ON PAIRWISE GENERALIZED ALPHA SEPARATION AXIOMS IN INTUITIONISTIC BIFUZZY TOPOLOGICAL SPACES

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Abstract: The aim of this work is to investigate several types of pairwise $G\alpha$ T_K , $K = 0, 1, 2, \dots$ separation axioms in Intuitionistic bifuzzy topological spaces, based on the characterizations of pairwise separation axioms in bitopological spaces. Intuitionistic bifuzzy pairwise $G\alpha$ -regular spaces, pairwise $G\alpha$ -normal spaces are also studied and their relationships with other separation axioms are discussed.

AMS Subject Classification: 54D10, 54E55, 03E72

Key Words: intuitionistic bifuzzy topological spaces, intuitionistic bifuzzy pairwise $G\alpha$ T_K , $K = 0, 1, 2, \dots$ separation axioms, intuitionistic bifuzzy pairwise $G\alpha$ -regular spaces, intuitionistic bifuzzy pairwise $G\alpha$ -normal spaces

1. Introduction

The triple (X, τ_1, τ_2) , where X is a non empty set and τ_1, τ_2 are topologies on X is called a bitopological space. Two topologies on the same set may separate the

Received: August 30, 2013

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points of the set to a different degree. J.C. Kelly [13] initiated the systematic study of such spaces and several other authors have contributed to the subsequent development of various bitopological properties. Zadeh [15] introduced the concept of fuzzy sets. After that there have been a number of generalizations of this fundamental concept. Atanassov [1] introduced the notion of intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [7] introduced the notion of intuitionistic fuzzy topological spaces. The separation axioms stipulate the degree in which distinct points or closed sets are separated by open sets. These axioms are statements about the richness of the topology. Fuzzy separation axioms and fuzzy continuity in fuzzy bitopological spaces was given by A.S. Abu Safiya, A. A. Fora, and M. W. Warner [3]. Pairwise separation axioms in *IFTS* were introduced by Sadik Bayhan and Dogan Coker [5,6]. It is the aim of this paper to give more properties of pairwise separation axioms introduced by Sadik Bayhan. In chapter [3] Intuitionistic bifuzzy pairwise $G\alpha T_K$ spaces is introduced and their interrelationship are studied. In chapter [4] Intuitionistic bifuzzy pairwise $G\alpha$ -regular spaces, Intuitionistic bifuzzy pairwise $G\alpha$ -normal spaces are studied as a development of pairwise separation axioms in bitopological spaces by T.M. Nour. We shall use P - to denote pairwise, P - $G\alpha T_K$ spaces for pairwise generalized alpha T_K spaces, $K = 0, 1, 2, \dots$ P -regular to denote pairwise regular and P -normal to denote pairwise normal.

2. Preliminaries

Definition 2.1. [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (*IFS* in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \}$ where the functions $\mu_A : X \rightarrow [0, 1]$ and $\gamma_A : X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) for each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Denote by $IFS(X)$, the set of all intuitionistic fuzzy sets in X .

Definition 2.2. [7] An intuitionistic fuzzy topology (*IFT* in short) on X is a family τ of intuitionistic fuzzy sets in X satisfying the following axioms

- (i) $0, 1 \in \tau$;
- (ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;
- (iii) $\cup G_i \in \tau$ for arbitrary family $\{G_i \mid i \in I\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (*IFTS* in short) and any intuitionistic fuzzy sets in τ is known as an intuitionistic fuzzy open set (*IFOS* in short) in X .

Definition 2.3. [1] Let A and B be IFS's of the form $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x) \rangle : x \in X\}$. Then

- (a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$
- (b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
- (c) $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x) \rangle : x \in X\}$
- (d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle : x \in X\}$
- (e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle : x \in X\}$
- (f) $0 = \{\langle x, 0, 1 \rangle : x \in X\}$ and $1 = \{\langle x, 1, 0 \rangle : x \in X\}$

Definition 2.4. [12] An *IFS* A of an *IFTS* (X, τ) is said to be an intuitionistic fuzzy generalized α closed set (*IFG α CS* in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an *IF α OS* in X .

Definition 2.5. [4] Let X be a non-empty set and $x \in X$, a fixed element in X . If $\alpha \in (0, 1)$ and $\beta \in (0, 1)$ are fixed real number such that $\alpha + \beta \leq 1$ then the *IFS* $x_{(\alpha, \beta)} = (x_\alpha, 1 - x_{(1-\beta)})$ is called an intuitionistic fuzzy point in X (*IFP*, in short) in X , and x is called its support. Two *IFPs* are said to be distinct if there supports are distinct.

Let $x_{(\alpha, \beta)}$ be an *IFP* in X and $A = (\mu_A, \gamma_A)$ be an *IFS* in X then $x_{(\alpha, \beta)}$ is belong to A if $\alpha \leq \mu_A(x)$, $\gamma_A \geq \gamma_A(x)$.

Definition 2.6. [5] Let $\langle a, b \rangle$, $\langle a_1, b_1 \rangle$, $\langle a_2, b_2 \rangle$ and $\langle a_i, b_i \rangle (i \in J)$ be intuitionistic fuzzy pairs. Then we define

- (i) $\langle a_1, b_1 \rangle \leq \langle a_2, b_2 \rangle$ if and only if $a_1 \leq a_2$ and $b_1 \geq b_2$,
- (ii) $\langle a_1, b_1 \rangle = \langle a_2, b_2 \rangle$ if and only if $a_1 = a_2$ and $b_1 = b_2$,
- (iii) if $\vee \langle a_i, b_i \rangle = \langle \vee a_i, \wedge b_i \rangle$ and $\wedge \langle a_i, b_i \rangle = \langle \wedge a_i, \vee b_i \rangle$,
- (iv) $\overline{\langle a, b \rangle} = \langle b, a \rangle$,
- (v) $1 = \langle 1, 0 \rangle$ and $0 = \langle 0, 1 \rangle$.

Definition 2.7. [4] Let (X, τ_1, τ_2) be a bitopological space. Then it is called

- (i) pairwise T_0 if for all x and y in X , there exist U in τ_1 with $x \in U, y \notin U$ or there exist V in τ_2 with $y \in V, x \notin V$.
- (ii) pairwise T_1 if for all x and y in X , there exist U in τ_1 with $x \in U, y \notin U$ and there exist V in τ_2 with $y \in V, x \notin V$.
- (iii) pairwise T_2 if and only if for each pair of distinct fuzzy points $x_{r,s}$ and $y_{p,q}$ in X , there exist U in τ_1 with $x_{r,s} \in U$ and there exist V in τ_2 with $y_{p,q} \in V$ such that $U \cap V = 0$.

Definition 2.8. [4] An *IFTS* (X, τ) is called

- (i) T_0 if for all $x, y \in X, x \neq y, \exists U = (\mu_U, \gamma_U), V = (\mu_V, \gamma_V) \in \tau$ such that $U(x) = \tilde{1}, U(y) = \tilde{0}$ or $V(y) = \tilde{1}, V(x) = \tilde{0}$.
- (ii) T_1 for all $x, y \in X, x \neq y, \exists U = (\mu_U, \gamma_U), V = (\mu_V, \gamma_V) \in \tau$ such that $U(x) = \tilde{1}, U(y) = \tilde{0}$ and $V(y) = \tilde{1}, V(x) = \tilde{0}$.
- (iii) T_2 if for all pair of distinct intuitionistic fuzzy points $x_{r,s}$ and $y_{p,q}$ in X , $\exists U = (\mu_U, \gamma_U), V = (\mu_V, \gamma_V) \in \tau$ such that $x_{r,s} \in U, y_{p,q} \in V$ and $U \cap V = 0$.

Definition 2.9. [14] If $\{A_i = (\mu_{A_i}, \gamma_{A_i}), i \in J\}$ is a family of *IFSs* in X_i , then their product is defined as the *IFS* in $\prod X_i$ given by $\prod A_i = (\prod \mu_{A_i}, \prod \gamma_{A_i})$ where $\prod \mu_{A_i} = \inf(\mu_{A_i}(x_i)) \forall x = \prod x_i \in X$ and $\prod \gamma_{A_i} = \sup(\gamma_{A_i}(x_i))$.

Definition 2.10. [4] An *IFS* N in an *IFTS* (X, τ) is called an intuitionistic fuzzy neighbourhood (*IFN*, in short) of an *IFP* $x_{(\alpha, \beta)}$ if $\exists U \in \tau$ such that $x_{(\alpha, \beta)} \in U \subseteq N$.

Proposition 2.1. [4] Let (X, τ) be an *IFTS*. Then an *IFS* A in X is an *IFOS* if and only if A is an *IFN* of each of *IFP* $x_{(\alpha, \beta)} \in A$.

Proposition 2.2. [4] Let (X, τ_1, τ_2) be a *BFTS* space $\tau_{(\tau_1, \tau_2)} = \{(U, V^c) \mid U \in \tau_1, V \in \tau_2 \text{ and } U \subseteq V\}$. Then (X, τ) is an *IFTS* and $(\tau_{(\tau_1, \tau_2)})_1 = \tau_1, (\tau_{(\tau_1, \tau_2)})_2 = \tau_2$.

Remark 2.1. [4] Let (X, τ) be an *IFTS* and let $\tau_1 = \{\mu_A \mid \exists \gamma_A \in I^X \text{ such that } (\mu_A, \gamma_A) \in \tau\}, \tau_2 = \{1 - \gamma_A \mid \exists \mu_A \in I^X \text{ such that } (\mu_A, \gamma_A) \in \tau\}$.

3. Pairwise Generalized Alpha Separation Axioms in Intuitionistic BiFuzzy Topological Spaces

Definition 3.1. Let X be a nonempty set and τ_1, τ_2 be two intuitionistic fuzzy topologies on X . Then (X, τ_1, τ_2) is called an intuitionistic bifuzzy topological space (*IBFTS*, in short).

Definition 3.2. An *IFTS* (X, τ) is called

- (i) $G\alpha T_0$ if for all $x, y \in X, x \neq y$, there exist $G\alpha$ -open sets $U = (\mu_U, \gamma_U), V = (\mu_V, \gamma_V) \in \tau$ such that $U(x) = \tilde{1}, U(y) = \tilde{0}$ or $V(y) = \tilde{1}, V(x) = \tilde{0}$.
- (ii) $G\alpha T_1$ for all $x, y \in X, x \neq y$, there exist $G\alpha$ -open sets $U = (\mu_U, \gamma_U), V = (\mu_V, \gamma_V) \in \tau$ such that $U(x) = \tilde{1}, U(y) = \tilde{0}$ and $V(y) = \tilde{1}, V(x) = \tilde{0}$.
- (iii) $G\alpha T_2$ if for all pair of distinct intuitionistic fuzzy points $x_{r,s}$ and $y_{p,q}$ in X , there exist $G\alpha$ -open sets $U = (\mu_U, \gamma_U), V = (\mu_V, \gamma_V) \in \tau$ such that $x_{r,s} \in U, y_{p,q} \in V$ and $U \cap V = 0$.

Definition 3.3. (**Pairwise $G\alpha T_K$ Spaces**) An *IBFTS* (X, τ_1, τ_2) is called Pairwise $G\alpha T_K$, $K = 0, 1, 2$ if for any pair of distinct points x and y in X , there exist a τ_i - $G\alpha$ open set which contains one of them but not the other $i = 1, 2$.

Definition 3.4. An *IBFTS* (X, τ_1, τ_2) is called

- (i) pairwise $G\alpha T_0$ if for all $x, y \in X, x \neq y$, there exist τ_i -*IFG* α -open set $U = (\mu_U, \gamma_U)$ and τ_j -*IFG* α -open set $V = (\mu_V, \gamma_V)$ such that $U(x) = \tilde{1}, U(y) = \tilde{0}$ or $V(y) = \tilde{1}, V(x) = \tilde{0}, i, j = 1, 2, i \neq j$.
- (ii) pairwise $G\alpha T_1$ for all $x, y \in X, x \neq y$, there exist τ_i -*IFG* α -open set $U = (\mu_U, \gamma_U)$ and τ_j -*IFG* α -open set $V = (\mu_V, \gamma_V)$ such that $U(x) = \tilde{1}, U(y) = \tilde{0}$ and $V(y) = \tilde{1}, V(x) = \tilde{0}, i, j = 1, 2, i \neq j$.
- (iii) pairwise $G\alpha T_2$ if for any pair of distinct points $x_{r,s}$ and $y_{p,q}$ in X , there exist τ_i -*IFG* α -open set U and τ_j -*IFG* α -open set V such that $x_{r,s} \in U, y_{p,q} \in V$ and $U \cap V = 0, i, j = 1, 2, i \neq j$.

Proposition 3.1. Let $\{(X_i, \tau_{1i}, \tau_{2i}) : i \in J\}$ be a family of *IBFTS* and $(\prod X_i, \prod \tau_{1i}, \prod \tau_{2i})$ be their product *IBFTS*. Then $\{(\prod X_i, \prod \tau_{1i}, \prod \tau_{2i}) : i \in J\}$ is pairwise *IFG* αT_1 if and only if each $\{(X_i, \tau_{1i}, \tau_{2i})\}$ is pairwise *IFG* αT_1 for all $i \in J$.

Proof. Assume that $\{(X_i, \tau_{1i}, \tau_{2i}) : i \in J\}$ be pairwise $IFG\alpha T_1$. To show that

$\{(\prod X_i, \prod \tau_{1i}, \prod \tau_{2i}) : i \in J\}$ is pairwise $IFG\alpha T_1$. Choose $x, y \in X$ such that $x \neq y$. Let $x = \prod x_j, y = \prod y_j$ then $x_j \neq y_j$ for some $j \in J$. Now consider $x_j, y_j \in X_j$. Since $\{(X_i, \tau_{1i}, \tau_{2i}) : i \in J\}$ is pairwise $IFG\alpha T_1$ there exists intuitionistic fuzzy generalized alpha open sets $U_j = (\mu_{U_j}, \gamma_{U_j}) \in \tau_{1j}$ and $V_j = (\mu_{V_j}, \gamma_{V_j}) \in \tau_{2j}$ such that $U_j(x_j) = \tilde{1}, U_j(y_j) = \tilde{0}$ and $V_j(y_j) = \tilde{1}, V_j(x_j) = \tilde{0}$. Now consider the basic intuitionistic fuzzy generalized alpha open sets $U = \prod U_i$ and $V = \prod V_i$ where $U_i = V_i = (\tilde{1}, \tilde{0})$ for $i \neq j$ and $\overline{U_j} = U$ and $\overline{V_j} = V$ for $i = j$. Then $U = \prod \tau_{1i}$ and $V = \prod \tau_{2i}$ and hence $U(x) = \tilde{1}, U(y) = \tilde{0}$ and $V(y) = \tilde{1}, V(x) = \tilde{0}$. Hence $\{(\prod X_i, \prod \tau_{1i}, \prod \tau_{2i}) : i \in J\}$ is pairwise $IFG\alpha T_1$.

Conversely let $\{(\prod X_i, \prod \tau_{1i}, \prod \tau_{2i}) : i \in J\}$ is pairwise $IFG\alpha T_1$. To prove that $\{(X_i, \tau_{1i}, \tau_{2i}) : i \in J\}$ is pairwise $IFG\alpha T_1$. Let x_i, y_i then $x_i \neq y_i$. Now consider $x, y \in X$ such that $x = \prod x_j, y = \prod y_j$ where $x_j = y_j, j \neq i$ and $x_i = x_j, y_i = y_j$ for $i \neq j$. Then $x \neq y$, since $\{(\prod X_i, \prod \tau_{1i}, \prod \tau_{2i}) : i \in J\}$ is pairwise $IFG\alpha T_1$, there exists $U = (\mu_U, \gamma_U) \in \tau_1$ and $V = (\mu_V, \gamma_V) \in \tau_2$ such that $U(x) = \tilde{1}, U(y) = \tilde{0}$ and $V(y) = \tilde{1}, V(x) = \tilde{0}$. Consider the IFP $x_{r,s} \in U$ and $y_{p,q} \in V$. Then there exist basic intuitionistic fuzzy generalized alpha open sets $\prod U_j \in \tau_{1j}$ and $\prod V_j \in \tau_{2j}$ in X such that $x_{r,s} \in \prod U_j \subseteq U$ and $y_{p,q} \in \prod V_j \subseteq V$. Since $x_{r,s} \in \prod U_j \subseteq U, U(x) = \tilde{1} \Rightarrow \prod U_j(x) = \tilde{1} \Rightarrow (\inf \mu_{U_j}, \sup \gamma_{U_j})(x_j) = \tilde{1} \Rightarrow r \leq \inf \mu_{U_j}(x_j), s \geq \sup \gamma_{U_j}(x_j)$ implies $r \leq \mu_{U_j}(x_j), s \geq \gamma_{U_j}(x_j)$ and $U(y) = \tilde{0}$ implies $\prod U_j(y) = \tilde{0}$.

Similarly $y_{p,q} \in \prod V_j \subseteq V \Rightarrow p \leq \inf \mu_{V_j}(y_j), q \geq \sup \gamma_{V_j}(y_j)$. Also, since, $x_j = y_j$ for $i \neq j$. From the above we have $r \leq \mu_{U_i}(x_i), s \geq \gamma_{U_i}(x_i) \forall i \in J, i \neq j, p \leq \mu_{V_i}(y_i), q \geq \gamma_{V_i}(y_i) \forall i \in J, i \neq j$. Therefore, $U(y) = \tilde{0}$ implies $\prod U_j(y) = \tilde{0} \Rightarrow \inf \mu_{U_j}(y_j) = 0, \sup \gamma_{U_j}(y_j) = 1$ implies $\mu_{U_j}(y_j) = 0, \gamma_{U_j}(y_j) = 1$. This implies $\sup \mu_{U_j}(y_j) = 0, \inf \gamma_{U_j}(y_j) = 1$. That is, $U_j(y_j) = \tilde{0}$. Similarly we can show that $V_j(x_j) = \tilde{0}$. Thus we have $U_i(x_i) \geq r, U_i(y_i) = \tilde{0}$ and $V_i(y_i) \geq p, V_i(x_i) = \tilde{0}$. Consider, $\sup U_i = U_i \in \tau_{1i}$ and $\sup V_i = V_i \in \tau_{2i}$. Then $U_i(x_i) = \tilde{1}, U_i(y_i) = \tilde{0}, V_i(x_i) = \tilde{0}, V_i(y_i) = \tilde{1}$ showing that $(X_i, \tau_{1i}, \tau_{2i})$ is pairwise $IFG\alpha T_1$.

Theorem 3.1. Let $\{(X_i, \tau_{1i}, \tau_{2i}) : i \in J\}$ be a family of $IBFTS$ and $(\prod X_i, \prod \tau_{1i}, \prod \tau_{2i})$ be their product $IBFTS$. Then $\{(\prod X_i, \prod \tau_{1i}, \prod \tau_{2i}) : i \in J\}$ is pairwise $IFG\alpha T_0$ if and only if each $\{(X_i, \tau_{1i}, \tau_{2i})\}$ is pairwise $IFG\alpha T_0$ for all $i \in J$.

Proposition 3.2. If an $IFTS$ (X, τ) is $IFG\alpha T_1$ then (X, τ, τ_2) is pairwise $IFG\alpha T_1$.

Proof. Let (X, τ) be an $IFG\alpha T_1$ space. Then for all $x, y \in X$, $x \neq y$ there exist τ_1 - $IFG\alpha$ -open set $U = (\mu_U, \gamma_U)$ and τ_2 - $IFG\alpha$ -open set $V = (\mu_V, \gamma_V)$ such that $U(x) = \tilde{1}, U(y) = \tilde{0}$ and $V(y) = \tilde{1}, V(x) = \tilde{0}$. Since $U(x) = \tilde{1} \Rightarrow \mu_U(x) = 1, \gamma_U(x) = 0, U(y) = \tilde{0} \Rightarrow \mu_U(y) = 0, \gamma_U(y) = 1$. Similarly, $V(y) = \tilde{1} \Rightarrow \mu_V(y) = 1, \gamma_V(y) = 0, V(x) = \tilde{0} \Rightarrow \mu_V(x) = 0, \gamma_V(x) = 1$. Now $\mu_U(x) = 1 \Rightarrow x \in \mu_U \in \tau_1$ and $1 - \gamma_V(x) = 0 \Rightarrow x \in (1 - \gamma_V) \in \tau_2$ and also $\mu_U(x) = 1, \mu_U(y) = 0, 1 - \gamma_V(x) = 0$ and $1 - \gamma_V(y) = 0$ showing that (X, τ, τ_2) is pairwise $IFG\alpha T_1$.

Proposition 3.3. Every IBF Pairwise $G\alpha T_2$ is IBF Pairwise $G\alpha T_1$.

Proof. Suppose that (X, τ, τ_2) is Pairwise $G\alpha T_2$. Given any points $x_{r,s}$ and $y_{p,q}$ in X , there exist a τ_1 - $IFG\alpha$ -open set U containing $x_{r,s}$ and there exist a τ_2 - $IFG\alpha$ -open set V containing $y_{p,q}$ such that $U \cap V = \emptyset$. This implies $y_{p,q} \notin U$ and $x_{r,s} \notin V$. Hence (X, τ, τ_2) is Pairwise Pairwise $G\alpha T_1$.

Proposition 3.4. An $IFTS$ (X, τ) is $IFG\alpha T_2$ then the $IBFTS$ (X, τ, τ_2) is pairwise $IFG\alpha T_2$.

Proof. Given (X, τ) be an $IFG\alpha T_1$ space. To prove that (X, τ, τ_2) is pairwise $IFG\alpha T_2$. Let x_r, y_{1-p} be any two FP in X . Choose $(s, q) \in (0, 1)$, then $x_{r,s}$ and $y_{p,q}$ are district IFP 's in X . Since (X, τ) is $IFG\alpha T_2$ space there exist $G\alpha$ -open sets $U = (\mu_U, \gamma_U), V = (\mu_V, \gamma_V) \in \tau$ such that $x_{r,s} \in U, y_{p,q} \in V$ and $U \cap V = \emptyset$. Let $U = (\mu_U, \gamma_U), V = (\mu_V, \gamma_V)$, then $x_{r,s} \in U \Rightarrow \mu_U(x) \leq r, s \geq \gamma_U(x)$ and $y_{p,q} \in V \Rightarrow p \leq \mu_V(y), q \geq \gamma_V(y)$. Then $1 - q \leq 1 - \gamma_V(y)$,

$$\mu_U + \gamma_U \leq 1, \mu_V + \gamma_V \leq 1 \quad (3.1)$$

Now $U \cap V = \emptyset \Leftrightarrow \mu_U \wedge \mu_V = 0, \gamma_U \vee \gamma_V = 1$. From 3.1, $x_r \in \mu_U$ and $y_{1-q} \in 1 - \gamma_V$. We will show that $\mu_U \wedge (1 - \gamma_V) = 0$ as follows. We have $\gamma_U(x) < 1 \Rightarrow \gamma_V(x) = 1 \Rightarrow 1 - \gamma_V(x) = 0$ which implies that $(\mu_U \wedge (1 - \gamma_V))(x) = 0$. Further if $\gamma_V(y) < 1 \Rightarrow \gamma_U(y) = 1 \Rightarrow 1 - \gamma_U(y) = 0$ since $\mu_U(y) + \gamma_U(y) \leq 1$, which implies that $(\mu_U \wedge (1 - \gamma_V))(y) = 0$. Now take $z \in X$ such that $z \neq x, y$. If $\mu_U(z) = 0$ there is nothing to prove. If $\mu_U(z) \neq 0$ then $\gamma_U(z) < 1 \Rightarrow \gamma_V(z) = 1 \Rightarrow (1 - \gamma_V)(z) = 0$ which implies that $(\mu_U \wedge (1 - \gamma_V))(z) = 0$. Therefore $\mu_U \in \tau_1, 1 - \gamma_V \in \tau_2$ such that $x_r \in \mu_U, y_{1-q} \in 1 - \gamma_V$ and $(\mu_U \wedge (1 - \gamma_V)) = 0$. Hence, (X, τ, τ_2) is IBF pairwise $IFG\alpha T_2$.

Proposition 3.5. If $f : (X, \tau) \rightarrow (Y, \sigma)$ be $IFG\alpha$ continuous mapping, then $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $IBFG\alpha$ continuous.

Proof. Let $U \in \sigma_1$ then there exists $V \in I^Y$ such that $(U, V) \in \sigma$. Since f is $IFG\alpha$ continuous, $f^{-1}(U, V) \in \tau$. This implies $(f^{-1}(U), f^{-1}(V)) \in \tau \Rightarrow$

$f^{-1}(U) \in \tau_{\alpha_1} \Rightarrow f^{-1}(U)$ is $IFG\alpha$ open in τ_1 . Let $V_1 \in \sigma_2$ then there exists $U_1 \in I^Y$ such that $(U_1, V_1^c) \in \sigma$. Since f is $IFG\alpha$ continuous, $f^{-1}(U_1, V_1^c) \in \tau$. This implies $(f^{-1}(U_1), f^{-1}(V_1^c)) \in \tau \Rightarrow f^{-1}(V) \in \tau_2$. Thus f is $IBFG\alpha$ continuous.

4. IF Pairwise $G\alpha$ Regular and Pairwise $G\alpha$ Normal Spaces

Definition 4.1. (Pairwise Regular) An IBF topological space (X, τ_1, τ_2) is said to be IF pairwise regular if for each IFP $x_{r,s} \in X$ and each τ_i closed set F not containing $x_{r,s}$, there exist a τ_i open set U and a τ_j open set V such that $x_{r,s} \in U, F \subset V, U \cap V = \emptyset, i, j = 1, 2$ and $i \neq j$.

Definition 4.2. (Pairwise $G\alpha$ Regular) An IBF topological space (X, τ_1, τ_2) is said to be IF pairwise $G\alpha$ regular if for each IFP $x_{r,s} \in X$ and each τ_i closed set F not containing $x_{r,s}$, there exist a τ_i - $G\alpha$ open set U and a τ_j - $G\alpha$ open set V such that $x_{r,s} \in U, F \subset V, U \cap V = \emptyset, i, j = 1, 2$ and $i \neq j$.

Proposition 4.1. An IBF topological space (X, τ_1, τ_2) is said to be IF pairwise $G\alpha$ regular if for each IFP $x_{r,s} \in X$ and each τ_i closed set K not containing $x_{r,s}$, there exist a τ_i - $G\alpha$ open set U such that $x_{r,s} \in U$ and τ_j - $G\alpha cl(U) \cap K = \emptyset$.

Proof. Assume that (X, τ_1, τ_2) is said to be IF pairwise $G\alpha$ regular. Then from definition we have for each IFP $x_{r,s} \in X$ and each τ_i closed set K , there exist a τ_i - $G\alpha$ open set U and a τ_j - $G\alpha$ open set V such that $x_{r,s} \in U, K \subseteq V, U \cap V = \emptyset, i, j = 1, 2$ and $i \neq j$. Since $\overline{K}$ is a τ_i closed set containing $x_{r,s}$, then by definition there is a τ_i - $G\alpha$ open set U such that $x_{r,s} \in U \subseteq \tau_j$ - $G\alpha cl(U) \subseteq \overline{K}$. Hence τ_j - $G\alpha cl(U) \cap K = \emptyset$.

Conversely, let K be any τ_i closed set and $x_{r,s} \notin K$. Then there exists a τ_i - $G\alpha$ open set U such that $x_{r,s} \in U$ and τ_j - $G\alpha cl(U) \cap K = \emptyset$. Let $V = \overline{\tau_j$ - $G\alpha cl(U)}$. Then V is a τ_j - $G\alpha$ open set V such that $K \subseteq V, x_{r,s} \in U$ and $U \cap V = \emptyset$. Thus (X, τ_1, τ_2) is said to be IF pairwise $G\alpha$ regular.

Definition 4.3. A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be IF pairwise $G\alpha$ closed if the functions $f : (X, \tau_1) \rightarrow (Y, \sigma_1)$ and $f : (X, \tau_2) \rightarrow (Y, \sigma_2)$ are $IFG\alpha$ closed.

Definition 4.4. A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be IF pairwise $G\alpha$ continuous if the functions $f : (X, \tau_1) \rightarrow (Y, \sigma_1)$ and $f : (X, \tau_2) \rightarrow (Y, \sigma_2)$ are $IFG\alpha$ continuous.

Proposition 4.2. Let f be a pairwise continuous and $G\alpha$ closed function

of a space (X, τ_1, τ_2) on to a space (Y, σ_1, σ_2) . If (X, τ_1, τ_2) is pairwise regular, $f(y_{p,q})$ is τ_1 -compact and τ_2 , for each IFP $y_{p,q} \in Y$, then (Y, σ_1, σ_2) is pairwise $G\alpha$ regular.

Proof. Let K be a σ_i -closed subset of Y and let $y \in \overline{K}$. Then $f^{-1}(K)$ is a τ_i closed subset of X and $f^{-1}(K) \cap f^{-1}(y_{p,q}) = \emptyset$. Since X is IF pairwise regular, for each $x_{r,s} \in f^{-1}(y_{p,q})$ there exist τ_j open set U_k and τ_i open set V_k such that $f^{-1}(K) \subseteq U_k$, $x_{r,s} \in V_k$ and $U_k \cap V_k = \emptyset$, $i, j = 1, 2$ and $i \neq j$. The family $\{V_k : x_{r,s} \in f^{-1}(y_{p,q})\}$ is a τ_i -open cover of $f^{-1}(y_{p,q})$. Since $f^{-1}(y_{p,q})$ is τ_i -compact, there exist a finite sub cover of $\{V_k : x_{r,s} \in f^{-1}(y_{p,q})\}$ say $V_k, k = 1, 2, 3, \dots, n$ such that $f^{-1}(y_{p,q}) \subseteq \bigcup_{k=1}^n V_k$. Let $V = \bigcup_{k=1}^n V_k$ and $U = \bigcap_{k=1}^n U_k$. Then V is τ_i open and U is τ_j open. Also, $f^{-1}(y_{p,q}) \subseteq V, f^{-1}(K) \subseteq U$. Since f is pairwise $G\alpha$ closed, there exist $V \in G\alpha O(Y, \sigma_i)$ and $U \in G\alpha O(Y, \sigma_j)$, $y_{p,q} \subseteq V, K \subseteq U, f^{-1}(V) \subset V$ and $f^{-1}(U) \subset U, i, j = 1, 2$ and $i \neq j$. Since U and V are disjoint sets $U \cap V = \emptyset$. Hence (Y, σ_1, σ_2) is pairwise $G\alpha$ regular.

Definition 4.5. (Pairwise Normal) An IFB topological space (X, τ_1, τ_2) is said to be IF pairwise normal if for each τ_i closed set A and τ_j closed set B such that $A \cap B = \emptyset$, there exist sets U and V such that U is τ_j open set, V is τ_i open set, $A \subset U, B \subset V, U \cap V = \emptyset, i, j = 1, 2$ and $i \neq j$.

Definition 4.6. (Pairwise $G\alpha$ Normal) A space (X, τ_1, τ_2) is said to be pairwise $G\alpha$ normal if for any τ_i closed set A and τ_j closed set B disjoint from A , there exist τ_j - $G\alpha$ open set U and τ_i - $G\alpha$ open set V such that $A \subset U, B \subset V, U \cap V = \emptyset, i, j = 1, 2$ and $i \neq j$.

Proposition 4.3. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise continuous and pairwise $G\alpha$ closed surjection. If (X, τ_1, τ_2) is pairwise normal then (Y, σ_1, σ_2) is pairwise $G\alpha$ normal.

Proof. Assume that (X, τ_1, τ_2) is pairwise normal. Let A and B be two disjoint subsets of Y such that A is σ_i -closed and B is σ_j -closed. Since f is pairwise continuous, $f^{-1}(A)$ is τ_i -closed and $f^{-1}(B)$ is τ_j -closed sets such that $f^{-1}(A) \cap f^{-1}(B) = \emptyset$. Since X is pairwise normal, there exist disjoint sets U and V such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V, U$ is τ_i -open and V is τ_j -open. Now f is pairwise $G\alpha$ closed, there exist σ_j - $G\alpha$ open set L and σ_i - $G\alpha$ open set M such that $A \subset L, B \subset M$ with $f^{-1}(L) \subseteq U$ and $f^{-1}(M) \subseteq V$. Since $U \cap V = \emptyset$, we have $L \cap M = \emptyset$. Hence (Y, σ_1, σ_2) is pairwise $G\alpha$ normal.

5. Conclusion

Thus, we have discussed the nature of some pairwise $G\alpha$ T_K separation axioms via bifuzzy $G\alpha$ open sets in *IFTS*. Pairwise $G\alpha$ regular, pairwise $G\alpha$ normal spaces are studied and their inter relations are proved. This may be very useful when we study about how open sets are separated by closed sets in intuitionistic bifuzzy topological spaces. This study may bring an intuition among the researchers to develop pairwise separation axioms in other generalized closed sets in intuitionistic bifuzzy Topological Spaces.

Acknowledgments

The author pays her sincere thanks to the unknown referees for their guidance to improve their work.

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