International Journal of Pure and Applied Mathematics
Volume 92 No. 1 2014, 13-26
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v92i1.2

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ABOUT THE PROBLEM OF GROUP PERSECUTION IN LINEAR DIFFERENTIAL GAMES WITH A SIMPLE MATRIX AND STATE CONSTRAINTS

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Abstract: Consider two dependent problems of evasion one evader from many pursuers. In the first problem assumed, that the all players have a simple motion, among the pursuers are both participants the maximum speed of which coincide with the maximum speed of evader and parties who maximum speeds strictly less than the maximum speed of evader while evader is not leaves the convex compact set with non-empty interior. In the second problem consider linear dipendent problem of persecution one evader by group pursuers, provided that the matrix of the system is the product of a function and identity matrix, among the pursuers are both participants, which the set of admissible controls, is a sphere with center at the origin, coincides with the set of admissible controls the evader, and the pursuers with fewer opportunities and evader does not leave the confines of a convex cone with vertex at the origin. We prove that if the number of pursuers, opportunities coincide with opportunities the evader, less than the dimension space, then the pursuers with fewer features do not affect the solvability evasion problem.

AMS Subject Classification: 49N70, 49N75, 49N90
Key Words: differential games, evasion from many pursuers, state constraints, incomplete information

Received: September 5, 2013

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1. Introduction

Differential games of two players, first considered in the book of Isaacs [1], now present wide field of research [2]-[11]. Methods were developed for solving various classes of game problems: Isaaks' method, based on the analysis of a certain partial differential equation and its characteristics, Krasovskii's method of extremal guidans, Pontryagin's method and others. A natural generalization of differential games pursuit-evasion of two persons are games with a group of pursuers and one or several evaders [12]-[15]. These games are interesting from the theoretical point of view, they cannot be treated by theory for two-person games. One reason for this is, that the union sets of the reachability of all pursuers and the union of all target sets are sets, is non-convex and, furthermore, is not connected. On the other hand, there are some applications of these games to the problems of motion vehicles, collisions of avoidance for ships and others. Among a large number of papers, devoted to the pursuit-evasion games with a group of pursuers, mention the works [16]-[33]. Works [24], [26]-[31] is devoted to model problems, where all players have a simple motion. In work [24] proved the possibility of evasion evader from any number of pursuers, provided, that maximum at a rate speed of the evader greater than the maximum at a rate of speed of any pursuer. The generalization of this work to a wide class of problems is work [25]. The task of pursuit-evasion with equal opportunities to all the participants considered in [26], [27], [29]. In [28] proved the possibility of evasion the evader from a group of pursuers, if all participants have equal opportunities, number of pursuers less than the dimension of space, evader does not leave the confines of a convex compact set with non-empty interior. In [32], [33] proved the possibility of evasion evader from group of pursuers in linear differential game with a simple matrix if the number of pursuers less then dimension of space and evader does not leave the confines of a convex cone. Problem of evasion group of evaders from a group of pursuers in various productions considered in [14], [16], [30].

This work is devoted to two non-stationary problems of evasion of one the evader from a group of pursuers. In the first problem assumed, that the all players have a simple motion, among the pursuers are both participants the maximum speed of which coincide with the maximum speed of evader and parties who maximum speeds strictly less than the maximum speed of evader while evader is not leaves the convex compact set with non-empty interior. In the second problem consider linear dependent problem of persecution one evader by group pursuers , provided that the matrix of the system is the product of a function and identity matrix, among the pursuers are both participants, which the set of admissible controls, is a sphere with center at the origin, coincides with the set of admissible controls the evader, and the pursuers with fewer opportunities and evader does not leave the confines of a convex cone with vertex at the origin.

We prove that if the number of pursuers, opportunities coincide with opportunities the evader, less than the dimension space, then the pursuers with fewer features do not affect the solvability evasion problem.

2. Non-Stationary Problem with a Simple Motion

2.1. Statement of the Problem

In space $R^k (k \geq 2)$ we consider differential game $n+1$ objects: n pursuers $P_1, \dots, P_n$ and evader E.

The law of motion of each of the pursuers P_i has the form

$$\dot{x}_i = b(t)u_i, \;\; \|u_i\| \leq \alpha_i,$$

where $\alpha_j = 1$ for all $j = 1, \dots, m < n \;\; \alpha_j < 1$ for all $j = m+1, \dots, n$.

The law of motion of evader E has the form

$$\dot{y} = b(t)v, \;\; \|v\| \leq 1.$$

At $t = t_0$ set the initial position of pursuers $x_1^0, \dots, x_n^0$ and the initial position of evader y^0, and $x_i^0 \neq y^0, i = 1, \dots, n$.

Here $i = 1, \dots, n, x_i, y, u_i, v \in R^k$, $b : [t_0, \infty) \to R^1$ – measurable function.

It is further assumed, that evader E in the course of the game does not leave a convex set D ($D \subset R^k$) with non-empty interior.

Let σ – a partition $t_0 < t_1 < \dots < t_s < \dots$, of interval $[t_0, \infty)$, has no finite accumulation points.

Definition 1. The piecewise-program strategy V of player E, a given on $[t_0, \infty)$, appropriate partitioning σ, mean a family mappings $\{c^l\}_{l=0}^{\infty}$, that put to conformity values

$$(t_l, x_1(t_l), \dots, x_n(t_l), y(t_l))$$

measurable function $v = v_l(t)$, defined for $t \in [t_l, t_{l+1})$ and such that $\|v_l(t)\| \leq 1, \; y(t) \in D, \; t \in [t_l, t_{l+1})$.

We denote this game by $\Gamma(n)$.

Definition 2. We say, in game $\Gamma(n)$ evasion occurs from meeting, if there partitioning σ of interval $[t_0, \infty)$ which has no finite condensation points, strategy V of evader E, which corresponds to partitioning σ such that for all trajectories $x_1(t), \dots, x_n(t)$ of pursuers $P_1, \dots, P_n$ takes place

$$x_i(t) \neq y(t), \ t \geq t_0, \quad i = 1, \dots, n,$$

where $y(t)$ – implemented in this situation, the trajectory of the evader E.

We denote by Int$D$ interior of the set D.

2.2. The Theorem about Evasion

Theorem 1. *Let $y^0 \in \mathrm{Int}D$, b — is a function, which is bounded on any compact and $m < k$. Then in game $\Gamma(n)$ occurs evasion of meeting from any initial positions.*

Proof. Inasmuch as $y^0 \in \mathrm{Int}D$, there is $D_r(q)$ – a sphere of radius r with center q such that $y^0 \in \mathrm{Int}D_r(q) \subset D$. Assume further that ε – distance from y^0 to border $D_r(q)$, $I_l = [t_0 + l - 1, t_0 + l], b_l > 0$ such that $|b(t)| \leq b_l$ for all $t \in I_l$ $(l = 1, 2, \dots,)$

$$\Omega_j(\tau) = \left\{ t > \tau \ : \ \int\limits_{\tau}^{t} |b(s)| ds = \frac{\varepsilon}{j+1} \right\}, \ j = 1, 2, \dots, .$$

Note, that if $t \in \Omega(\tau)$ $\tau, t \in I_l$ for some l, that

$$\frac{\varepsilon}{j+1} = \int\limits_{\tau}^{t} |b(s)| ds \leq b_l(t - \tau).$$

Therefore

$$t - \tau \geq \frac{\varepsilon}{b_l(j+1)}. \tag{1}$$

For each segment I_l define a partition σ_l this segment and natural number m_l follows. Consider the segment I_1. Let $\tau_0^1 = t_0, \quad j = 1, 2 \dots,$

$$\tau_j^1 = \begin{cases} \inf\{t > \tau_{j-1}^1, \ t \in \Omega_j(\tau_{j-1}^1)\}, \ \text{if } \tau_j^1 < t_0 + 1 \ \ \Omega_j(\tau_{j-1}^1) \neq \emptyset, \\ t_0 + 1 \text{ otherwise} \end{cases}$$

Then we assume $m_1 = \min\{j : \tau_j^1 = t_0 + 1\}, \sigma_1 = \{\tau_0^1, \dots, \tau_{m_1}^1\}$. Now consider the segment I_2. Let $\tau_0^2 = t_0 + 1$. For all $j = 1, 2 \dots,$

$$\tau_j^2 = \inf\{t > \tau_{j-1}^2,\ t \in \Omega_{j+m_1}(\tau_{j-1}^2)\},$$

if

$$\tau_j^2 < t_0 + 2 \text{ and } \Omega_{j+m_1}(\tau_{j-1}^2) \neq \emptyset$$

$\tau_j^2 = t_0 + 2$, if the relevant conditions are not met.

Then we assume

$$m_2 = m_1 + \min\{j : \tau_j^2 = t_0 + 2\},\ \sigma_2 = \{\tau_0^2, \dots, \tau_{m_2-m_1}^2\}.$$

Assume, that the partitioning is already defined σ_{l-1} of segment I_{l-1} and number m_{l-1}. Consider the segment I_l. Let $\tau_0^l = t_0 + l - 1$. For all $j = 1, 2 \dots,$

$$\tau_j^l = \inf\{t > \tau_{j-1}^2,\ t \in \Omega_{j+m_{l-1}}(\tau_{j-1}^l)\},$$

if

$$\tau_j^l < t_0 + l \quad \Omega_{j+m_{l-1}}(\tau_{j-1}^l) \neq \emptyset$$

and $\tau_j^l = t_0 + l$, if the relevant conditions are not met.

Then we assume

$$m_l = m_{l-1} + \min\{j : \tau_j^l = t_0 + l\},\ \sigma_l = \{\tau_0^l, \dots, \tau_{m_l-m_{l-1}}^2\}.$$

Note, that due to (1) numbers m_l exist for all l. As a partition σ of interval $[t_0, \infty)$ take such a partition, that constriction σ to any segment I_l coincides with σ_l. Let $\sigma = \{t_0 < t_1 < \dots < t_r < \dots\}$.

We introduce the following notation

$$M_j(t) = \int\limits_{t_j}^{t} |b(s)|ds,$$

$$\delta_j = \min_i \Big(\sqrt{\|x_i(t_j) - y(t_j)\|^2 + M_j(t_{j+1})} - M_j(t_{j+1})\Big),$$

$$\Delta_j = r - \frac{\varepsilon}{j+1} - \sqrt{\Big(r - \frac{\varepsilon}{j}\Big)^2 + \Big(\frac{\varepsilon}{j+1}\Big)^2},$$

$$\gamma_j = \frac{1}{2}\min\{\delta_j, \Delta_j\},\ a_i(t_j) = x_i(t_j) - y(t_j).$$

Note, that $\Delta_j > 0, \delta_j > 0$ if $x_i(t_j) \neq y(t_j)$.

We set the strategy V of evader E to $[t_j, t_{j+1})$ follows. Let v_j — is vector, which satisfies the system

$$(v_j, x_l(t_j) - y(t_j)) = 0, l = 1, \ldots, m, (v_j, y(t_j) - q) \leq 0, \|v_j\| = 1.$$

inasmuch as $m < k$, that the system has a solution.

Assume further $v_j(t, t_j, v_j, \gamma_j)$ — is the control of the evader E, which guarantees to it evading his pursuers $P_{m+1}, \ldots, P_n$ $[t_j, t_{j+1})$ and such that

$$\|y(t) - \overline{y}(t)\| < \gamma_j \text{ for all } t \in [t_j, t_{j+1}),$$

where $\overline{y}(t) = y(t_j) + v_j \int\limits_{t_j}^{t} |b(s)| ds$, $y(t)$ — is trajectory of the evader E, control which meets $v_j(t, t_j, v_j, \gamma_j)$. by virtue of [25], [26] such control E exists. We assume control of the evader E in game $\Gamma(n)$ to $[t_j, t_{j+1})$ equal to $v_j^0(t) = v_j(t, t_j, v_j, \gamma_j)$.

We show that V is a strategy of evasion. Consider the segment $[t_j, t_{j+1}]$. Let $u_i(t)$ — arbitrary control pursuer P_i,

$$\hat{u}_i(t) = \begin{cases} \frac{1}{M_j(t)} \int\limits_{t_j}^{t} b(s) u_i(s) ds, \text{ if } M_j(t) \neq 0 \\ 0, \text{ if } M_j(t) = 0 \end{cases}$$

Then $\|\hat{u}_i(t)\| \leq 1$ for all $t \in [t_j, t_{j+1})$.

By the triangle inequality, we have

$$\|x_i(t) - y(t)\| \geq \|x_i(t) - \overline{y}(t)\| - \|y(t) - \overline{y}(t)\|.$$

Estimate the first term.

$$\|x_i(t) - \overline{y}(t)\| = \|x_i(t_j) - y(t_j) - M_j(t) v_j + M_j(t) \hat{u}_i(t)\| \geq$$
$$\geq \|a_i(t_j) - M_j(t) v_j\| - M_j(t) =$$
$$= \sqrt{\|a_i(t_j)\|^2 - 2M_j(t)(a_i(t_j), v_j) + M_j^2(t)} - M_j(t) =$$
$$= \sqrt{\|a_i(t_j)\|^2 + M_j^2(t)} - M_j(t) \geq \delta_j.$$

Since

$$\|y(t) - \overline{y}(t)\| \leq \gamma_j \leq \frac{1}{2}\delta_j,$$

then

$$\|x_i(t) - y(t)\| \geq \delta_j - \frac{1}{2}\delta_j = \frac{1}{2}\delta_j.$$

From the last inequality implies that if capture does not occur until the time t_j, then it does not happen on $[t_j, t_{j+1})$. Since $x_i^0 \neq y^0$ for all i, then $y(t) \neq x_i(t)$ for all $i, t \geq t_0$.

We show that the evader E does not leave the confines of the set D. Let us prove that if the inequality $\|y(t_j) - q\| \leq r - \frac{\varepsilon}{j}$, then

$$\|y(t) - q\| \leq r - \frac{\varepsilon}{j+1} \text{ for all } t \in [t_j, t_{j+1}).$$

Indeed,

$$\|y(t) - q\| \leq \|y(t) - \overline{y}(t)\| + \|\overline{y}(t) - q\|.$$

Since $\|y(t) - \overline{y}(t)\| < \gamma_j$ and

$$\|\overline{y}(t) - q\| = \|y(t_j) + M_j(t)v_j - q\| =$$
$$= \sqrt{\|y(t_j) - q\|^2 + 2(v_j, y(t_j) - q)M_j(t) + M_j^2(t)} \leq$$
$$\leq \sqrt{\left(r - \frac{\varepsilon}{j}\right)^2 + M_j^2(t)} \leq \sqrt{\left(r - \frac{\varepsilon}{j}\right)^2 + \left(\frac{\varepsilon}{j+1}\right)^2},$$

$$\|y(t) - q\| \leq \gamma_j + \sqrt{\left(r - \frac{\varepsilon}{j}\right)^2 + \left(\frac{\varepsilon}{j+1}\right)^2} \leq$$
$$\leq \Delta_j + \sqrt{\left(r - \frac{\varepsilon}{j}\right)^2 + \left(\frac{\varepsilon}{j+1}\right)^2} = r - \frac{\varepsilon}{j+1}.$$

Thus proved that $y(t) \in D_r(q) \subset D$ for all $t \geq t_0$. The theorem is proved. □

Remark. Note that the case $k = 2, m = 1$, D — is circle, $b(t) = 1$ for all t considered in [29], where proposed a different approach to the selection of parameters.

3. Evasion in a Cone in a Linear Problem with a Simple Matrix

3.1. Statement of the Problem

In space $R^k (k \geq 2)$ we consider differential game $n + 1$ objects: n pursuers $P_1, \ldots, P_n$ and evader E.

The law of motion of each of the pursuers P_i has the form

$$\dot{x}_i = a(t)x_i + u_i, \; \|u_i\| \leq \alpha_i, \tag{2}$$

where $\alpha_j = 1$ for all $j = 1, \dots, m < n$ and $\alpha_j < 1$ for all $j = m+1, \dots, n$.

The law of motion of evader E has the form

$$\dot{y} = a(t)y + v, \ \|v\| \leq 1. \tag{3}$$

At $t = t_0$ set the initial position of pursuers $x_1^0, \dots, x_n^0$ and the initial position of evader y^0, and $x_i^0 \neq y^0, i = 1, \dots, n$.

Here $a : [t_0, \infty) \to R^1$ – measurable function.

It is further assumed, that evader E in the game does not leave limits of convex cone

$$D = \{y : \ y \in R^k, \ \ (p_j, y) \leq 0, j = 1, \dots r\},$$

where $p_1, \dots p_r$ – the unit vectors R^k such that $\mathrm{Int}D \neq \emptyset$.

The evader uses a piecewise-program strategies.

3.2. The Theorem about Evasion

Theorem 2. *Let $y^0 \in \mathrm{Int}D$, a — is a function, which is bounded on any compact and $m < k$. Then in game $\Gamma(n)$ occurs evasion of meeting from any initial positions.*

Proof. Consider the segment $I_l = [t_0 + l - 1, t_0 + l], t_l = t_0 + l - 1, l = 1, 2, \dots,$ In systems (2), (3) we make the change variables

$$x_i = e^{\int\limits_{t_l}^{t} a(s)ds} w_i^l, \ \ y = e^{\int\limits_{t_l}^{t} a(s)ds} z^l.$$

Get systems

$$\dot{w}_i^l = e^{-\int\limits_{t_l}^{t} a(s)ds} u_i, \ \ \dot{z}^l = e^{-\int\limits_{t_l}^{t} a(s)ds} v. \tag{4}$$

Note that $x_i(\tau) = y(\tau)$ under some $i, \tau \in I_l$ if and only if, when $w_i^l(\tau) = z^l(\tau)$. Furthermore, $y(t) \in D$ if and only if, when $z^l(t) \in D$. Assume further

$$b_l(t) = e^{-\int\limits_{t_l}^{t} a(s)ds}, K_l = e^{-\int\limits_{t_l}^{t_{l+1}} a(s)ds},$$

$D_r(q)$ – a sphere of radius r with center q such that $y^0 \in D_r(q) \subset D$, ε – distance from y^0 to border $D_r(q)$, $q_1 = K_1 q, r_1 = K_1 r, \varepsilon_1 = K_1 \varepsilon$, $q_l = K_l q_{l-1}$, $r_l = K_l r_{l-1}$, $l \geq 2$. Note that $b_l(t) > 0$ for all $t \in I_l$.

For segment I_1 by ε_1, and function b_1 define number m_1 and partition σ_1 by the scheme previous section. For segment I_2 by $\varepsilon_2 = \frac{K_2\varepsilon_1}{m_1+2}$ and function b_2 define number m_2 and partition σ_2 and so on. For segment I_l by $\varepsilon_l = \frac{K_l\varepsilon_{l-1}}{m_{l-1}+2}$ and function b_l define number m_l and partition σ_l. As a partition σ of interval $[t_0, \infty)$ take such a partition σ, whose restriction to any segment I_l coincides with σ_l.

Consider the segment I_l and the corresponding partition $\sigma_l = \{t_l = \tau_0^l < \tau_1^l < \cdots < \tau_s^l = t_{l+1}\}$. Let $\tau_j^l, \tau_{j+1}^l \in I_l$, v_j^l — is a vector satisfying the system

$$(v_j^l, z^l(\tau_j^l) - w_i^l(\tau_j^l)) = 0, \ i = 1, \dots, m, \ (v_j^l, q_l - z^l(\tau_j^l)) \geq 0, \ \|v_j^l\| = 1.$$

Since $m < k$, v_j^l is always exists. Assume further $v_j^l(t, \tau_j^l, v_j^l, \gamma_j)$ is the control of the evader E to segment $[\tau_j^l, \tau_{j+1}^l)$, which guarantee him evasion in this segment from the pursuers $P_{m+1}, \dots, P_n$ in the game, described (4) and is such that

$$\|z_j^l(t) - \overline{z}_j^l(t)\| < \gamma_j \text{ for all } t \in [\tau_j^l, \tau_{j+1}^l],$$

where $\overline{z}_j^l(t) = z_j^l(t_j^l) + v_j^l \cdot \int\limits_{t_j^l}^{t} b(s)ds$, $z_j^l(t)$ — the trajectory of the evader E, which is meets a control $v_j^l(t, \tau_j, v_j^l, \gamma_j)$. By virtue of [24], [25] such control exists.

We put control of the evader E in the game $\Gamma(n)$ to $[\tau_j^l, \tau_{j+1}^l)$ equals $v_j^0(t) = v_i^l(t, \tau_j^l, v_j^l, \gamma_j)$.

Let us show that for all l and all $t \in I_l$ we have the inequalities

$$\|z^l(t) - q_l\| \leq r_l - \frac{\varepsilon_l}{j+1}, \ t \in [\tau_j, \tau_{j+1}). \tag{5}$$

Consider the segment $I_1 = [t_0, t_1]$ and partition σ_1 of this segment. Then

$$\|z^1(t_0) - q_1\| = \|K_1 y_0 - q_1\| = \|K_1 y_0 - K_1 q\| =$$
$$= K_1(r - \varepsilon) = r_1 - \varepsilon_1.$$

Let $t \in [\tau_0^1, \tau_1^1]$. Define

$$M_j^l(t) = \int\limits_{\tau_j^l}^{t} b_l(s)ds, \ a_i^l(\tau_j^l) = z^l(\tau_j^l) - w_i^l(\tau_j^l).$$

Then

$$\|\overline{z}^1(t) - q_1\| = \|\overline{z}^1(t_0) + M_1^1(t)v_1^1 - q_1\| =$$

$$= \sqrt{\|\overline{z}(t_0) - q_1\|^2 + 2M_1^1(t)(\overline{z}(t_0) - q_1, v_1^1) + \left(M_1^1(t)\right)^2} \leq$$
$$\leq \sqrt{(r_1 - \varepsilon_1)^2 + \left(\frac{\varepsilon_1}{2}\right)^2}.$$

Therefore

$$\|z^1(t) - q_1\| \leq \|z^1(t) - \overline{z}^1(t)\| + \|\overline{z}^1(t) - q_1\| \leq$$
$$\leq \gamma_j + \sqrt{(r_1 - \varepsilon_1)^2 + \left(\frac{\varepsilon_1}{2}\right)^2} \leq$$
$$\leq \Delta_j + \sqrt{(r_1 - \varepsilon_1)^2 + \left(\frac{\varepsilon_1}{2}\right)^2} = r_1 - \frac{\varepsilon_1}{2}.$$

Further proof of (5) for I_1 is similar proof of the corresponding inequality in the previous section.

Assume that the inequality (5) proved for all $I_l, l \leq s$. We prove this inequality for the interval I_{s+1}. By assumption we have the inequality

$$\|z^s(t_s) - q_s\| \leq r_s - \frac{\varepsilon_s}{m_s + 2}.$$

Then

$$\|z^{s+1}(t_s) - q_{s+1}\| = \|K_{s+1}(z(t_s) - q_s)\| = K_{s+1}\|z^s(t_s) - q_s\| \leq$$
$$\leq K_{s+1}\big(r_s - \frac{\varepsilon_s}{m_s + 2}\big) = r_{s+1} - \varepsilon_{s+1}.$$

Therefore the proof of (5) for I_{s+1} similar to the proof of inequality (5) for I_1. Therefore, $z^l(t) \in D_{r_l}(q_l)$ for $t \in I_l$ and for all l.

Let us prove that $D_{r_l}(q_l) \subset D$. Note that

$$q_l = k_l q_{l-1} = K_l K_{l-1} q_{l_2} = \cdots = K_l \cdot K_{l-1} \cdot \ldots K_1 q = e^{-\int_{t_0}^{t_{l+1}} a(s)ds} q.$$

Similarly, $r_l = e^{-\int_{t_0}^{t_{l+1}} a(s)ds} r$. Let $c \in D_{r_l}(q_l)$. Represent c in the form $c = \hat{c}e^{-\int_{t_0}^{t_{l+1}} a(s)ds}$. Then

$$\|c - q_l\| = e^{-\int_{t_0}^{t_{l+1}} a(s)ds} \|\hat{c} - q\| \leq e^{-\int_{t_0}^{t_{l+1}} a(s)ds} \cdot r.$$

Therefore, $\hat{c} \in D_r(q) \subset D$. In that D is a cone, that $c \in D$. Then $D_{r_l}(q_l) \subset D$ for all l.

We show that strategy V guarantees evasion.

Let us show that $z^l(t) \neq w_i^l(t)$ for all $i, t \in I_l$. Let $\tau_j^l, \tau_{j+1}^l \in \sigma_l$. From the system (4) we have

$$z^l(t) = z^l(\tau_j^l) + \int_{\tau_j^l}^{t} b_l(s)ds \cdot v_j^l,$$

$$w_i^l(t) = w_i^l(\tau_j^l) + \int_{\tau_j^l}^{t} b_l(s)u_i(s)ds.$$

Therefore for all $t \in [\tau_j^l, \tau_{j+1}^l)$

$$\begin{aligned}\|z^l(t) - w_i^l(t)\| &\geq \|z^l(\tau_j^l) + M_j^l(t)v_j^l - w_i^l(\tau_j^l)\| - M_j^l(t) = \\ &= \sqrt{(a_i^l(\tau_j^l))^2 + 2M_j^l(t)(v_j^l, a_i^l(\tau_j^l)) + (M_j^l(t))^2} - M_j^l(t) = \\ &= \sqrt{(a_i^l(\tau_j^l))^2 - (M_j^l(t))^2} - M_j^l(t) > 0 \text{ if } a_i^l(\tau_j^l) \neq 0.\end{aligned}$$

Therefore, if the capture did not occur until τ_j^l, it does not happen on a segment $[\tau_j^l, \tau_{j+1}^l]$. In that $z^0 \neq w_i^0$ for all i, it is proved that $z^l(t) \neq w_i^l(t)$ for all $i, l, t \in I_l$.

Therefore, the strategy V is a strategy of evasion. The theorem is proved. □

References

[1] R. Isaacs, *Differential Games*, John Wiley and Sons, New York (1965).

[2] A. Blaquiere, F. Gerard, G. Leitmann, *Quantitative and Qualitative Differential Games*, Academic Press, New York (1969).

[3] N. N. Krasovskii, *Game Problems of Contact of Motions* (in Russian), Nauka, Moscow (1970).

[4] A. Friedman, *Differential Games*, John Wiley and Sons, New York, (1971).

[5] N. N. Krasovskii, A. I. Subbotin, *Positional Differential Games* (in Russian), Nauka, Moscow (1974).

[6] O. Hajek, *Pursuit Games*, Academic Press, New York (1975).

[7] G. Leitmann, *Cooperative and Noncooperative Many-Player Differential Games*, Springer-Verlag, Vienna, Austria (1974).

[8] L. A. Petrosyan, *Differential Games of Pursuit* (in Russian), Leningrad University Press, Leningrad (1977).

[9] F. L. Chernousko, A. A. Melikyan, *Game Problems of Control and Search* (in Russian), Nauka, Moscow (1978).

[10] A. I. Subbotin, A. G. Chentsov, *Optimization of a Guarantee in Problems of Control* (in Russian), Nauka, Moscow (1981).

[11] L. S. Pontryagin, Linear Differential Games of Evasion (in Russian), *USSR Academy of Sciences, Proceedings of the Mathematical Institut*, **112** (1971), 30-63.

[12] A. A. Chikrii, *Conflict controlled prosesses*,Kluwer Acad. Publ.,Boston, Lomdon, Dordrecht (1997).

[13] N. L. Grigorenko, *Mathematical methods of control a few dynamic processes* (in Russian), Moscow State University, Moscow (1990).

[14] A. I. Blagodatskikh, N.N. Petrov, *Conflict interaction of groups of controlled objects* (in Russian), Udmurt State University, Izhevsk (2009).

[15] N. Yu. Satimov, B.B. Rikhsiev, *Methods of solving the problem of avoiding encounter in mathematical control theory* (in Russian), Fan, Tashkent (2000).

[16] P. Borowko, W. Rzymovski,*Avoidance of many pursuers in the simple motion case.* J. Math. Analys Appl. **111**, (1985), 535-546.

[17] W. Rzymovski, On the game of $n + 1$ cars, *J. Math. Analys Appl.*, **99**, (1984), 109-122.

[18] P. Hagedorn, J. V. Breakwell, A Differential Game with Two Pursuers and One Evader, *Journal of Optimization Theory and Applications*, **18**, (1976), 15-29.

[19] M. A. Foley, W. E. Schmitendorf, A Class of Differential Games with Two Pursuers versus One Evader, *IEEE Transactions on Automatic Control*, **AC-19**, No.3(1976), 294-298.

[20] W. Chodun, Differential games of evasion with many pursuers, *J. Math. Analys and Appl.,* **142**, (1989), 370-389.

[21] D. M. Stipanovic, A.Melikyan, N. Hovakimyan, Guaranted Strategies for Nonlinear Melti-Plaer Pursuit-Evasion Games, *Jnternational Game Theory Review,* **12**, No.1(2010), 1-17.

[22] W. Chodun, Avoidance of Many Pursuers in Differential Games Described by Differential Inclusions, *J. Math. Analys. Appl.*, **135**, (1998), 581-590.

[23] E. F. Mishchenko, M. S. Nikol'skii, N. Satimov, A Problem of Escaping Contact in Many-Person Differential Games (in Russian), *USSR Academy of Sciences, Proceedings of the Mathematical Institute,* **143**, (1977), 105-128.

[24] F. L. Chernous'ko, The one problem of evasion from many pursuers (in Russian), *Prikl. Mat. Mekh.*, **40**, No.1(1976), 14-24.

[25] F. L. Chernous'ko, V.L. Zak, On Differential Games of Evasion from Many Pursuers, *J. Optimiz. Theory Appl.*, **46**, No.4(1985), 461-470.

[26] B. N. Pshenichnyi, Simple pursuit by a few objects (in Russian), *Kibernetika*, No.3(1976), 145-146.

[27] N. L. Grigorenko, The game of simple pursuit-evasion of pursuit group and one evader (in Russian), *Moscow University Computational Mathematics and Cybernetics*, No.1(1983), 41-47.

[28] R. P. Ivanov, Simple pursuit in a compact set (in Russian), *Doklady Akademii Nauk SSSR*, **254**, No.6(1978), 1318-1321.

[29] N. N. Petrov, On the nonstationary problem of group pursuit with phase constraints (in Russian), *Mathematical game theory and applications*, **2**, No.4(2010), 74-83.

[30] N. N. Petrov, N. Nikandr. Petrov, About differential game "cossacks-robbers" (in Russian), *Differ. Uravn.,* **19**, No.8(1983), 1366-1374.

[31] W. Rzymovski, The two results of solving one problem of evasion from many pursuers, *Bull. Acad. Sci. Ser. math.*, **28**, No.3-4(1980), 155-159.

[32] N. N. Petrov, The one linear problem of evasion from many pursuers (in Russian), *Izvestiya RAN. Teoriya i sistemy upravleniya*, No.1(1988), 41-43.

[33] I. N. Shuravina, About the one problem of evasion in cone (in Russian), *Vestn. Udmurt. Univ. Mat. Mekh. Komp. Nauki*, No.1(2012), 50-59.