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LEFT-RIGHT REGULAR ELEMENTS IN $Hyp_G(2)$

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Abstract: The concepts of a left-right regular element are important role in semigroup theory. In this paper we characterize left-right regular elements of the set of all generalized hypersubstitutions of type $\tau = (2)$.

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1. Introduction

All idempotent elements and all regular elements of the set of all generalized hypersubstitutions of type $\tau = (2)$ were studied by W. Puninagool and S. Leeratanavalee [2],[5]. In this paper we used the concept of idempotent elements as a tool to characterize left-right regular elements.

First, we recall the definition of a generalized hypersubstitution and some properties. A generalized hypersubstitution of type $\tau = (n_i)_{i \in I}$ is a mapping σ which maps each n_i-ary operation symbol to the set $W_\tau(X)$ of all terms of type τ built up by operation symbols from $\{f_i | i \in I\}$ where f_i is n_i-ary

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and variables from a countably infinite alphabet $X := \{x_1, x_2, x_3, ...\}$ which dose not necessarily preserve the arity. We denote the set of all generalized hypersubstitutions of type τ by $Hyp_G(\tau)$. To define a binary operation on $Hyp_G(\tau)$, we define at first the concept of *generalized superposition of terms* $S_m : W_\tau(X)^{m+1} \longrightarrow W_\tau(X)$ by the following steps:

(i) If $t = x_j, 1 \leq j \leq m$, then $S^m(x_j, t_1, ..., t_m) := t_j$.

(ii) If $t = x_j, m < j \in N$, then $S^m(x_j, t_1, .., t_m) := x_j$.

(iii) If $t = f_i(s_1, .., s_{n_i})$, then $S^m(t, t_1, ..., t_m) := f_i(S^m(s_1, t_1, ..., t_m),$ $.., (S^m(s_{n_i}, t_1, ..., t_m))$.

We extend a generalized hypersubstitution σ to a mapping $\hat{\sigma} : W_\tau(X) \longrightarrow W_\tau(X)$ inductively defined as follows:

(i) $\hat{\sigma}[x] := x \in X$,

(ii) $\hat{\sigma}[f_i(t_i, ..., t_{n_i})] := S^{n_i}(\sigma(f_i), \hat{\sigma}[t_1], ..., \hat{\sigma}[t_{n_i}])$, for any n_i-ary operation symbol f_i supposed that $\hat{\sigma}[t_j], 1 \leq j \leq n_j$ are already defined.

Then we define a binary operation $\circ_G$ on $Hyp_G(\tau)$ by $\sigma_1 \circ_G \sigma_2 := \hat{\sigma}_1 \circ \sigma_2$ where $\circ$ denotes the usual composition of mapping and $\sigma_1, \sigma_2 \in Hyp_G(\tau)$. Let σ_{id} be the hypersubstitution which maps each n_i-ary operation symbol f_i to the term $f_i(x_1, ..., x_{n_i})$. It turns out that $\underline{Hyp_G(\tau)} = (Hyp_G(\tau); \circ_G, \sigma_{id})$ is a monoid and σ_{id} is the identity element.

Proposition 1.1. ([1]) *For arbitrary terms $t, t_1, ..., t_n \in W_\tau(X)$ and for arbitrary generalized hypersubstitutions $\sigma, \sigma_1, \sigma_2$ we have*

(i) $S^n(\sigma[t], \sigma[t_1], ..., \sigma[t_n]) = \hat{\sigma}[S^n(t, t_1, ..., t_n)]$,

(ii) $(\hat{\sigma}_1 \circ \sigma_2)\hat{} = \hat{\sigma}_1 \circ \hat{\sigma}_2$.

Proposition 1.2. ([1]) *$\underline{Hyp_G(\tau)} = (Hyp_G(\tau); \circ_G, \sigma_{id})$ is a monoid and the set of all hypersubstitutions of type τ forms a submonoid of $\underline{Hyp_G(\tau)}$.*

For more details on generalized hypersubstitutions see [1], [3], [4].

2. Left-Right Regular Elements

In this section we characterize left-right regular elements of $Hyp_G(2)$ the set of all generalized hypersubstitutions of type $\tau = (2)$, i.e. we have only one binary operation symbol, say f. The generalized hypersubstitution σ which maps f to the term t is denoted by σ_t. First, we recall the definition of a left(right) regular element.

Definition 2.1. *An element a of a semigroup S is called left(right) regular if there exists $x \in S$ such that $xaa = a(aax = a)$.*

Then we need the definition of the variable count or the length of a term t, denoted by $vb(t)$, is the total number of occurrences of variables in t (including multiplicities).

In [5], we introduce some notations. For $s, f(c,d) \in W_{(2)}(X)$, $x_i, x_j \in X, i, j \in N$ and $S \subseteq W_{(2)}(X) \setminus X$, we denote :

$leftmost(s)$:= the first variable (from the left) that occurs in s,

$rightmost(s)$:= the last variable (from the left) that occurs in s,

$W^G_{(2)}(\{x_1\}) := \{s \in W_{(2)}(X) | x_1 \in var(s), x_2 \notin var(s)\}$,

$W^G_{(2)}(\{x_2\}) := \{s \in W_{(2)}(X) | x_2 \in var(s), x_1 \notin var(s)\}$,

$W(\{x_1\}) := W^G_{(2)}(\{x_1\}) \backslash \{x_1\}$,

$W(\{x_2\}) := W^G_{(2)}(\{x_2\}) \backslash \{x_2\}$,

$W^G := \{t \in W_{(2)}(X) | t \notin X, x_1, x_2 \notin var(t)\}$,

$W^G_{(2)}(\{x_1, x_2\}) := \{t \in W_{(2)}(X) | x_1, x_2 \in var(t)\}$,

$E^G(\{x_1, x_2\}) := \{\sigma_t \in Hyp_G(2) | t \in W^G_{(2)}(\{x_1, x_2\})\}$.

We define $Lp(t), Rp(t)$ by induction as follows :

(i) If $t = f(x, t_2), t_2 \in W_{(2)}(X)$, then $Lp(t) = f$.

(ii) If $t = f(t_1, x), t_1 \in W_{(2)}(X)$, then $Rp(t) = f$.

(iii) If $t = f(t_1, t_2), t_1 \notin X$, then $Lp(t) = f(Lp(t_1))$.

(iv) If $t = f(t_1, t_2), t_2 \notin X$, then $Rp(t) = f(Rp(t_2))$.

The f count of $Lp(t), Rp(t)$ we denoted by $length(Lp(t))$ and $length(Rp(t))$.

Then we have the following lemmas which are useful for characterize the generalized hypersubstitutions which are not left regular.

Lemma 2.1. *Let $s \in W(\{x_1\})$. If $leftmost(s) = x_m$ where $m > 2$, then $x_1, x_2 \notin var(\sigma_s^2(f))$.*

Lemma 2.2. *Let $s \in W(\{x_2\})$. If $rightmost(s) = x_m$ where $m > 2$, then $x_1, x_2 \notin var(\sigma_s^2(f))$.*

Lemma 2.3. *Let $u \in W_{(2)}(X), \sigma_t \in Hyp_G(2)$ and $x = x_1$ or $x = x_2$. If $x \notin var(u)$, then $x \notin var(\hat{\sigma}_t[u])$ (x is not a variable occurring in the term $(\sigma_t \circ_G \sigma_u)(f)$).*

Lemma 2.4. *Let $\sigma_{f(c,d)} \in Hyp_G(2) \backslash \{\sigma_{id}, \sigma_{f(x_2,x_1)}\}$ and $u \in W_{(2)}(X) \backslash X$. If $\sigma_{f(c,d)} \in E^G(\{x_1, x_2\})$, then the term w corresponding to the composition $\sigma_{f(c,d)} \circ_G \sigma_u$ is longer than u.*

Lemma 2.5. *If $f(c,d) \in W(\{x_1\}) \cup W(\{x_2\}) \cup W^G(x_1 \notin var(f(c,d))$ or $x_2 \notin var(f(c,d)))$, then for any $u, v \in W_{(2)}(X)$ the term w corresponding to $\sigma_{f(c,d)} \circ_G \sigma_{f(u,v)}$ is in $W(\{x_1\}) \cup W(\{x_2\}) \cup W^G$.*

Lemma 2.6. *Let $s, u, v \notin X$ and $\sigma_s \circ_G \sigma_u = \sigma_v$. Then the following statements hold.*

(i) *If $leftmost(s) = x_1$, then*
$length(Lp(v)) = length(Lp(s))length(Lp(u))$.
If $leftmost(s) = x_2$, then
$length(Lp(v)) = length(Lp(s))length(Rp(u))$.

(ii) *If $rightmost(s) = x_1$, then*
$length(Rp(v)) = length(Rp(s))length(Lp(u))$.
If $rightmost(s) = x_2$, then
$length(Rp(v)) = length(Rp(s))length(Rp(u))$.

For more details see [5].

Proposition 2.1. *If σ_t is idempotent, then σ_t is left(right) regular.*

Proof. Let σ_t be idempotent. Then $\sigma_t^3 = \sigma_t$ and so σ_t is left(right) regular. □

It is clear that for all idempotent elements is left(right) regular. Then we consider only the cases of non idempotent elements. From [5], W. Puninagool and S. Leeratanavalee characterized non idempotent elements of $Hyp_G(2)$ into three cases.

Case 1: Let $i, j \in N$ with $i > 2$ and $j \neq 2$. Then $\sigma_{f(x_2,x_j)}$ and $\sigma_{f(x_i,x_1)}$ are not idempotent.

Case 2: Let $t \in W_{(2)}(X) \setminus X$. Then the following statements hold:

(i) If $x_2 \in var(t)$ where $var(t)$ denotes the set of all variables occurring in t, then $\sigma_{f(x_1,t)}$ is not idempotent.

(ii) If $x_1 \in var(t)$, then $\sigma_{f(t,x_2)}$ is not idempotent.

(iii) $\sigma_{f(t,x_1)}$ and $\sigma_{f(x_2,t)}$ are not idempotent.

(iv) If $x_1 \in var(t)$ or $x_2 \in var(t)$, then $\sigma_{f(x_m,t)}$ and $\sigma_{f(t,x_m)}$ are not idempotent where $m \in N$ with $m > 2$.

Case 3: Let $t_1, t_2 \in W_{(2)}(X) \setminus X$. If $x_1 \in var(t_1) \cup var(t_2)$ or $x_2 \in var(t_1) \cup var(t_2)$, then $\sigma_{f(t_1,t_2)}$ is not idempotent.

For Case 1, we get $\sigma_{f(x_2,x_1)}$, $\sigma_{f(x_2,x_m)}$ and $\sigma_{f(x_m,x_1)}$ where $m > 2$. First, we will consider $\sigma_{f(x_2,x_1)}$.

Proposition 2.2. $\sigma_{f(x_2,x_1)}$ *is left(right) regular.*

Proof. Since $\sigma^3_{f(x_2,x_1)} = \sigma_{f(x_2,x_1)}$, thus $\sigma_{f(x_2,x_1)}$ is left(right) regular. □

Next, we will consider $\sigma_{f(x_2,x_m)}$ where $m \in N$ with $m > 2$.

Proposition 2.3. $\sigma_{f(x_2,x_m)}$ *where $m \in N$ with $m > 2$ is not left(right) regular.*

Proof. Let $m \in N$ with $m > 2$. Then $\sigma^2_{f(x_2,x_m)}(f) = f(x_m, x_m)$. Since $x_2 \notin f(x_m, x_m)$ and by Lemma 2.3, we get $\sigma_u \circ_G \sigma_{f(x_2,x_m)} \circ_G \sigma_{f(x_2,x_m)} \neq \sigma_{f(x_2,x_m)}$ for all $\sigma_u \in Hyp_G(2)$. So $\sigma_{f(x_2,x_m)}$ is not left regular. Since $x_1, x_2 \notin f(x_m, x_m)$, thus $\sigma_{f(x_2,x_m)} \circ_G \sigma_{f(x_2,x_m)} \circ_G \sigma_u \neq \sigma_{f(x_2,x_m)}$ for all $\sigma_u \in Hyp_G(2)$. So $\sigma_{f(x_2,x_m)}$ is not right regular. □

Finally, we will consider $\sigma_{f(x_m,x_1)}$ where $m \in N$ with $m > 2$.

Proposition 2.4. $\sigma_{f(x_m,x_1)}$ *where $m \in N$ with $m > 2$ is not left(right) regular.*

Proof. The proof is similar to the proof of Proposition 2.3. □

Next, we will characterize left regular elements of $Hyp_G(2)$ in Case 2 and Case 3.

Proposition 2.5. *Let $t \in W_{(2)}(X) \setminus X$. Then the following statements hold:*

(i) If $x_2 \in var(t)$, then $\sigma_{f(x_1,t)}$ is not left regular.

(ii) *If* $x_1 \in var(t)$, *then* $\sigma_{f(t,x_2)}$ *is not left regular.*

(iii) $\sigma_{f(t,x_1)}$ *and* $\sigma_{f(x_2,t)}$ *are not left regular.*

(iv) *If* $x_1 \in var(t)$ *or* $x_2 \in var(t)$, *then* $\sigma_{f(x_m,t)}$ *and* $\sigma_{f(t,x_m)}$ *are not left regular where* $m \in N$ *with* $m > 2$.

Proof. (i) Let $x_2 \in var(t)$. Suppose that $\sigma_{f(x_1,t)}$ is left regular. Then there exists $\sigma_u \in Hyp_G(2)$ such that $\sigma_u \circ_G \sigma_{f(x_1,t)} \circ_G \sigma_{f(x_1,t)} = \sigma_{f(x_1,t)}$. We get $u \notin X$. Since $x_1, x_2 \in var(f(x_1,t))$ and $\sigma_u \circ_G \sigma_{f(x_1,t)} \circ_G \sigma_{f(x_1,t)} = \sigma_{f(x_1,t)}$, thus by Lemma 2.5 we get $x_1, x_2 \in var(u)$. Put $s = (\sigma_{f(x_1,t)} \circ_G \sigma_{f(x_1,t)})(f)$. By Lemma 2.4, we get $vb(s) > vb(f(x_1,t))$. From $\sigma_u \circ_G \sigma_{f(x_1,t)} \circ_G \sigma_{f(x_1,t)} = \sigma_{f(x_1,t)}$ and $s = (\sigma_{f(x_1,t)} \circ_G \sigma_{f(x_1,t)})(f)$, we get $\sigma_u \circ_G \sigma_s = \sigma_{f(x_1,t)}$. If $\sigma_u = \sigma_{id}$ or $\sigma_u = \sigma_{f(x_2,x_1)}$, then $vb(s) = vb(f(x_1,t))$, which contradicts to $vb(s) > vb(f(x_1,t))$. If $\sigma_u \neq \sigma_{id}, \sigma_{f(x_2,x_1)}$, then by Lemma 2.4, we get $vb(f(x_1,t)) > vb(s)$ which contradicts to $vb(s) > vb(f(x_1,t))$.

(ii) The proof is similar to the proof of (i).

(iii) Suppose that $\sigma_{f(t,x_1)}$ is left regular. Then there exists $\sigma_u \in Hyp_G(2)$ such that $\sigma_u \circ_G \sigma_{f(t,x_1)} \circ_G \sigma_{f(t,x_1)} = \sigma_{f(t,x_1)}$. We get $u \notin X$. Since $t \notin X$, thus $length(Lp(f(t,x_1))) \geq 2$. Since $rightmost(f(t,x_1)) = x_1$, then from $\sigma_u \circ_G \sigma_{f(t,x_1)} \circ_G \sigma_{f(t,x_1)} = \sigma_{f(t,x_1)}$ we get $rightmost(u) = x_1$ or $rightmost(u) = x_2$. If $rightmost(u) = x_1$, then from $\sigma_u \circ_G \sigma_{f(t,x_1)} \circ_G \sigma_{f(t,x_1)} = \sigma_{f(t,x_1)}$ we get $leftmost(f(t,x_1)) = x_1$. By Lemma 2.6, we get

$$\begin{aligned} &length(Rp((\sigma_u \circ_G \sigma_{f(t,x_1)} \circ_G \sigma_{f(t,x_1)})(f))) \\ &= length(Rp(u))length(Lp((\sigma_{f(t,x_1)} \circ_G \sigma_{f(t,x_1)})(f))) \\ &= length(Rp(u))length(Lp(f(t,x_1)))length(Lp(f(t,x_1))) \\ &> length(Rp(f(t,x_1))), \end{aligned}$$

which contradicts to $\sigma_u \circ_G \sigma_{f(t,x_1)} \circ_G \sigma_{f(t,x_1)} = \sigma_{f(t,x_1)}$. If $rightmost(u) = x_2$, then by Lemma 2.6 we get

$$\begin{aligned} &length(Rp((\sigma_u \circ_G \sigma_{f(t,x_1)} \circ_G \sigma_{f(t,x_1)})(f))) \\ &= length(Rp(u))length(Rp((\sigma_{f(t,x_1)} \circ_G \sigma_{f(t,x_1)})(f))) \\ &= length(Rp(u))length(Rp(f(t,x_1)))length(Lp(f(t,x_1))) \\ &> length(Rp(f(t,x_1))), \end{aligned}$$

which contradicts to $\sigma_u \circ_G \sigma_{f(t,x_1)} \circ_G \sigma_{f(t,x_1)} = \sigma_{f(t,x_1)}$.

By the same way we can show that $\sigma_{f(x_2,t)}$ is not left regular.

(iv) Suppose that $\sigma_{f(t,x_m)}$ is left regular where $m \in N$ with $m > 2$. Then there exists $\sigma_u \in Hyp_G(2)$ such that $\sigma_u \circ_G \sigma_{f(t,x_m)} \circ_G \sigma_{f(t,x_m)} = \sigma_{f(t,x_m)}$.

We get $u \notin X$. Since $t \notin X$, thus $length(Lp(\sigma_{f(t,x_m)}(f))) > 2$. We will consider three cases.

Case 1 : $x_1, x_2 \in var(t)$. The proof of this case is similar to the proof of (i).

Case 2 : $x_1 \in var(t), x_2 \notin var(t)$. If $leftmost(t) = x_m$ where $m \in N$ with $m > 2$, then by Lemma 2.1 we get $x_1 \notin var(\sigma^2_{f(t,x_m)}(f))$. By Lemma 2.3, we get $x_1 \notin var((\sigma_u \circ_G \sigma^2_{f(t,x_m)})(f))$, which contradicts to $\sigma_u \circ_G \sigma_{f(t,x_m)} \circ_G \sigma_{f(t,x_m)} = \sigma_{f(t,x_m)}$. If $leftmost(t) = x_1$, then from $\sigma_u \circ_G \sigma_{f(t,x_m)} \circ_G \sigma_{f(t,x_m)} = \sigma_{f(t,x_m)}$ we get $leftmost(u) = x_1$ or $leftmost(u) = x_2$. If $leftmost(u) = x_1$, then by Lemma 2.6 we get

$$\begin{aligned}
&length(Lp((\sigma_u \circ_G \sigma_{f(t,x_m)} \circ_G \sigma_{f(t,x_m)})(f))) \\
&\quad = length(Lp(u))length(Lp((\sigma_{f(t,x_m)} \circ_G \sigma_{f(t,x_m)})(f))) \\
&\quad = length(Lp(u))length(Lp(f(t,x_m)))length(Lp(f(t,x_m))) \\
&\quad > length(Lp(f(t,x_m))),
\end{aligned}$$

which contradicts to $\sigma_u \circ_G \sigma_{f(t,x_m)} \circ_G \sigma_{f(t,x_m)} = \sigma_{f(t,x_m)}$. If $leftmost(u) = x_2$, then $leftmost((\sigma_u \circ_G \sigma_{f(t,x_m)} \circ_G \sigma_{f(t,x_m)})(f)) = x_m$, which contradicts to $\sigma_u \circ_G \sigma_{f(t,x_m)} \circ_G \sigma_{f(t,x_m)} = \sigma_{f(t,x_m)}$.

Case 3 : $x_1 \notin var(t), x_2 \in var(t)$. By Lemma 2.2, we get $x_2 \notin var(\sigma^2_{f(t,x_m)}(f))$. By Lemma 2.3, we get $x_2 \notin var((\sigma_u \circ_G \sigma^2_{f(t,x_m)})(f))$ which contradicts to $\sigma_u \circ_G \sigma_{f(t,x_m)} \circ_G \sigma_{f(t,x_m)} = \sigma_{f(t,x_m)}$.

By the same way we can show that $\sigma_{f(x_m,t_1)}$ is not left regular. □

Proposition 2.6. *Let $t_1, t_2 \in W_{(2)}(X) \setminus X$. If $x_1 \in var(t_1) \cup var(t_2)$ or $x_2 \in var(t_1) \cup var(t_2)$, then $\sigma_{f(t_1,t_2)}$ is not left regular.*

Proof. Let $t = f(t_1, t_2)$. Since $t_1 \notin X$, thus $length(Lp(t)) \geq 2$. Since $t_2 \notin X$, thus $length(Rp(t)) \geq 2$. Suppose that σ_t is left regular. Then there exists $\sigma_u \in Hyp_G(2)$ such that

$$\sigma_u \circ_G \sigma_t \circ_G \sigma_t = \sigma_t. \tag{1}$$

We get $u \notin X$. We will consider three cases.

Case 1 : $x_1, x_2 \in var(t)$. The proof of this case is similar to the proof of Proposition 2.5 (i).

Case 2 : $x_1 \in var(t), x_2 \notin var(t)$. If $leftmost(t) = x_m$ where $m \in N$ with $m > 2$, then by Lemma 2.1 we get $x_1 \notin var((\sigma_t \circ_G \sigma_t)(f))$. By Lemma 2.3, we get $x_1 \notin var((\sigma_u \circ_G \sigma_t \circ_G \sigma_t)(f))$, which contradicts to (1). If $leftmost(t) = x_1$, then from (1) we get $leftmost(u) = x_1$ or $leftmost(u) = x_2$. If $leftmost(u) = x_1$, then by Lemma 2.6 we get

$$\begin{aligned} length(Lp((\sigma_u \circ_G \sigma_t \circ_G \sigma_t)(f))) &= length(Lp(u))length(Lp((\sigma_t \circ_G \sigma_t)(f))) \\ &= length(Lp(u))length(Lp(t))length(Lp(t)) \\ &> length(Lp(t)), \end{aligned}$$

which contradicts to (1). If $leftmost(u) = x_2$, then by (1) we get $rightmost(t) = x_1$. By Lemma 2.6, we get

$$\begin{aligned} length(Lp((\sigma_u \circ_G \sigma_t \circ_G \sigma_t)(f))) &= length(Lp(u))length(Rp((\sigma_t \circ_G \sigma_t)(f))) \\ &= length(Lp(u))length(Rp(t))length(Lp(t)) \\ &> length(Lp(t)), \end{aligned}$$

which contradicts to (1).

Case 3 : $x_1 \notin var(t), x_2 \in var(t)$. The proof of this case is similar to the proof of Case 2. □

Finally, we will characterize right regular elements of $Hyp_G(2)$ in Case 2 and Case 3.

Proposition 2.7. *Let $t \in W_{(2)}(X) \setminus X$. Then the following statements hold:*

(i) If $x_2 \in var(t)$, then $\sigma_{f(x_1,t)}$ is not right regular.

(ii) If $x_1 \in var(t)$, then $\sigma_{f(t,x_2)}$ is not right regular.

(iii) $\sigma_{f(t,x_1)}$ and $\sigma_{f(x_2,t)}$ are not right regular.

(iv) If $x_1 \in var(t)$ or $x_2 \in var(t)$, then $\sigma_{f(x_m,t)}$ and $\sigma_{f(t,x_m)}$ are not right regular where $m \in N$ with $m > 2$.

Proof. (i) Let $x_2 \in var(t)$. Then $vb((\sigma_{f(x_1,t)} \circ_G \sigma_{f(x_1,t)})(f)) > vb(f(x_1,t))$. So $\sigma_{f(x_1,t)} \circ_G \sigma_{f(x_1,t)} \circ_G \sigma_s \neq \sigma_{f(x_1,t)}$ for all $\sigma_s \in Hyp_G(2)$. Thus $\sigma_{f(x_1,t)}$ is not right regular.

The proof of (ii), (iii) and (iv) are similar to the proof of (i). □

Proposition 2.8. *Let $t_1, t_2 \in W_{(2)}(X) \setminus X$. If $x_1 \in var(t_1) \cup var(t_2)$ or $x_2 \in var(t_1) \cup var(t_2)$, then $\sigma_{f(t_1,t_2)}$ is not right regular.*

Proof. The proof is similar to the proof of Proposition 2.7. □

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