International Journal of Pure and Applied Mathematics
Volume 92 No. 1 2014, 109-112
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v92i1.9

ijpam.eu

ON THE DIOPHANTINE EQUATION $7^x + 31^y = z^2$

Banyat Sroysang
Department of Mathematics and Statistics
Faculty of Science and Technology
Thammasat University
Rangsit Center, Pathumthani 12121, THAILAND

Abstract: In this paper, we show that the Diophantine equation $7^x + 31^y = z^2$ has no non-negative integer solution where x, y and z are non-negative integers.

AMS Subject Classification: 11D61
Key Words: exponential Diophantine equation

1. Introduction

In 2012, Sroysang [19] showed that the Diophantine equation $31^x + 32^y = z^2$ has no non-negative integer solution where x, y and z are non-negative integers.

In 2013, Rabago [6] showed that $(1, 1, 6)$ is a unique non-negative integer solution (x, y, z) for both two Diophantine equation $5^x + 31^y = z^2$ and $7^x + 29^y = z^2$ where x, y and z are non-negative integers. In the same year, Sroysang [15, 17] showed that both two Diophantine equations $5^x + 7^y = z^2$ and $7^x + 8^y = z^2$ have no non-negative integer solution where x, y and z are non-negative integers. Many Diophantine equations were solved [1, 2, 4, 7, 8, 9, 10, 11, 12, 13, 14, 16, 18, 20, 21].

In this paper, we show that the Diophantine equation $7^x + 31^y = z^2$ has no non-negative integer solution where x, y and z are non-negative integers.

Received: January 14, 2014

2. Preliminaries

Proposition 2.1. (see [5], Catalan's Conjecture) $(3, 2, 2, 3)$ *is a unique solution* (a, b, x, y) *for the Diophantine equation* $a^x - b^y = 1$ *where* a, b, x *and* y *are integers such that* $\min\{a, b, x, y\} > 1$.

Lemma 2.2. [15, 17] *The Diophantine equation* $7^x + 1 = z^2$ *has no non-negative integer solution where* x *and* z *are non-negative integers.*

Lemma 2.3. [19] *The Diophantine equation* $1 + 31^y = z^2$ *has no non-negative integer solution where* y *and* z *are non-negative integers.*

3. Main Results

Theorem 3.1. *The Diophantine equation* $7^x + 31^y = z^2$ *has no non-negative integer solution where* x, y *and* z *are non-negative integers.*

Proof. Suppose that there are non-negative integers x, y and z such that $7^x + 31^y = z^2$. By Lemma 2.2 and 2.3, we obtain that $x \geq 1$ and $y \geq 1$. This implies that z is even. It follows that $z^2 \equiv 0 \pmod 3$ or $z^2 \equiv 1 \pmod 3$. Note that $7^x \equiv 1 \pmod 3$ and $31^x \equiv 1 \pmod 3$. Thus, $z^2 \equiv 2 \pmod 3$. This is a contradiction. □

Corollary 3.2. *Let* k *be a positive integer. Then the Diophantine equation* $7^x + 31^y = w^{2k+2}$ *has no non-negative integer solution where* x, y *and* w *are non-negative integers.*

Proof. Suppose that there are non-negative integers x, y and w such that $7^x + 31^y = w^{2k+2}$. Let $z = w^{k+1}$. It follows that $7^x + 31^y = z^2$. This is a contradiction with Theorem 3.1. □

Corollary 3.3. *The Diophantine equation* $49^u + 31^y = z^2$ *has no non-negative integer solution where* u, y *and* z *are non-negative integers.*

Proof. Suppose that there are non-negative integers u, y and z such that $49^u + 31^y = z^2$. Let $x = 2u$. It follows that $49^x + 31^y = z^2$. This is a contradiction with Theorem 3.1. □

Corollary 3.4. *Let* k *be a positive integer. Then the Diophantine equation* $49^x + 31^y = w^{2k+2}$ *has no non-negative integer solution where* x, y *and* w *are non-negative integers.*

Proof. Suppose that there are non-negative integers x, y and w such that $49^x + 31^y = w^{2k+2}$. Let $z = w^{k+1}$. It follows that $49^x + 31^y = z^2$. This is a contradiction with Corollary 3.3. □

References

[1] D. Acu, On a Diophantine equation $2^x + 5^y = z^2$, *Gen. Math.*, **15** (2007), 145−148.

[2] S. Chotchaisthit, On the Diophantine equation $2^x + 11^y = z^2$, *Maejo Int. J. Sci. Technol.*, **7** (2013), 291−293.

[3] S. Chotchaisthit, On the Diophantine equation $4^x + p^y = z^2$ where p is a prime number, *Amer. J. Math. Sci.*, **1** (2012), 191−193.

[4] S. Chotchaisthit, On the Diophantine equation $p^x + (p+1)^y = z^2$ where p is a Mersenne prime, *Int. J. Pure Appl. Math.*, **88** (2013), 169−172.

[5] P. Mihailescu, Primary cycolotomic units and a proof of Catalan's conjecture, *J. Reine Angew. Math.*, **27** (2004), 167−195.

[6] J. F. T. Rabago, More on the Diophantine equation of type $p^x + q^y = z^2$, *Int. J. Math. Sci. Comp.*, **3** (2013), 15−16.

[7] J. F. T. Rabago, On an open problem by B. Sroysang, *Konuralp J. Math.*, **1** (2013), 30−32.

[8] J. F. T. Rabago, On two Diophantine equations $3^x + 19^y = z^2$ and $3^x + 91^y = z^2$, *Int. J. Math. Sci. Comp.*, **3** (2013), 28−29.

[9] B. Sroysang, More on the Diophantine equation $2^x + 3^y = z^2$, *Int. J. Pure Appl. Math.*, **84** (2013), 133−137.

[10] B. Sroysang, More on the Diophantine equation $2^x + 19^y = z^2$, *Int. J. Pure Appl. Math.*, **88** (2013), 157−160.

[11] B. Sroysang, More on the Diophantine equation $2^x + 37^y = z^2$, *Int. J. Pure Appl. Math.*, **89** (2013), 275−278.

[12] B. Sroysang, More on the Diophantine equation $8^x + 19^y = z^2$, *Int. J. Pure Appl. Math.*, **81** (2012), 601−604.

[13] B. Sroysang, On the Diophantine equation $3^x + 5^y = z^2$, *Int. J. Pure Appl. Math.*, **81** (2012), 605–608.

[14] B. Sroysang, On the Diophantine equation $3^x + 17^y = z^2$, *Int. J. Pure Appl. Math.*, **89** (2013), 111–114.

[15] B. Sroysang, On the Diophantine equation $5^x + 7^y = z^2$, *Int. J. Pure Appl. Math.*, **89** (2013), 115–118.

[16] B. Sroysang, On the Diophantine equation $5^x + 23^y = z^2$, *Int. J. Pure Appl. Math.*, **89** (2013), 119–122.

[17] B. Sroysang, On the Diophantine equation $7^x + 8^y = z^2$, *Int. J. Pure Appl. Math.*, **84** (2013), 111–114.

[18] B. Sroysang, On the Diophantine equation $23^x + 32^y = z^2$, *Int. J. Pure Appl. Math.*, **84** (2013), 231–234.

[19] B. Sroysang, On the Diophantine equation $31^x + 32^y = z^2$, *Int. J. Pure Appl. Math.*, **81** (2012), 609–612.

[20] B. Sroysang, On the Diophantine equation $47^x + 49^y = z^2$, *Int. J. Pure Appl. Math.*, **89** (2013), 279–282.

[21] B. Sroysang, On the Diophantine equation $89^x + 91^y = z^2$, *Int. J. Pure Appl. Math.*, **89** (2013), 283–286.