

$$\int_0^{d(fv,gw)} \phi(t)dt \le \psi\Big(\int_0^{M(v,w)} \phi(t)dt\Big)$$

where

$$\begin{aligned}M(v,w) &= \max\Big\{d(kv,hw), d(kv,fv), d(gw,hw),\\ &\qquad \frac{1}{2}[d(kv,gw) + d(hv,fv)]\Big\}\\ &= \max\Big\{d(u,u), d(u,u), d(gw,u), \frac{1}{2}[d(u,gw) + d(u,u)]\Big\}\\ &= d(u,gw)\end{aligned}$$

$$\int_0^{d(u,gw)} \phi(t)dt \le \psi\Big(\int_0^{d(u,gw)} \phi(t)dt\Big).$$

If $d(u, gw) \neq 0$, then

$$\int_0^{d(u,gw)} \phi(t)dt \leq \psi\Big(\int_0^{d(u,gw)} \phi(t)dt\Big) < \int_0^{d(u,gw)} \phi(t)dt \quad \text{a contradiction.}$$

Therefore

$$d(u, gw) = 0 \quad \Rightarrow \quad gw = u = hw\,. \tag{2.9}$$

Hence w is a coincidence point of (g, h).
As the pairs (g, h) and (f, k) are weakly compatible therefore

$$\left.\begin{aligned} ghw &= hgw \\ kfv &= fkv \end{aligned}\right\} \tag{2.10}$$

$$u = gw = hw = kv = fv \tag{2.11}$$

Then

$$\left.\begin{aligned} ku &= kfv = fkv = fu \\ hu &= hgw = ghw = gu \end{aligned}\right\} \tag{2.12}$$

If $gu \neq u$, then from (ii), (2.11) and (2.12),

$$\int_0^{d(u,gu)} \phi(t)dt = \int_0^{d(fv,gu)} \phi(t)dt \leq \psi\Big(\int_0^{M(v,u)} \phi(t)dt\Big)$$

where

$$\begin{aligned} M(v, u) &= \max\Big\{d(kv, hu), d(kv, fv), d(gu, hu), \\ &\qquad \frac{1}{2}[d(kv, gu) + d(hu, fv)]\Big\} \\ &= \max\Big\{d(u, gu), d(u, u), d(gu, gu), \\ &\qquad \frac{1}{2}[d(u, gu) + d(gu, u)]\Big\} \\ &= d(u, gu) \end{aligned}$$

$$\int_0^{d(u,gu)} \phi(t)dt \leq \psi\Big(\int_0^{d(u,gu)} \phi(t)dt\Big)$$
$$< \int_0^{d(u,gu)} \phi(t)dt, \quad \text{a contradiction}$$

Therefore $gu = u = hu$. Similarly $fu = u = ku$. Then evidently, from (2.12), u is a common fixed point of f, g, h and k.

Uniqueness. For uniqueness of u, let if possible, we assume that u and u' $(u \neq u')$ are common fixed point of f, g, h and k By (ii), we have

$$\int_0^{d(u,u')} \phi(t)dt = \int_0^{d(fu,gu')} \phi(t)dt \leq \psi\Big(\int_0^{M(u,u')} \phi(t)dt\Big)$$

where

$$M(u,u') = \max\Big\{d(ku,hu'), d(ku,fu), d(gu',hu'), \frac{1}{2}[d(ku,gu') + d(hu',fu)]\Big\}$$
$$= \max\Big\{d(u,u'), d(u,u), d(u',u'), \frac{1}{2}[d(u,u') + d(u',u)]\Big\}$$

$$= d(u,u')$$
$$\int_0^{d(u,u')} \phi(t)dt \leq \psi\Big(\int_0^{d(u,u')} \phi(t)dt\Big)$$
$$< \int_0^{d(u,u')} \phi(t)dt, \quad \text{a contradiction}$$

Therefore $u = u'$.
Thus u is the unique common fixed point of f, g, h and k Hence the theorem. □

Corollary 2.2. *Let (X, d) be a metric space. Suppose that the maps f, g and h are self-maps of X satisfying the following conditions:*

(i) $f(X) \subseteq h(X)$, $g(X) \subseteq h(X)$

(ii) *For all $x, y \in X$, there exists a right continuous function $\psi : [0, \infty) \to [0, \infty)$, $\psi(0) = 0$ and $\psi(s) < s$ for $s > 0$ such that*

$$\int_0^{d(fx,gy)} \phi(t)dt \leq \psi\left(\int_0^{M(x,y)} \phi(t)dt\right), \tag{2.13}$$

where $\phi : R^+ \to R^+$ is a lebesgue integrable mapping which is summable, nonnegative and such that

$$\int_0^{\varepsilon} \phi(t)dt > 0 \quad \text{for each } \varepsilon > 0 \tag{2.14}$$

$$M(x,y) = \max\Big\{d(hx,hy), d(hx,fx), d(gy,hy), \frac{1}{2}[d(hx,gy) + d(hy,fx)]\Big\}$$

(1) *The pairs (f, h) or (g, h) satisfy (E.A.) property.*

(2) *The pairs (f, h) and (g, h) are weakly compatible. If $h(X)$ is closed. Then f, g and h have a unique common fixed point.*

Proof. By taking $k = h$ in Theorem 2.1, we get the result. □

Corollary 2.3. *Let (X, d) be a metric space. Suppose that the maps g and h are self maps on X satisfying the following conditions:*

(i) $g(X) \subseteq h(X)$

(ii) *For all $x, y \in X$, there exists a right continuous function $\psi : [0, \infty) \to [0, \infty)$, $\psi(0) = 0$ and $\psi(s) < s$ for $s > 0$ such that*

$$\int_0^{d(gx,hy)} \phi(t)dt \leq \psi\left(\int_0^{M(x,y)} \phi(t)dt\right) \tag{2.15}$$

where $\phi : R^+ \to R^+$ is a lebesgue integrable mapping which is summable, non-negative and such that

$$\int_0^{\varepsilon} \phi(t)dt > 0 \quad \text{for each} \quad \varepsilon > 0 \tag{2.16}$$

$$M(x,y) = \max\Big\{d(hx,hy), d(hx,gx), d(hy,gy), \frac{1}{2}[d(hx,gy) + d(hy,gx)]\Big\} \tag{2.17}$$

(1) *The pair (g,h) satisfy (E.A.) property,*

(2) *The pair (g,h) is weakly compatible*

If $h(X)$ is closed, then (g,h) have a unique common fixed point.

Proof. By taking $k = h$ and $f = g$ in Theorem 2.1, we get the result. □

Remark 2.4. If $\phi(t) = 1$, then Theorem 2.1 of this paper reduces to Theorem 3.1 of [6].

3.

Theorem 3.1. *Let (X,d) be a metric space. Suppose that the mappings f, g, h and k are four self-maps of X satisfying the following conditions:*

(i) *$f(X) \subseteq h(X)$, $g(X) \subseteq k(X)$.*

(ii) *For all $x, y \in X$, there exists a right continuous function $\psi : [0,\infty) \to [0,\infty)$, $\psi(0) = 0$ and $\psi(s) < s$ for $s > 0$, such that*

$$\int_0^{d(fx,gy)} \phi(t)dt \le \psi\Big(\int_0^{M(x,y)} \phi(t)dt\Big) \tag{3.1}$$

where $\phi : R^+ \to R^+$ is a lebesgue integrable mapping which is summable, nonnegative and such that:

$$\int_0^{\varepsilon} \phi(t)dt > 0 \quad \text{for each } \varepsilon > 0 \tag{3.2}$$

$$M(x,y) = \max\Big\{d(kx,hy), d(kx,fx), d(gy,hy), \frac{1}{2}[d(kx,gy) + d(hy,fx)]\Big\}$$

(1) *The pairs (f,k) or (g,h) satisfy (CLR_f) or (CLR_g)-property*

(2) *If the pairs (f,k) and (g,h) are weakly compable, then f, g, h and k have a unique common fixed point.*

Proof. First suppose that the pair (f,k) satisfy (CLR$_f$)-property so there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} kx_n = fx \tag{3.3}$$

Since $f(X) \subseteq h(X)$, so there exists a sequence $\{y_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} hy_n = fx. \tag{3.4}$$

We claim that $\lim_{n\to\infty} gy_n = fx$. For this put $x = x_n$ $y = y_n$ in (ii)

$$\int_0^{d(fx_n,gy_n)} \phi(t)dt \le \psi\left(\int_0^{M(x_n,y_n)} \phi(t)dt\right)$$

$$M(x_n,y_n) = \max\Big\{d(kx_n,hy_n), d(kx_n,fx_n), d(gy_n,hy_n), \frac{1}{2}[d(kx_n,gy_n)+d(hy_n,fx_n)]\Big\}$$

$$\lim_{n\to\infty} d(kx_n,hy_n) = 0 = \lim_{n\to\infty} d(kx_n,fx_n) = \lim_{n\to\infty} d(hy_n,fx_n)$$
$$\lim_{n\to\infty} d(fx_n,gy_n) = d(fx,gy_n) =) \lim_{n\to\infty} d(kx_n,gy_n) = \lim_{n\to\infty} d(gy_n,hy_n).$$

Taking limit as $n\to\infty$, we get

$$\begin{aligned}\lim_{n\to\infty}\int_0^{d(fx_n,gy_n)} \phi(t)dt &\le \psi\left(\int_0^{d(fx,gy_n)} \phi(t)dt\right),\\ \int_0^{d(fx,gy_n)} \phi(t)dt &\le \psi\left(\int_0^{d(fx,gy_n)} \phi(t)dt\right)\\ &< \int_0^{d(fx,gy_n)} \phi(t)dt, \quad \text{a contradiction}\end{aligned}$$

Therefore

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} kx_n = \lim_{n\to\infty} gy_n = \lim_{n\to\infty} hy_n = fx \tag{3.5}$$

Since $f(X) \subseteq h(X)$, so there exists a point $v \in X$ such that $fx = hv$, we claim that $gv = hv = fx$.
Put $x = x_n$, $y = v$ in (ii), we have

$$\int_0^{d(fx_n,gv)} \phi(t)dt \leq \psi\Big(\int_0^{M(x_n,v)} \phi(t)dt\Big)$$

$$M(x_n, v) = \max\Big\{d(kx_n, hv), d(kx_n, fx_n), d(gv, hv), \frac{1}{2}[d(kx_n, gv) + d(hv, fx_n)]\Big\}$$

Taking limit as $n \to \infty$,

$$\lim_{n\to\infty} \int_0^{d(fx_n,gv)} \phi(t)dt \leq \lim_{n\to\infty} \psi\Big(\int_0^{M(x_n,v)} \phi(t)dt\Big)$$

$$\lim_{n\to\infty} d(fx_n, gv) = \lim_{n\to\infty} d(kx_n, gv) = d(fx, gv) = d(gv, hv),$$
$$\lim_{n\to\infty} d(kx_n, hv) = \lim_{n\to\infty} d(kx_n, fx_n) = \lim_{n\to\infty} d(hv, fx_n) = 0.$$

$$\int_0^{d(fx,gv)} \phi(t)dt \leq \psi\Big(\int_0^{d(fx,gv)} \phi(t)dt\Big) \tag{3.6}$$

If $d(fx, gv) \neq 0$, then $d(fx, gv) > 0$.

$$\int_0^{d(fx,gv)} \phi(t)dt \leq \psi\Big(\int_0^{d(fx,gv)} \phi(t)dt\Big) < \int_0^{d(fx,gv)} \phi(t)dt, \quad \text{a contradiction.}$$

Therefore

$$fx = gv \tag{3.7}$$
$$gv = hv = fx = w \quad \text{(say)}$$

Hence v is a coincidence point of (g, h).
As the pair (g, h) is weakly compatible, therefore

$$\left.\begin{aligned} ghv &= hgv \\ gw &= hw \end{aligned}\right\} \tag{3.8}$$

Since $g(X) \subseteq k(X)$, so there exists a point $u \in X$, such that

$$gv = ku\,. \tag{3.9}$$

We claim that $ku = fu = w$.
Put $x = u$, $y = v$ in (ii)

$$\int_0^{d(fu,gv)} \phi(t)dt \le \psi\Big(\int_0^{M(u,v)} \phi(t)dt\Big)$$

where

$$\begin{aligned} M(u,v) &= \max\Big\{d(ku,hv), d(ku,fu), d(gv,hv) \\ &\qquad \frac{1}{2}[d(ku,gv)+d(hv,fu)]\Big\} \\ &= \max\Big\{d(gv,gv), d(w,fu), d(gv,gv), \\ &\qquad \frac{1}{2}[d(gv,gv)+d(w,fu)\Big\} \end{aligned}$$

$$\begin{aligned} \int_0^{d(fu,w)} \phi(t)dt &\le \psi\Big(\int_0^{d(fu,w)} \phi(t)dt\Big) \\ &< \int_0^{d(fu,w)} \phi(t)dt, \quad \text{a contradiction.} \end{aligned}$$

Therefore

$$fu = ku = w\,. \tag{3.10}$$

Hence u is a coincidence point of (f,k).
As the pair (f,k) is weakly compatible.

$$\left.\begin{aligned} fku &= kfu \\ fw &= kw \end{aligned}\right\} \tag{3.11}$$

Now, we show that $fw = w$. If possible, suppose that $fw \ne w$ then

$$\int_0^{d(fw,w)} \phi(t)dt = \int_0^{d(fw,gv)} \phi(t)dt \le \psi\Big(\int_0^{M(w,v)} \phi(t)dt\Big)$$

where

$$M(w,v) = \max\Big\{d(kw,hv), d(kw,fw), d(gv,hv), \frac{1}{2}[d(kw,gv)+d(hv,fw)]\Big\}$$
$$= \max\Big\{d(fw,w), d(fw,fw), d(w,w), \frac{1}{2}[d(fw,w)+d(w,fw)]\Big\}$$

$$\int_0^{d(fw,w)} \phi(t)dt \le \psi\Big(\int_0^{d(fw,w)} \phi(t)dt\Big) < \int_0^{d(fw,w)} \phi(t)dt, \quad \text{a contradiction.}$$

Therefore $fw = w$, similarly $gw = w$.

Then evidently, from (3.8) and (3.11) f, g, h and k have a common fixed point.

The uniqueness of the common fixed point follows easily from condition (ii). □

Example 3.2. Let $X = [1, 11)$ be equipped with the usual metric space

$$d(x, y) = |x - y|.$$

Let f, g, h and k be self-maps of X, defined by:

$$f(x) = \begin{cases} 1 & \text{if } x \in \{1\} \cup [3, 11) \\ x & \text{if } x \in (1, 3) \end{cases}, \quad g(x) = \begin{cases} 1 & \text{if } x \in \{1\} \cup [3, 11) \\ x+1 & \text{if } x \in (1, 3) \end{cases},$$

$$h(x) = \begin{cases} 1 & \text{if } x = 1 \\ 2 & \text{if } x \in (1, 3] \\ x-2 & \text{if } x \in (3, 11) \end{cases}, \quad k(x) = \begin{cases} 1 & \text{if } x = 1 \\ 3 & \text{if } x \in (1, 3] \\ \dfrac{3x-1}{8} & \text{if } x \in (3, 11) \end{cases}$$

Clearly $f(X) = [1, 3) \subseteq [1, 9) = h(X)$, $g(X) = \{1\} \cup (2, 4) \subseteq [1, 4] = k(X)$.
Let $x_n = \langle 1 \rangle$ and $y_n = \left\langle 3 + \frac{1}{n} \right\rangle$ be two sequences in X.

The pairs (f, k) and (g, h) satisfy (E.A) property. Also the pairs (f, k) or (g, h) satisfy (CLR_f) or (CLR_g)-property. All the conditions of Theorem 3.1 are satisfied.

1 is the unique common fixed point of f, g, h and k.

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