ON T-FUZZY ESSENTIAL IDEALS OF RINGS

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Abstract: In this paper our attempt is to extend the notion of essential fuzzy ideals of rings to T-fuzzy essential ideals. Various properties on T-fuzzy essential ideals are established. Some results are being discussed which establish necessary and sufficient condition between T-fuzzy essential ideal in fuzzy set theory and essential ideal in crisp set theory. Moreover it is proved that if μ, ν and σ are non-zero T-fuzzy left ideals of R such that $\mu \subseteq \nu \subseteq \sigma$, then $\mu \subseteq_{e(T)} \sigma$ if and only if $\mu \subseteq_{e(T)} \nu \subseteq_{e(T)} \sigma$.

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1. Introduction

The notion of fuzzy set was introduced by L.A. Zadeh [13] in 1965. Since then these ideas have been applied to other algebraic structures like groups, rings, modules, topologies and so on. The concept of fuzzy group was introduced by Rosenfeld [12] in 1971. The study of fuzzy sub rings and fuzzy ideals of rings was initiated by W. Liu [7] around 1982. Mukherjee and Sen [10], [11] characterized regular rings in terms of fuzzy ideals of rings. Abou-Zaid [1], Kumar et al [5], [6] and Malik et al [8] have carried out extensive work on fuzzy

ideals of rings. U. Medhi et al [9] studied essential fuzzy ideals of rings. The concept of t-norm (i.e. triangular norm) was studied by D Dubois and H Prade [4]. Dheena and Mohanraaj [3] studied T-fuzzy ideals of rings. In this paper we extend the notion of fuzzy essential ideals of rings to T-fuzzy essential ideals and various properties are being investigated.

2. Preliminaries

Throughout this paper R denotes a non-commutative ring with unity unless and otherwise stated.

Definition 2.1. A mapping $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a triangular norm [T-norm] if and only if it satisfies the following conditions:

$$\text{T1) } T(x, 1) = T(1, x) = 1, \forall x \in [0, 1].$$

$$\text{T2) If } x \geq x^*, y \geq y^* \text{ then } T(x, y) \geq T(x^*, y^*).$$

$$\text{T3) } T(x, y) = T(y, x), \forall x, y \in [0, 1].$$

$$\text{T4) } T(x, T(y, z)) = T(T(x, y), z), \forall x, y, z \in [0, 1].$$

Note. The T-norm minimum (min T-norm) is defined by $T(x, y) = \min(x, y)$. Some other T-norms are $T_p(x, y) = xy$, $T_n(x, y) = \max(x + y - 1, 0)$ and

$$T_w(x, y) = \begin{cases} x, & \text{if } y = 1 \\ y, & \text{if } x = 1 \\ 0, & \text{otherwise} \end{cases}$$

Every T-norm T^* satisfies the inequality:

$$T_w(x, y) \leq T^*(x, y) \leq \min(x, y) \text{ and } T^*(0, 0) = 0.$$

A triangular norm is called an Archimedean t-norm if it satisfies the following conditions:

$$\text{T5) } T \text{ is a continuous function (continuity)}$$

$$\text{T6) } T(x, x) < x, \forall x \in [0, 1] \text{ (subidempotency)}$$

Moreover, if T also satisfies (T7), where

$$\text{T7) } x_1 < x_2, y_1 < y_2 \Rightarrow T(x_1, y_1) < T(x_2, y_2) \text{ (strict monotnicity)}$$

is called a strict Archimedean t-norm.

Definition 2.2. A fuzzy subset μ of R is called a *fuzzy left ideal* if it satisfies the following properties:

- (i) $\mu(x - y) \geq \mu(x) \wedge \mu(y)$, for all $x, y \in R$
- (ii) $\mu(xy) \geq \mu(y)$, for all $x, y \in R$

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Definition 2.4. A fuzzy subset μ of R is called a *fuzzy ideal* if it satisfies the following properties:

- (i) $\mu(x - y) \geq \mu(x) \wedge \mu(y)$, for all $x, y \in R$.
- (ii) $\mu(xy) \geq \mu(x) \vee \mu(y)$, for all $x, y \in R$

Definition 2.5. (see [9]) Let μ be a fuzzy subset of a non-empty set X . Then a *fuzzy point* x_t , $x \in X, t \in (0, 1]$ is defined as the fuzzy subset x_t of X such that $x_t(x) = t$, and $x_t(y) = 0$, for all $y \in X - x$. We write $x_t \in \mu$ if and only if $x \in \mu_t$.

Definition 2.6. (see [3]) Let μ, λ be two fuzzy subsets of a set X . A fuzzy subset $\mu \wedge \lambda$ is defined as $(\mu \wedge \lambda)(x) = T(\mu(x), \lambda(x))$.

Definition 2.7. (see [3]) Let μ, λ be two fuzzy subsets of a set X . The product of the fuzzy subsets μ and λ is defined as:

$$(\mu\lambda)(x) = \sup_{x=yz} T(\mu(y), \lambda(z))$$

Definition 2.8. (see [3]) A fuzzy subset μ of a ring R is called T-fuzzy left(right) ideal if:

- (i) $\mu(x - y) \geq T(\mu(x), \mu(y))$,
- (ii) $\mu(xy) \geq \mu(y)$ ($\mu(xy) \geq \mu(x)$) for all $x, y \in R$.

Notes. (1) If we take T-norm as min-norm T-fuzzy right coincides with fuzzy right ideal.

- (2) T-fuzzy ideal is both T-fuzzy right and left ideal.

Lemma 2.1. (see [3]) Every fuzzy right ideal of a ring R is a T -fuzzy right ideal.

Lemma 2.2. (see [3]) If A is a right ideal of a ring R , then χ_A is a T -fuzzy right ideal.

Theorem 2.1. (see [3]) If μ, λ are T -fuzzy right ideals of a ring R , then $\mu \wedge \lambda$ is a T -fuzzy right ideal of R .

Remark 2.1. (see [3]) Every T -fuzzy right ideal need not be a fuzzy ideal by the following example:

Example 1. Consider the ring $R = \{0, a, b, c\}$ with addition and multiplication operations defined as follows:

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

and

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	0	0	0
c	0	a	b	c

Now we define a fuzzy subset μ of R such that $\mu(0) = 1, \mu(a) = .3, \mu(b) = .5, \mu(c) = .4$

Then μ is a T -fuzzy ideal of R while it is not a fuzzy ideal of R as

$$\mu(b + c) = \mu(a) = .3 \not\geq \mu(b) \wedge \mu(c) = .5 \wedge .4.$$

Definition 2.9. (see [9]) A non zero fuzzy left ideal μ of R is called a essential fuzzy left ideal of R , denoted by $\mu \subseteq_e R$, if for every nonzero fuzzy left ideal δ of R , there exist $x(\neq 0) \in R$ such that $x_t \in \mu$ and $x_t \in \delta$, for some $t \in (0, 1]$.

Definition 2.10. (see [9]) Let μ and σ be two nonzero fuzzy left ideals of R such that $\mu \subseteq \sigma$. Then μ is called a essential fuzzy left ideal in σ , denoted by $\mu \subseteq_e \sigma$ if for every nonzero fuzzy left ideal δ of R satisfying $\delta \subseteq \sigma$, there exist $x(\neq 0) \in R$ such that $x_t \in \mu$ and $x_t \in \delta$, for some $t \in (0, 1]$.

3. T-Fuzzy Essential Ideal of Rings

Definition 3.1. A non-zero T-fuzzy left ideal μ of R is called a T-fuzzy essential left ideal of R , denoted by $\mu \subseteq_{e(T)} R$ if for every non-zero T-fuzzy left ideal σ of R , $\mu \wedge \sigma \neq \chi_0$ i.e there exists $x(\neq 0) \in R$ such that $T[\mu(x), \sigma(x)] \geq t$, for some $t \in (0, 1]$.

Example 2. Consider the ring $R = \{0, a, b, c\}$ with addition and multiplication operations defined as follows:

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

and

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	0	0	0
c	0	a	b	c

Now we define a fuzzy subset μ of R such that $\mu(0) = 1, \mu(a) = .4, \mu(b) = .6, \mu(c) = .5$.

Suppose σ is a non zero T-fuzzy left ideal of R . Then there exists $x(\neq 0) \in R$ such that $\sigma(x) = t(\neq 0)$, where $t \in (0, 1]$. Suppose $\sigma(a) \neq 0$.

Now we consider the product norm T_p defined by $T_p(s, t) = st$, where $s, t \in [0, 1]$.

Then clearly $T_p(\mu(a), \sigma(a)) = \mu(a)\sigma(a) \neq 0$. Hence μ is a T-fuzzy essential left ideal of R .

But μ is not a T-fuzzy essential under the norm $T = T_w$, where T_w is defined as follows:

$$T_w(s, t) = \max\{s + t - 1, 0\}, \text{ for the non zero T-fuzzy ideal } \sigma,$$

where $\sigma(0) = 1, \sigma(a) = .3, \sigma(b) = .5, \sigma(c) = .4$ is such that there exists no non zero x satisfying $T_w(\mu(x), \sigma(x)) \neq 0$.

Note. For the next results except theorems 3.2, 3.4, 3.5 of this section we consider T as an Archimedean t-norm.

Theorem 3.1. *Let μ be a T-fuzzy essential left ideal of R . Then μ is a T-fuzzy essential left ideal of R if and only if μ_t is an essential left ideal of R , for some $t \in (0, 1]$.*

Proof. Let μ be a T-fuzzy essential left ideal of R . Let A be any non-zero left ideal of R . Then χ_A is a non-zero T-fuzzy left ideal of R . Since $\mu \subseteq_{e(T)} R$, so we have $\mu \wedge \chi_A \neq \chi_0$, there exists $x(\neq 0) \in R$ such that $T[\mu(x), \chi_A(x)] \neq 0$. Thus, there exists $x(\neq 0) \in R$ such that $T[\mu(x), \chi_A(x)] \geq t$ for some $t \in (0, 1]$. Now, $\chi_A(x), \mu(x) \geq \min[\chi_A(x), \mu(x)] \geq T[\chi_A(x), \mu(x)] \geq t$. This implies $\mu(x) \geq t$ and $\chi_A(x) \geq t, t \neq 0$ and thus $x \in \mu_t \cap A$. Hence μ_t is an essential left ideal of R .

Conversely, let $\mu_t \subseteq_e R$. Then, for some $t \in (0, 1]$, μ_t is non-zero for which μ is non-zero. Let σ be any non-zero T-fuzzy left ideal of R . Then there exists $x(\neq 0) \in R$ such that $\sigma(x) = s(\neq 0)$. As μ_t is an essential left ideal of R , there exists $y(\neq 0) \in R$ such that $y \in \mu_t \cap \sigma_s$. This implies $\mu(y) \geq t$, $\sigma(y) \geq s$ and therefore $\mu(y) \geq t_0, \sigma(y) \geq t_0$ where $t_0 = \min\{s, t\}$. Also, $(\mu \wedge \sigma)(y) = T[\mu(y), \sigma(y)] \geq T(s, t) > T(0, 0) = 0$. Thus for $y(\neq 0) \in R$ we get $(\mu \wedge \sigma)(y) \neq 0$ i.e. $\mu \wedge \sigma \neq \chi_0$. Hence μ is a T-fuzzy essential left ideal of R . $\square$

Theorem 3.2. *A left ideal A of R is essential in R if and only if χ_A is a T-fuzzy essential left ideal of R .*

Proof. Let $A \subseteq_e R$. Then χ_A is a non-zero T-fuzzy left ideal of R . Let μ be any non-zero T-fuzzy left ideal of R . Then μ_t is a non-zero left ideal of R where $t \in (0, 1]$. Then there exists $x(\neq 0) \in R$ such that $x \in \mu_t \cap A$ i.e. $\mu(x) \geq t$ and $\chi_A(x) = 1$. Now, $(\chi_A \wedge \mu)(x) = T[\mu(x), \chi_A(x)] = T[\mu(x), 1] = \mu(x) \geq t > 0$. Hence $\chi_A \subseteq_{e(T)} R$.

Conversely, let A be a left ideal of R such that χ_A is T-fuzzy essential in R . Let B be any non-zero left ideal of R . Then χ_B is a non-zero T-fuzzy left ideal of R . This implies there exists $x(\neq 0) \in R$ such that $T[\chi_A(x), \chi_B(x)] \geq t$ for some $t \in (0, 1]$. Now $\chi_A(x), \chi_B(x) \geq \min[\chi_A(x), \chi_B(x)] \geq T[\chi_A(x), \chi_B(x)] \geq t$. Thus we get, $\chi_A(x) \geq t$ and $\chi_B(x) \geq t$. Thus we have, $\chi_A(x) = 1$ and $\chi_B(x) = 1$. This implies $x \in A$ and $x \in B$. Hence A is an essential left ideal of R . $\square$

Lemma 3.1. *A T-fuzzy left ideal μ of R is a T-fuzzy essential left ideal of R , if and only if μ is a T-fuzzy essential left ideal of χ_R .*

Proof. Let $\mu \subseteq_{e(T)} R$. Let $\sigma \subseteq \chi_R$ be any non-zero T-fuzzy left ideal of R . Since $\mu \subseteq_{e(T)} R$, so we have $\mu \wedge \sigma \neq \chi_0$. Therefore $\mu \subseteq_{e(T)} \chi_R$.

Conversely, let $\mu \subseteq_{e(T)} \chi_R$. Let σ be any non-zero T-fuzzy left ideal of R . Then clearly $\sigma \subseteq \chi_R$. Also $\mu \subseteq_{e(T)} \chi_R$ and therefore $\mu \wedge \sigma \neq \chi_0$. Hence $\mu \subseteq_{e(T)} R$. $\square$

Definition 3.2. Let μ and σ be two non-zero T-fuzzy left ideals of R such that $\mu \subseteq \sigma$. Then μ is called a T-fuzzy essential left ideal in σ , denoted by $\mu \subseteq_{e(T)} \sigma$, if for every non-zero T-fuzzy left ideal θ of R satisfying $\delta \subseteq \sigma$, $\mu \wedge \delta \neq \chi_0$ i.e. there exists $x(\neq 0) \in R$ such that $T[\mu(x), \delta(x)] \geq t$, for some $t \in (0, 1]$.

Theorem 3.3. If A and B are two non-zero left ideals of R , then A is an essential left ideal of B if and only if χ_A is a T-fuzzy essential left ideal in χ_B .

Proof. Let $A \subseteq_e B$. Then $A \subseteq B$ and therefore $\chi_A \subseteq \chi_B$. Let σ be any non-zero T-fuzzy ideal with $\sigma \subseteq \chi_B$. Then σ_t is a non-zero left ideal and $\sigma_t \subseteq B$, for some $t \in (0, 1]$. This implies there exists $x(\neq 0) \in R$ such that $x \in \sigma_t \cap A$. Therefore $\sigma(x) \geq t$ and $\chi_A(x) = 1$. Now $T[\sigma(x), \chi_A(x)] = T[\sigma(x), 1] = \sigma(x) \geq t$. Thus we have, $\sigma \wedge \chi_A \neq \chi_0$. Hence $\chi_A \subseteq_{e(T)} \chi_B$.

Conversely, let $\chi_A \subseteq_{e(T)} \chi_B$. Then $A \subseteq B$. Let C be a non-zero left ideal of R with $C \subseteq B$. Then χ_C is non-zero and $\chi_C \subseteq \chi_B$. Since $\chi_A \subseteq_{e(T)} \chi_B$, we get $\chi_A \wedge \chi_C \neq \chi_0$. This implies there exists $x(\neq 0) \in R$ such that $T[\chi_A(x), \chi_C(x)] \geq t$ for some $t \in (0, 1]$.

$$\begin{aligned} \text{Now } \chi_A(x), \chi_C(x) &\geq \min[\chi_A(x), \chi_C(x)] \\ &\geq T[\chi_A(x), \chi_C(x)] \\ &\geq t \end{aligned}$$

Thus $\chi_A(x) \geq t$ and $\chi_C(x) \geq t$. Thus $x \in A$ and $x \in C$. Hence $A \subseteq_e B$. $\square$

Theorem 3.4. Let μ and σ be two non-zero T-fuzzy left ideals of R such that $\mu \subseteq \sigma$. Then $\mu \subseteq_{e(T)} \sigma$ if and only if $\mu_t \subseteq_e \sigma_t$, for some $t \in (0, 1]$.

Proof. Let $\mu \subseteq_{e(T)} \sigma$. Since $\mu \subseteq \sigma$, we have $\mu_t \subseteq \sigma_t$, for some $t \in (0, 1]$. Let A be any non-zero left ideal of σ_t .

A fuzzy subset δ of R is defined such that

$$\delta(x) = \begin{cases} \sigma(x), & \text{if } x \in A \\ 0, & \text{if } x \notin A. \end{cases}$$

Then δ is a non-zero T-fuzzy left ideal of R such that $\delta \subseteq \sigma$. Thus there exists $x(\neq 0) \in R$ such that $T[\mu(x), \delta(x)] \geq t$ for some $t \in (0, 1]$.

Now $\mu(x), \delta(x) \geq \min[\mu(x), \delta(x)] \geq T[\mu(x), \delta(x)] \geq t$.

Thus $\exists x (\neq 0) \in R$ such that $x \in \mu_t \cap \delta_t$. Hence $\mu_t \subseteq_e \sigma_t$.

Conversely, let $\mu_t \subseteq_e \sigma_t$, for some $t \in (0, 1]$. Let δ be a non-zero T-fuzzy left ideal of R with $\delta \subseteq \sigma$. Then δ_t is a non-zero left ideal of R with $\delta_t \subseteq \sigma_t$, for all $t \in (0, 1]$. Since $\mu_t \subseteq_e \sigma_t$, for some $t \in (0, 1]$. So, there exists $x (\neq 0) \in R$ such that $x \in \mu_t \cap \delta_t$. This implies $\mu(x) \geq t$ and $\delta(x) \geq t$. Now, $(\mu \wedge \delta)(x) = T[\mu(x), \delta(x)] \geq T[t, t] > T[0, 0] = 0$ [since T satisfies Archeamediam property]. Thus we have, $\mu \wedge \delta \neq \chi_0$. Hence $\mu \subseteq_{e(T)} \sigma$. $\square$

Lemma 3.2. (see [3]) Let T be a t -norm. Then $T(T(a, b), T(c, d)) = T(T(a, c), T(b, d))$, for all $a, b, c, d \in [0, 1]$.

Theorem 3.5. Let μ and σ be two non-zero T-fuzzy left ideals of R such that μ is a T-fuzzy essential left ideal in σ . Then for any T-fuzzy left ideal δ of R , $\mu \wedge \delta \subseteq_{e(T)} \sigma \wedge \delta$.

Proof. Since $\mu \subseteq \sigma$, so $\mu \wedge \delta \subseteq \sigma \wedge \delta$. Let η be any non-zero T-fuzzy left ideal of R such that $\eta \subseteq \sigma \wedge \delta$. Then η is a non-zero T-fuzzy left ideal of R such that $\eta \subseteq \sigma$ and $\eta \subseteq \delta$. This implies $\exists x (\neq 0) \in R$ such that $T[\mu(x), \eta(x)] \geq t$.

Now $\mu(x), \eta(x) \geq \min[\mu(x), \eta(x)] \geq T[\mu(x), \eta(x)] \geq t$.

Also $\delta(x) \geq \eta(x)$, for all $x \in R$ this implies $\delta(x) \geq \eta(x) \geq t$, for all $x \in R$.

Therefore

$$\begin{aligned} [(\mu \wedge \delta) \wedge \eta](x) &= T[(\mu \wedge \delta)(x), \eta(x)] \\ &= T[T(\mu(x), \delta(x)), \eta(x)] \\ &= T[\mu(x), T(\delta(x), \eta(x))] \\ &= T[\mu(x), T(\eta(x), \delta(x))] \\ &= T[T(\mu(x), \eta(x)), \delta(x)] \\ &\geq T[T(\mu(x), \eta(x)), t] \\ &\geq T(t, t) > T(0, 0) = 0, \end{aligned}$$

since T satisfies Archeamediam property.

Thus we get $(\mu \wedge \delta) \wedge \eta \neq \chi_0$. Hence $\mu \wedge \delta \subseteq_{e(T)} \sigma \wedge \delta$. $\square$

Theorem 3.6. Let μ, ν and σ be non-zero T-fuzzy left ideals of R such that $\mu \subseteq \nu \subseteq \sigma$. Then $\mu \subseteq_{e(T)} \sigma$ if and only if $\mu \subseteq_{e(T)} \nu \subseteq_{e(T)} \sigma$.

Proof. Let $\mu \subseteq_{e(T)} \sigma$. Let δ be a non-zero T-fuzzy left ideal of ν . Then δ is also a non-zero T-fuzzy left ideal of σ . Thus there exists $x (\neq 0) \in R$ such that $T[\mu(x), \delta(x)] \geq t$, for some $t \in (0, 1]$. Consequently $\mu \subseteq_{e(T)} \nu$. Again, let ξ be

any non-zero T-fuzzy left ideal of σ . Since $\mu \subseteq_{e(T)} \sigma$, so there exists $x(\neq 0) \in R$ such that $T[\mu(x), \delta(x)] \geq t$, for some $t \in (0, 1]$ and this implies $\nu \subseteq_{e(T)} \sigma$.

Conversely, let $\mu \subseteq_{e(T)} \nu \subseteq_{e(T)} \sigma$. Let η be a non-zero T-fuzzy left ideal of σ . Since $\nu \subseteq_{e(T)} \sigma$, so there exists $x(\neq 0) \in R$ such that $T[\nu(x), \eta(x)] \geq t$, for some $t \in (0, 1]$. Also $\eta \wedge \nu$ is a non-zero T-fuzzy left ideal of ν and $\mu \subseteq_{e(T)} \nu$. This implies there exists $y(\neq 0) \in R$ such that $T[\mu(y), (\nu \wedge \eta)(y)] \geq s$, for some $s \in (0, 1]$.

Now $\mu(y), (\nu \wedge \eta)(y) \geq \min[\mu(y), (\nu \wedge \eta)(y)] \geq T[\mu(y), (\nu \wedge \eta)(y)] \geq s$.

Also $\nu(y), \eta(y) \geq \min[\nu(y), \eta(y)] \geq T[\nu(y), \eta(y)] \geq s$.

Thus $(\mu \wedge \eta)(y) = T[\mu(y), \eta(y)] \geq T(s, s) > T(0, 0) = 0$. This implies there exists $y(\neq 0) \in R$ such that $T[\mu(y), \eta(y)] \geq s$, for some $s \in (0, 1]$.

Thus we have $\mu \wedge \eta \neq \chi_0$. Hence $\mu \subseteq_{e(T)} \sigma$. □

References

- [1] Abou-Zaid S. On fuzzy ideals and fuzzy quotient rings of a ring. *Fuzzy sets and systems*, 59(1993), 205-210.
- [2] Chatters A.W and Hazarnavis C.R. Rings with chain condition. *Pitman Advanced Publishing program*, (1980).
- [3] Dheena P. and Mohanraaj G., T-fuzzy Ideals in Rings, *International Journal of Computational Cognition*, Volume 9, No.2(june 2011), 98-101.
- [4] Dubois D. and Prade H., *Theory and Applications, Fuzzy sets and systems*, Academic Press, New York (1978).
- [5] Kumar R. Fuzzy ideals and fuzzy semi-prime ideals. Some ring theoretic analogues, *Information Sci.* 46(1992), 43-52.
- [6] Kumar R. Certain fuzzy ideals of rings redefined. *Fuzzy sets and systems*, 46(1992), 251-260.
- [7] Liu. W Fuzzy invariant subgroups and fuzzy ideals. *Fuzzy sets and systems*, 8(1982), 133-139.
- [8] Malik D.S. Fuzzy ideals of artinian rings. *Fuzzy sets and systems*, 46(1990), 111-115.
- [9] Medhi.U, Rajkhowa. K. K, Barthakur. L. K. and Saikia H.K. On fuzzy essential ideals of rings. *Advances in fuzzy sets and systems*, Vol.3(No.3)(2008), 287-299.

- [10] Mukharjee T.K. and Sen M.K. Prime fuzzy ideals in rings. Fuzzy sets and systems, 32(1989), 337-341.
- [11] Mukharjee T. K. and Sen M. K. Rings with chain conditions Fuzzy sets and systems, 39(1991), 117-123.
- [12] Rosenfeld A. Fuzzy groups. J. Math. anal. Appl. 35(1971), 512-517.
- [13] Zadeh L.A. Fuzzy sets. Information and Control, 8(1965), 338-353.