International Journal of Pure and Applied Mathematics
Volume 91 No. 1 2014, 13-22
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v91i1.2

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ON $M_X\alpha\delta\left\langle\widetilde{H}\right\rangle$ IN M-STRUCTURES

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Abstract: V. Kokilavani and P. Basker in [16] introduced the concepts of $M_X\alpha\delta$-closed sets in M-structures. In this paper we introduce $M_X\alpha\delta$-continuous, $M_X\alpha\delta$-irresolute and $M_X\alpha\delta\left\langle\widetilde{H}\right\rangle$. Further, we derive some properties of $M_X\alpha\delta\left\langle\widetilde{H}\right\rangle$ and Pasting Lemma for $M_X\alpha\delta$-irresolute functions.

AMS Subject Classification: 05C15
Key Words: $M_X\alpha\delta$-closed sets, $M_X\alpha\delta$-open sets, $M_X\alpha\delta$-continuous maps, $M_X\alpha\delta$-irresolute maps, $M_X\alpha\delta\left\langle\widetilde{H}\right\rangle$

1. Introduction

In 1950, H. Maki, J. Umehara and T. Noiri [3] introduced the notions of minimal structure and minimal space. Also they introduced the notion of m_X-open set and m_X-closed set and characterize those sets using m_X-cl and m_X-int operators respectively. Further they introduced m-continuous functions [11] and studied some of its basic properties. They achieved many important results compatible by the general topology case. Some other results about minimal

Received: November 19, 2012

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spaces can be found in [4-11]. For easy understanding of the material incorporated in this paper we recall some basic definitions. For details on the following notions we refer to [4], [3] and [7].

The notion of $M_X\alpha\delta$-closed set was introduced and studied by V.Kokilavani and P. Basker [13]. In this paper we introduce $M_X\alpha\delta$ -continuous, $M_X\alpha\delta$-irresolute and $M_X\alpha\delta\left\langle\widetilde{H}\right\rangle$. Further, We obtain some characterizations and some properties.

2. Preliminaries

In this section, we introduce the M-structure and define some important subsets associated to the M-structure and the relation between them.

Definition 2.1. (see [3]) Let X be a nonempty set and let $m_X \subseteq P(X)$, where $P(X)$ denote the power set of X. Where m_X is an M-structure (or a minimal structure) on X, if φ and X belong to m_X.

The members of the minimal structure m_X are called m_X-open sets, and the pair $(X,\ m_X)$ is called an m-space. The complement of m_X-open set is said to be m_X-closed.

Definition 2.2. (see [3]) Let X be a nonempty set and m_X an M-structure on X. For a subset A of X, m_X-closure of A and m_X-interior of A are defined as follows:

- $m_XCl(A) = \bigcap\{F : A \subseteq F,\ X - F \in m_X\}$
- $m_XInt(A) = \bigcup\{F : U \subseteq A,\ U \in m_X\}$

Lemma 2.3. (see [3]) *Let X be a nonempty set and m_X an M-structure on X. For subsets A and B of X, the following properties hold:*

1. $m_XCl(X - A) = X - m_XInt(A)$ *and* $m_XInt(X - A) = X - m_XCl(A)$.
2. *If* $X - A \in m_X,$ *then* $m_XCl(A) = A$ *and if* $A \in m_X$ *then* $m_XInt(A) = A$.
3. $m_XCl(\varphi) = \varphi$, $m_XCl(X) = X,$ $m_XInt(\varphi) = \varphi$ *and* $m_XInt(X) = X$
4. *If* $A \subseteq B$ *then* $m_XCl(A) \subseteq m_XCl(B)$ *and* $m_XInt(A) \subseteq m_XInt(B)$.
5. $A \subseteq m_XCl(A)$ *and* $m_XInt(A) \subseteq A$.
6. $m_XCl(m_XCl(A)) = m_XCl(A)$ *and* $m_XInt(m_XInt(A)) = m_XInt(A)$.

7. $m_XInt(A\cap B) = (m_XInt(A))\cap(m_XInt(B))$ and $(m_XInt(A))\cup(m_XInt(B)) \subseteq m_XInt(A\cup B)$.

8. $m_XCl(A\cup B) = (m_XCl(A))\cup(m_XCl(B))$ and $m_XCl(A\cap B) \subseteq (m_XCl(A)) \cap(m_XCl(B))$.

Lemma 2.4. (see [7]) *Let $(X,\ m_X)$ be an m-space and A a subset of X. Then $x \in m_XCl(A)$ if and only if $U \cap A \neq \varphi$ for every $U \in m_X$ containing x.*

Definition 2.5. (see [10]) A minimal structure m_X on a nonempty set X is said to have the property B if the union of any family of subsets belonging to m_X belongs to m_X.

Remark 2.6. A minimal structure m_X with the property B coincides with a generalized topology on the sense of Lugojan.

Lemma 2.7. (see [5]) *Let X be a nonempty set and m_X an M-structure on X satisfying the property B. For a subset A of X, the following property hold:*

1. $A \in m_X$ iff $m_XInt(A) = A$

2. $A \in m_X$ iff $m_XCl(A) = A$

3. $m_XInt(A) \in m_X$ and $m_XCl(A) \in m_X$.

3. $M_X\alpha\delta$-Closed Sets

Definition 3.1 (see [16]) A subset A of an m-space $(X,\ m_X)$ is called

- $m_X\alpha$-open set if $A \subseteq m_XInt(m_XCl(m_XInt(A)))$
- m_X α-closed set if $m_XCl(m_XInt(m_XCl(A))) \subseteq A$.
- m_X-regular open set if $A = m_XInt(m_XCl(A))$.

The $m_X\delta$-interior of a subset is the union of all m_X-regular open set of X contained in A and is denoted by $m_XInt_\delta(A)$. The subset A is called $m_X\delta$-open if $A = m_XInt_\delta(A)$, *i.e.*, a set is $m_X\delta$-open if it is the union of regular open sets. the complement of a $m_X\delta$-open is called $m_X\delta$-closed. Alternatively, a set A $\subseteq (X,\ m_X)$ is called $m_X\delta$-closed if $A = m_XCl_\delta(A)$. Here

$$m_XCl_\delta(A) = \left\{x/x \in U \in\Rightarrow m_XInt(m_XCl(A)\bigcap A \neq \varphi\right\}.$$

Definition 3.2 (see [16]) A subset A of an m-space $(X,\ m_X)$ is called an

- $m_X\alpha g$-closed set if $m_X\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $m_X\alpha$-open in $(X,\ m_X)$.
- $M_X\alpha\delta$-closed set if $m_X Cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is $m_X\alpha g$-open in $(X,\ m_X)$.

Example 3.3 (see [16]) Let $X = \{a,\ b,\ c\}$. Define M-structure on X as follows: $m_X = \{\varphi,\ X,\ \{a\}\}$. Then $m_X\alpha$-open$= \{\varphi,\ X,\ \{a\},\ \{a,\ b\},\ \{a,\ c\}\}$, $m_X\delta$-open $= \{\varphi,\ X\}$, m$_X\alpha g$-open$= \{\varphi,\ X,\ \{a\},\ \{a,\ b\},\ \{a,\ c\}\}$ and $M_X\alpha\delta = \{\varphi,\ X,\ \{a\}\}$.

Example 3.4 (see [16]) Let $X = \{a,\ b,\ c\}$. Define M-structure on X as follows:
$m_X = \{\varphi,\ X,\ \{a\},\ \{a,\ b\}\}$. *Then* m$_X\alpha$-open$= \{\varphi,\ X,\ \{a\},\ \{a,\ b\},\ \{a,\ c\}\}$, $m_X\alpha g$-open$= \{\varphi,\ X,\ \{a\},\ \{a,\ b\},\ \{a,\ c\}\}$ and $M_X\alpha\delta = \{\varphi,\ X,\ \{a\}\}$.

4. Properties of $M_X\alpha\delta\left\langle\widetilde{H}\right\rangle$

Definition 4.1. A function $f : (X,\ m_X) \longrightarrow (Y,\ m_Y)$ is called

- $M_X\alpha\delta$-continuous if $f^{-1}(V)$ is $M_X\alpha\delta$-closed in $(X,\ m_X)$ for every m_X-closed set V of $(Y,\ m_Y)$.,
- $M_X\alpha\delta$-irresolute if $f^{-1}(V)$ is $M_X\alpha\delta$-closed in $(X,\ m_X)$ for every $M_X\alpha\delta$-closed set V of $(Y,\ m_Y)$.

Definition 4.2. A function $f : (X,\ m_X) \longrightarrow (Y,\ m_Y)$ is called $M_X\alpha\delta$-homeomorphism (briefly. $M_X\alpha\delta\left\langle\widetilde{H}\right\rangle$) if both f and f^{-1} are $M_X\alpha\delta$-irresolute and f is bijective.

Lemma 4.3. *Let $B \subseteq H \subseteq (X,\ m_X)$ and let m_X/H be the relative m-spaces of H.*

1. *If B is $M_X\alpha\delta$-closed in $(X,\ m_X)$, then B is $M_X\alpha\delta$-closed relative to H.*
2. *If B is $M_X\alpha\delta$-closed in a subspace $(H,\ m_X/H)$ and if H is clopen in $(X,\ m_X)$.*

Note that a subset H of m-space $(X,\ m_X)$ is $M_X\alpha\delta$-clopen if and only if H is open and $M_X\alpha\delta$-closed. We prepare some notations. Let $f : X \longrightarrow Y$ be a function and H a subset of $(X,\ m_X)$. Let $f/H : H \longrightarrow Y$ be the restriction of f to H. We define a function $r_{(H,K(f))} : H \longrightarrow K$ by $r_{(H,K(f))}(x) = f(x)$ for any $x \in H$, where $K = f(H)$. Then, $f/H = j \circ r_{(H,K(f))}$ holds, where $j : K \longrightarrow Y$ is an inclusion.

Theorem 4.4. *Let H and K be subset of $(X,\ m_X)$ and $(Y,\ m_Y)$ respectively.*

1. *If $f : (X,\ m_X) \longrightarrow (Y,\ m_Y)$ is $M_X\alpha\delta$-irresolute and if H is a $M_X\alpha\delta$-clopen subset of $(X,\ m_X)$, then the restriction $f/H : (H,\ m_X/H) \longrightarrow (Y,\ m_Y)$ is $M_X\alpha\delta$-irresolute.*
2. *Suppose that K is a $M_X\alpha\delta$-clopen subset of $(Y,\ m_Y)$. A function $K : (X,\ m_X) \longrightarrow (K,\ m_Y/K)$ is $M_X\alpha\delta$ -irresolute if and only if $j\circ k$:$(X,\ m_X) \longrightarrow (Y,\ m_Y)$ is $M_X\alpha\delta$-irresolute, where $j : (K,\ m_Y/K) \longrightarrow (Y,\ m_Y)$ is an inclusion.*
3. *If $f : (X,\ m_X) \longrightarrow (Y,\ m_Y)$ is a $M_X\alpha\delta\left\langle \widetilde{H}\right\rangle$ such that $f(H) = K$ and K are $M_X\alpha\delta$-clopen subset, then $r_{(H,K(f))} :\ (H,\ m_X/H) \longrightarrow (K,\ m_Y/H)$ is also $M_X\alpha\delta\left\langle \widetilde{H}\right\rangle$.*

Proof. (a) Let F be a $M_X\alpha\delta$-closed set of $(Y,\ m_Y)$. Since f is $M_X\alpha\delta$-irresolute, $(f/H)^{-1} = f^{-1}\cap H$, H is $M_X\alpha\delta$-closed, $(f/H)^{-1}(F)$ is $M_X\alpha\delta$-closed in $(H,\ m_X/H)$ by Lemma 3.3(a). Therefore f/H is $M_X\alpha\delta$-irresolute.

(b) Necessity: Let F be a $M_X\alpha\delta$-closed set of $(Y,\ m_Y)$. Then $(j\circ k)^{-1}(F) = k^{-1}(j^{-1}(F)) = k^{-1}(F\cap H)$ is a $M_X\alpha\delta$-closed in $(K,\ m_Y/K)$ by Lemma 3.3(a). Therefore $j \circ k$:$(X,\ m_X) \longrightarrow (Y,\ m_Y)$ is $M_X\alpha\delta$-irresolute.

Sufficiency: Let V be a $M_X\alpha\delta$-closed set of $(K, m_Y/K)$. By Lemma 3.3(b), $(j \circ k)^{-1}(V) = k^{-1}(j^{-1}(V)) = k^{-1}(F \cap V) = k^{-1}(F)$ is $M_X\alpha\delta$-closed in $(X,\ m_X)$. Therefore k is $M_X\alpha\delta$-irresolute.

(c) First, it suffices to prove $r_{(H,K(f))} : (H,\ m_X/H) \longrightarrow (K,\ \sigma/K)$ is $M_X\alpha\delta$-irresolute. Let F be a $M_X\alpha\delta$-closed subset of

$$(Y,\ m_Y).(j\circ r_{(H,K(f))})^{-1}(F) = (f/H)^{-1}(F) = (f/H)^{-1}(F\cap K) = f^{-1}(F\cap K) = f^{-1}(F)\cap K$$

is $M_X\alpha\delta$-closed in $(H,\ m_X/H)$ and hence $j\circ r_{(H,K(f))}$ is $M_X\alpha\delta$-irresolute. By (b) $r_{(H,K(f))}$ is $M_X\alpha\delta$-irresolute.

Next we show that $(r_{(H,K(f))})^{-1} : (K,\ m_Y/K) \longrightarrow (H,\ m_X/H)$ is $M_X\alpha\delta$-irresolute. Since $(r_{(H,K(f))})^{-1} = r_{(H,K(f^{-1}))}$ and since f^{-1} is $M_X\alpha\delta$-irresolute, then using the first argument above for f^{-1} we have $(r_{(H,K(f))})^{-1}$ is $M_X\alpha\delta$-irresolute. Therefore $r_{(H,K(f))}$ is a $M_X\alpha\delta\left\langle\widetilde{H}\right\rangle$. By using Theorem 3.4, for a $M_X\alpha\delta$-clopen subset H of $(X,\ m_X)$, we have a homomorphism called restriction $(r_H)^\star : M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(X,\ H;\ m_X) \longrightarrow M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(H, m_X/H)$ as follows: $(r_H)^\star(f) = r_{(H,K(f))}$ for any $f \in M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(X,\ H;\ m_X)$. To prove that $(r_H)^\star$ is onto we prepare the following:

Lemma 4.5. (Pasting Lemma for $M_X\alpha\delta$-Irresolute Functions) *Let $(X,\ m_X)$ be a m-space such that $X = A \cup B$ where A and B are $M_X\alpha\delta$-clopen subsets. Let $f : (A,\ m_X/A) \longrightarrow (Y,\ m_Y)$ and $g : (B,\ m_X/B) \longrightarrow (Y,\ m_Y)$ be $M_X\alpha\delta$-irresolute functions such that $f(x) = g(x)$ for every $X \in A\bigcap B$. Then the combination $f\nabla g(x) = f(x)$ for any $x \in A$ and $f\nabla g(y) = g(y)$ for any $y \in B$.*

Proof. Let F be $M_X\alpha\delta$-closed set of $(Y,\ m_Y)$. Then $(f\nabla g)^{-1}(F) = f^{-1}(F)\bigcup g^{-1}(F)$,
$f^{-1}(F) \in M_X\alpha\delta C(X,\ m_X)$ by using Lemma. It follows from Theorem 3.17 [2] that $f^{-1}(F)\bigcup g^{-1}(F)$ is $M_X\alpha\delta$-closed in $(X,\ m_X)$. Therefore $(f\nabla g)^{-1}(F)$ is $M_X\alpha\delta$-closed in $(X,\ m_X)$ and hence $f\nabla g$ is $M_X\alpha\delta$-irresolute.

Theorem 4.6. *If H is a $M_X\alpha\delta$-clopen subset of $(X,\ m_X)$, then $(r_H)^\star : M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(X, H;\ m_X) \longrightarrow M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(H,\ m_X/H)$ is an onto homomorphism.*

Proof. Let $k \in M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(H,\ m_X/H)$. By Theorem 4.4, $J_1 \circ k : (H,\ m_X/H) \longrightarrow (X, m_X)$ is $M_X\alpha\delta$-irresolute, where $J_1 : (H,\ m_X/H) \longrightarrow (X,\ m_X)$ is an inclusion. Similarly it is shown that $J_2 \circ l_{X/H} : (X/H,\ m_X(X/H)) \longrightarrow (X,\ m_X)$ is $M_X\alpha\delta$-irresolute, where $J_2 : (X/H,\ m_X(X/H)) \longrightarrow (X,\ m_X)$ is an inclusion.

By using Lemma 4.5, the combination $(J_1 \circ k)\nabla(J_2 \circ l_{X/H}) : (X,\ m_X) \longrightarrow (X,\ m_X)$, say k_1 is $M_X\alpha\delta$-irresolute. It is easily shown that $k_1(x) = k(x)$ for any $x \in H$ and k_1 is bijective and $k_1^{-1} = (J_1 \circ k^{-1})\nabla(J_2 \circ l_{X/H}) : (X,\ m_X) \longrightarrow (X,\ m_X)$ is also $M_X\alpha\delta$-irresolute. Therefore $k_1 : (X,\ m_X) \longrightarrow (X,\ m_X)$ is the required $M_X\alpha\delta\left\langle\widetilde{H}\right\rangle$ and $(r_H)^\star(k_1) = k$ holds and hence $(r_H)^\star$ is onto.

We define an equivalence relation R on $M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(X, H;\ m_X)$ as follows: fRh if and only if $f(x) = h(x)$ for any $x \in H$. Let $[f]$ be the equivalence class of f. Let $H = \Big\{f/f \in M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(X,\ H;\ m_X)$ and $f(x) = x$ for any $x \in H$.

Then, $H = ker(r_H)^\star$ and this is normal subgroup of $M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(X,\ H;\ m_X)$.

The factor group of $M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(X,\ H;\ m_X)$ by H is

$$M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(X,\ H;\ m_X)/H = \left\{fH/f \in M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(X,\ H;\ m_X)\right\},$$

where $fH = \{\mu(f,k)/k \in H\} = [f]$. Since $(r_H)^\star$ is onto by Theorem 3.6, then the relation between the groups $M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(X,\ H;\ m_X)$ is investigated as follows:

Theorem 4.7. *If H is $M_X\alpha\delta$-clopen subset of $(X,\ m_X)$, then*

$$M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(H,\ m_X/H)$$

is isomorphic to the factor group $M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(H,\ H;\ m_X)/H$.

Proof. By Theorem,

$$(r_H)^\star : M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(H,\ H;\ m_X) \longrightarrow M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(H,\ m_X/H)$$

is an onto homomorphism. Thus, we have the required isomorphism,

$$M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(H,\ \tau/H) \cong M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(H,\ H;\ m_X)/H.$$

Theorem 4.8. *If $\alpha : (X,\ m_X) \longrightarrow (Y,\ m_Y)$ is a $M_X\alpha\delta\left\langle\widetilde{H}\right\rangle$ such that $\alpha(H) = K$, then there is an isomorphism, $\alpha^\star : M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(H,\ H;\ m_X) \longrightarrow M_X\alpha\delta\left\langle\widetilde{H}\right\rangle(H,\ H;\ m_X)$.*

Proof. The isomorphism $\alpha^\star$ is defined by $\alpha^\star(f) = \alpha \circ f \circ \alpha^{-1}$. Let $(X/R,\ m_X/R)$ be the quotient m-space of $(X,\ m_X)$ by an equivalence relation R on X and let $\pi : (X,\ m_X) \longrightarrow$(X/R, m$_X/R$) be the canonical projection.

Definition 4.9. A space (X, m_X) is called $M_X\alpha\delta$-connected if X cannot be expressed as the disjoint union of two non-empty $M_X\alpha\delta$-closed sets.

Definition 4.10. A function $f : (X,\ m_X) \longrightarrow (Y,\ m_Y)$ is called $M_X\alpha\delta$-closed if $f(F)$ is $M_X\alpha\delta$-closed in $(Y,\ m_Y)$ for every closed set F of $(X,\ m_X)$.

Definition 4.11. A space $(X,\ m_X)$ is called $\star T^{\star M_X\alpha\delta}$-space if every $M_X\alpha\delta$-closed set is m_X-closed.

Theorem 4.12. *Let F be a subset of $(X/R,\ m_X/R)$. If*

$$\pi : (X,\ m_X) \longrightarrow (X/R,\ m_X/R)$$

is $M_X\alpha\delta$-closed function and $\pi^{-1}(F)$ is $M_X\alpha\delta$-closed in $(X,\ m_X)$, then F is $M_X\alpha\delta$-closed, where $(X,\ m_X)$ is a $\star T^{\star M_X\alpha\delta}$-space.

Proof. Let $\pi^{-1}(F)$ is $M_X\alpha\delta$-closed in $(X,\ m_X)$. Then $\pi^{-1}(F)$ is closed in $(X,\ m_X), since$ (X, m_X) is $\star T^{\star M_X\alpha\delta}$-space. Then $\pi(\pi^{-1}(F)) = F$ is $M_X\alpha\delta$-closed in $(X/R,\ m_X/R)$, since π is a $M_X\alpha\delta$-closed map.

Theorem 4.13. *If $\pi : (X,\ m_X) \longrightarrow (X/R,\ m_X/R)$ is $M_X\alpha\delta$-continuous and the subset F is $M_X\alpha\delta$-closed $(X/R,\ m_X/R)$, then $\pi^{-1}(F)$ is $M_X\alpha\delta$-closed in $(X,\ m_X)$.*

Proof. Let F be a closed set in $(X/R,\ m_X/R)$. Then F is $M_X\alpha\delta$-closed in $(X/R,\ m_X/R)$. Since π is $M_X\alpha\delta$-continuous, Then $\pi^{-1}(F)$ is $M_X\alpha\delta$-closed in $(X,\ m_X)$.

Theorem 4.14. *If the bijective map $\pi : (X,\ m_X) \longrightarrow (X/R,\ m_X/R)$ is $M_X\alpha\delta$-continuous and $(X,\ m_X)$ is $M_X\alpha\delta$-connected, then $(X/R,\ m_X/R)$ is $M_X\alpha\delta$-connected.*

Proof. Suppose that $(X/R,\ m_X/R)$ is not $M_X\alpha\delta$-connected. Therefore $X/R = A \cup B$, where A and B are $M_X\alpha\delta$-closed set. Then $\pi^{-1}(A)$ and $\pi^{-1}(B)$ are $M_X\alpha\delta$-closed in $(X,\ m_X)$ such that $X = \pi^{-1}(A) \cup \pi^{-1}(B)$. Therefore $(X,\ m_X)$ is not $M_X\alpha\delta$-connected. It is a contradiction. Therefore $(X/R,\ m_X/R)$ is $M_X\alpha\delta$-connected.

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