International Journal of Pure and Applied Mathematics
Volume 92 No. 4 2014, 471-480
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v92i4.3

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A NOTE ON UPPER AND LOWER WEAKLY QUASI CONTINUOUS FUZZY MULTIFUNCTIONS

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Abstract: We have already introduced upper and lower (weakly) quasi continuous fuzzy multifunctions in [3] ([3]). In [8], Malakar introduced fuzzy θ-continuous multifunctions. Again Mukherjee and Malakar have introduced fuzzy almost continuous multifunctions [9]. In this paper we have established a mutual relationships among these fuzzy multifunctions.

AMS Subject Classification: 54A40, 54C99
Key Words: fuzzy almost regular space, fuzzy almost preopen multifunction, θ-cluster point of a fuzzy set

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This paper is a continuation of [3]. A fuzzy multifunction is a function which carries a point of an ordinary topological space X to a fuzzy set in a fuzzy topological space Y according to Papageorgiou [13] in 1985. After introducing fuzzy multifunction by Papageorgiou, a good many researchers have been inspired to study it and a number of fuzzy multifunctions have been introduced and studied. Papageorgiou defined fuzzy upper and lower inverse of a fuzzy multifunctions. But the definition of lower inverse was not suitable for further study so that Mukherjee and Malakar [9] redefined it suitably in 1991 and this

Received: August 3, 2013

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new definition of lower inverse has been accepted to many of the researchers. In this paper we take the definition of upper inverse by Papageorgiou and the definition of lower inverse by Mukherjee and Malakar.

Throughout this paper, by (X, τ) or simply by X we shall mean an ordinary topological space, while (Y, τ_Y) or simply Y stands for a fuzzy topological space (fts, for short) in the sense of Chang [4]. The support of a fuzzy set A in Y will be denoted as $suppA$ [17] and is defined by $suppA = \{y \in Y : A(y) \neq 0\}$. A fuzzy point [15] with the singleton support $y \in Y$ and the value α $(0 < \alpha \leq 1)$ at y will be denoted by y_α. 0_Y and 1_Y are the constant fuzzy sets taking respectively the constant values 0 and 1 on Y. The complement of a fuzzy set A in Y will be denoted by $1 - A$ [17]. For two fuzzy sets A and B in Y, we write $A \leq B$ iff $A(y) \leq B(y)$, for each $y \in Y$, while we write AqB to mean A is quasi-coincident (q-coincident, for short) with B [15] if there exists $y \in Y$ such that $A(y) + B(y) > 1$; the negation of AqB is written as $A\overline{q}B$. clA and $intA$ of a set A in X (respectively, a fuzzy set A [17] in Y) respectively stand for the closure and interior of A in X (respectively, in Y). A fuzzy set A in Y is called fuzzy regular open [1] if $intclA = A$. The complement of a fuzzy regular open set is called fuzzy regular closed [1]. A fuzzy set B is called a quasi neighbourhood (q-nbd, for short) of a fuzzy set A in an fts Y if there is a fuzzy open set U in Y such that $AqU \leq B$. If, in addition, B is fuzzy open (regular open) then B is called a fuzzy open (regular open) q-nbd of A. In particular, a fuzzy (open) set B in Y is a fuzzy open (resp. regular open) q-nbd of a fuzzy point y_α in Y if $y_\alpha qU \leq B$, for some fuzzy open set U in Y. For a subset A of a topological space X, the θ-interior of A, denoted by $\theta - intA$ [16], is defined to be the set of those points x of X such that there exists an open nbd U_x of x with $clU_x \subseteq A$. A set A (or a fuzzy set A) in a topological space X (in an fts Y) is said to be semiopen [7] (fuzzy semiopen [1]) if there exists an open set (respectively a fuzzy open set) U in X (in Y) such that $U \subseteq A \subseteq clU$ (resp. $U \leq A \leq clU$), or equivalently, if $A \subseteq clintA$ (resp. $A \leq clintA$). By $SO(X)$ (resp. $FSO(Y)$) we mean the set of all semiopen (resp. fuzzy semiopen) sets in X (in Y). The complement of a semiopen set (resp. fuzzy semiopen set) in X (resp. in Y) is called a semiclosed (fuzzy semiclosed) set. We mean semiclosure (resp. fuzzy semiclosure) of a set A in X (resp. of a fuzzy set A in Y), to be written as $sclA$, which is the union of all points (resp. fuzzy points) x in X (resp. y_α in Y) such that for every semiopen set (resp. fuzzy semiopen set) U in X (in Y) with $x \in U$ (resp. $y_\alpha qU$), it follows that $U \bigcap A \neq \phi$ [5] (resp. UqA [6]). A set A in X (resp. a fuzzy set A in Y) is semiclosed (fuzzy semiclosed) iff $A = sclA$. The union of all semiopen (resp. fuzzy semiopen) sets in X (resp. in Y) contained in a set (resp. fuzzy set) A is called the semi-interior (resp.

fuzzy semi-interior) of A, denoted by $sintA$. It follows from definition that a set (resp. a fuzzy set) A is semiopen (resp. fuzzy semiopen) iff $A = sintA$. A set B in X is called a seminbd of a set A in X if there exists a semiopen set U in X such that $A \subseteq U \subseteq B$.

Let us now recall the following definition from [13].

Definition 1. Let (X, τ) and (Y, τ_Y) be respectively an ordinary topological space and an fts. We say that $F : X \to Y$ is a fuzzy multifunction if corresponding to each $x \in X$, $F(x)$ is a unique fuzzy set in Y.

Henceforth by $F : X \to Y$ we shall mean a fuzzy multifunction in the above sense.

Definition 2. (see [13] and [9]) For a fuzzy multifunction $F : X \to Y$, the upper inverse F^+ and lower inverse F^- are defined as follows:

For any fuzzy set A in Y, $F^+(A) = \{x \in X : F(x) \leq A\}$ and $F^-(A) = \{x \in X : F(x)qA\}$.

There is a following relationship between the upper and the lower inverses of a fuzzy multifunction.

Theorem 3. (see [9]) *For a fuzzy multifunction* $F : X \to Y$, *we have* $F^-(1 - A) = X - F^+(A)$, *for any fuzzy set* A *in* Y.

We now recall the following two definitions for ready references.

Definition 4. (see [3]) A fuzzy multifunction $F : X \to Y$ is said to be:

(a) fuzzy upper weakly quasi continuous (f.u.w.q.c., for short) (resp. fuzzy upper quasi continuous, f.u.q.c., for short) at a point $x \in X$ if for each open set U in X containing x and each fuzzy open set V in Y containing $F(x)$, there exists a non-empty open set G in X such that $G \subseteq U$ and $F(G) \leq clV$ (resp. $F(G) \leq V$),

(b) fuzzy lower weakly quasi continuous (f.l.w.q.c., for short) (resp. fuzzy lower quasi continuous, f.l.q.c., for short) at a point $x \in X$ if for each open set U in X containing x and each fuzzy open set V in Y with $F(x)qV$, there exists a non-empty open set G in X such that $G \subseteq U$ and $F(g)qclV$ (resp. $F(g)qV$), for each $g \in G$,

(c) f.u.w.q.c. (f.l.w.q.c.) on X if F has the corresponding property at each point x of X.

Definition 5. (see [2]) A fuzzy multifunction $F : X \to Y$ is said to be:

(a) fuzzy upper almost quasi continuous (f.u.a.q.c., for short) at a point $x \in X$, if for each open set U in X containing x and each fuzzy open set V in

Y containing $F(x)$, there exists a non-empty open set G in X such that $G \subseteq X$ and $F(G) \leq sclV$,

(b) fuzzy lower almost quasi continuous (f.l.a.q.c., for short) at a point $x \in X$, if for each open set U in X containing X and each fuzzy open set V in Y with $F(x)qV$, there exists a non-empty open set G in X such that $G \subseteq U$ and $F(g)qsclV$, for all $g \in G$,

(c)f.u.a.q.c. (f.l.a.q.c.) on X if F has the corresponding property at each point x of X.

We now recall the following two theorems from [2].

Theorem 6. *A fuzzy multifunction $F : X \to Y$ is f.u.a.q.c. iff $F^+(V) \in SO(X)$, for every fuzzy regular open set V in Y.*

Theorem 7. *A fuzzy multifunction $F : X \to Y$ is f.l.a.q.c. iff $F^-(V) \in SO(X)$, for every fuzzy regular open set V in Y.*

We recall the following two theorems from [2] for ready references.

Theorem 8. *A fuzzy multifunction $F : X \to Y$ is f.u.a.q.c. at a point x of X iff for any fuzzy open set V in Y containing $F(x)$, there exists $U \in SO(X)$ with $x \in U$ such that $F(U) \leq sclV$.*

Theorem 9. *A fuzzy multifunction $F : X \to Y$ is f.l.a.q.c. at a point x of X iff $F^-(V) \in SO(X)$, for every fuzzy regular open set V in Y.*

Definition 10. (see [10]) An fts X is said to be fuzzy almost regular if for each fuzzy point x_α in X and for each fuzzy regular open q-nbd U of x_α, there exists a fuzzy regular open q-nbd V of x_α such that $clV \leq U$.

Theorem 11. *If $F : X \to Y$ is f.l.w.q.c. and Y is fuzzy almost regular then F is f.l.a.q.c.*

Proof. Let V be a fuzzy regular open set in Y and $x \in F^-(V)$. Then $F(x)qV$. Since Y is fuzzy almost regular, there exists a fuzzy regular open set W in Y such that $F(x)qW \leq clW \leq V$. Since F is f.l.w.q.c., there exists a semiopen set U_x in X containing x such that $F(u)qclW$, for all $u \in U_x \Rightarrow F(u)qV$, for all $u \in U_x$. Therefore, we have $x \in U_x \subseteq F^-(V)$. This implies that $F^-(V) \in SO(X)$ and hence by Theorem 9, F is f.l.a.q.c.

Definition 12. A fuzzy multifunction $F : X \to Y$ is said to be fuzzy almost propen if $F(U) \leq int(cl(F(U)))$ for every $U \in SO(X)$.

Theorem 13. *If $F : X \to Y$ is fuzzy upper weakly quasi continuous and fuzzy almost preopen, then it is fuzzy upper almost quasi continuous.*

Proof. For any $x \in X$ and any fuzzy open set V of Y containing $F(x)$, there exists $U \in SO(X)$ with $x \in U$ such that $F(U) \leq clV$ (as F is f.u.w.q.c.). Since F is fuzzy almost preopen, $F(U) \leq int(cl(F(U))) \leq int(clV) = sclV$ (as V is fuzzy open in Y) [2]. Hence by Theorem 8, F is f.u.a.q.c.

Theorem 14. *Let $F : X \to Y$ be a fuzzy multifunction such that $F(x)$ is fuzzy open in Y for each $x \in X$. Then the following are equivalent:*

(a) F is fuzzy lower quasi continuous.

(b) F is fuzzy lower almost quasi continuous.

(c) F is fuzzy lower weakly quasi continuous.

Proof. (a) $\Rightarrow$ (b) $\Rightarrow$ (c) are obvious. We only show that (c) $\Rightarrow$ (a).

(c) $\Rightarrow$ (a) : Let $x \in X$ and V be a fuzzy open set in Y such that $F(x)qV$. Since F is f.l.w.q.c., there exists $U \in SO(X)$ containing x such that $F(u)qclV$ for every $u \in U$. Since $F(u)$ is fuzzy open in Y by the given condition, $F(u)qV$ for every $u \in U$ and hence (a) follows.

Remark 15. It is clear that f.u.a.q.c. (f.u.q.c.) $\Rightarrow$ f.u.w.q.c. and f.l.a.q.c. (f.l.q.c.) $\Rightarrow$ f.l.w.q.c., though the converses are not always true, as the following examples clarify.

Example 16. Let $X = \{a, b, c\}$, $Y = [0, 1]$, $\tau = \{\phi, X\}$, $\tau_Y = \{0_Y, 1_Y, A, B\}$ where $A(y) = 0.35$, $B(y) = .04$ for all $y \in Y$. Let $F : (X, \tau) \to (Y, \tau_Y)$ be a fuzzy multifunction defined by $F(a) = A$, $F(b) = B$, $F(c) = C$ where $C(y) = 0.6$, for all $y \in Y$. Now 1_Y and B are only non-null fuzzy regular open sets in (Y, τ_Y) and $F^+(B = \{x \in X : F(x) \leq B\} = \{a, b\} \notin SO(X)$ and so F is not f.u.a.q.c. by Theorem 6. Now $F^+(clB) = F^+(1 \setminus B) = \{x \in X : F(x) \leq 1 \setminus B\} = X$. Therefore $F^+(B) \subseteq cl(int(F^+(clB)))$. Now $F^+(A) = \{a\}$. Again $F^+(clA) = F^+(1 \setminus B) = X$ and so $F^+(A) \subseteq cl(int(F^+(clA))$. Hence F is f.l.w.q.c.
Since F is not f.u.a.q.c. it is not f.u.q.c.

Example 17. Let $X = \{a, b, c\}$, $Y = [0, 1]$, $\tau = \{\phi, X, \{a\}\}$, $\tau_Y = \{0_Y, 1_Y, B\}$ where $B(y) = 0.4$, for all $y \in Y$. Then 1_Y and B are the only fuzzy regular open sets in (Y, τ_Y). let $F : (X, \tau) \to (Y, \tau_Y)$ be a fuzzy multifunction defined by $F(a) = A$, $F(b) = B$, $F(c) = C$ where $A(y) = 0.45$, $C(y) = 0.61$, for all $y \in Y$. Now $F^-(B) = \{x \in X : F(x)qB\} = \{c\} \notin SO(X)$. Hence F is not f.l.a.q.c. by Theorem 7. But $F^-(clB) = F^-(1 \setminus B) = \{x \in X : F(x)q(1 \setminus B)\} = \{a, c\}$. Therefore, $cl(int(F^-(clB))) = X$ and hence F is f.l.w.q.c.
Since F is not f.l.a.q.c. it is not f.l.q.c.

We now recall the following definition from [6] for ready reference.

Definition 18. A fuzzy multifunction $F : X \to Y$ is called:

(a) fuzzy upper θ-continuous (f.u.θ.c., for short) at some point $x_0 \in X$ iff for every fuzzy open set V in Y with $x_0 \in F^+(V)$, there exists an open nbd U of x_0 such that $clU \subseteq F^+(clV)$,

(b) fuzzy lower θ-continuous (f.l.θ.c., for short) at some point $x_0 \in X$ iff for every fuzzy open set V in Y with $x_0 \in F^-(V)$, there exists an open nbd U of x_0 such that $clU \subseteq F^-(clV)$.

We now recall following two theorems from [6].

Theorem 19. *For a fuzzy multifunction $F : X \to Y$, the following are equivalent:*

(a) F is f.l.θ.c. on X, for each fuzzy open set V in Y, $F^-(V) \subseteq \theta - int(F^-(clV))$,

(c) $\theta - cl(F^+(intV)) \subseteq F^+(V)$, for any fuzzy closed set V in Y.

Theorem 20. *For a fuzzy multifunction $F : X \to Y$, the following are equivalent:*

(a) F is f.u.θ.c. on X,

(b) for each fuzzy open set V in Y, $F^+(V) \subseteq \theta - int(F^+(clV))$,

(c) $\theta - int(F^-(intV)) \subseteq F^-(V)$, for any fuzzy closed set V in Y.

We now recall following two theorems from [3] for ready references.

Theorem 21. *For a fuzzy multifunction*

$$F : X \to Y, \quad F^+(G) \subseteq cl(int(F^+(clG))),$$

for every fuzzy open set G in Y.

Theorem 22. *For a fuzzy multifunction*

$$F : X \to Y, \quad F^-(G) \subseteq cl(int(F^-(clG))),$$

for every fuzzy open set G of Y.

Remark 23. It is clear from Definition 18 and Definition 4 that f.u.θ.c. $\Rightarrow$ f.u.w.q.c. and f.l.θ.c. $\Rightarrow$ f.l.w.q.c.. But the converses may not be true, in general, as the following examples clarify.

Example 24. Let $X = \{a,b,c\}$, $Y = [0,1]$, $\tau = \{\phi, X, \{a\}, \{c\}, \{a,c\}\}$, $\tau_Y = \{0_Y, 1_Y, A, B\}$ where $A(y) = 0.35$, $B(y) = 0.4$, for all $y \in Y$. Let $F : (X,\tau) \to (Y,\tau_Y)$ be a fuzzy multifunction defined by $F(a) = A$, $F(b) = B$, $F(c) = C$ where $C(y) = 0.61$, for all $y \in Y$. Now closed sets in X are ϕ, X, $\{b\}$, $\{b,c\}$, $\{a,b\}$. Now $F^+(A) = \{x \in X : F(x) \leq A\} = \{a\}$. Now $clA = 1 \setminus B$. Therefore $F^+(clA) = F^+(1 \setminus B) = X \setminus F^-(B)$. Now $F^-(B) = \{x \in X : F(x)qB\} = \{c\}$ Therefore $F^+(clA) = X \setminus \{c\} = \{a,b\}$. Therefore, $cl(int(F^+(clA))) = cl(int(\{a,b\})) = cl(\{a\}) = \{a,b\}$. Therefore $F^+(A) \subseteq cl(int(F^+(clA)))$.

$F^+(B) = \{x \in X : F(x) \leq B\} = \{a,b\}$. $F^+(clB) = F^+(1 \setminus B) = \{a,b\}$ (by above). Therefore, $cl(int(F^+(clB))) = \{a,b\}$. Therefore, $F^+(B) \subseteq cl(int(F^+(clB)))$. Hence F is f.u.w.q.c. by Theorem 21.

But $\theta - int(F^+(clB)) = \theta - int(\{a,b\} \not\ni b$. Indeed, the only open nbd of b in X is X and so $clX = X \nsubseteq \{a,b\}$. Therefore, $F^+(B) \nsubseteq \theta - int(F^+(clB))$. Hence F is not f.u.θ.c. by Theorem 20.

Example 25. Let $X = \{a,b,c\}$, $Y = [0,1]$, $\tau = \{\phi, X, \{a\}$, $\tau_Y = \{0_Y, 1_Y, B\}$ where $B(y) = 0.4$, for all $y \in Y$. Let $F : (X,\tau) \to (Y,\tau_Y)$ be a fuzzy multifunction defined by $F(a) = A$, $F(b) = B$, $F(c) = C$ where $A(y) = 0.45$, $C(y) = 0.61$, for all $y \in Y$. Now $F^-(B) = \{x \in X : F(x)qB\} = \{c\}$ and $cl(int(F^-(clB))) = cl(int(F^-(1 \setminus B))) = cl(int(X \setminus F^+(B)))$. Now $F^+(B) = \{x \in X : F(x) \leq B\} = \{b\}$. Therefore, $cl(int(F^-(clB))) = cl(int(X \setminus \{b\}))) = cl(int(\{a,c\})) = cl(\{a\}) = X$. $F^-(B) \subseteq cl(int(F^-(clB)))$. Hence F is f.l.w.q.c. by Theorem 21.

But $\theta - int(F^-(clB)) = \theta - int(\{a,c\}) \not\ni c$. Indeed, X is the only one nbd of c in X and so $clX = X \nsubseteq \{a,c\}$. Hence $F^-(B) \nsubseteq \theta - int(F^-(clB))$ where B is fuzzy open in Y. Hence F is not f.l.θ.c. by Theorem 19.

Remark 26. Fuzzy upper (lower) quasi continuity and fuzzy upper (lower) θ-continuity are independent notions, as the following four examples clarify.

Example 27. Let $X = \{a,b,c\}$, $Y = [0,1]$, $\tau = \{\phi, X, \{a\}, \{c\}, \{a,c\}\}$, $\tau_Y = \{0_Y, 1_Y, A, B\}$ where $A(y) = 0.35$, $B(y) = 0.4$, for all $y \in Y$. Let $F : (X,\tau) \to (Y,\tau_Y)$ be a fuzzy multifunction defined by $F(a) = A$, $F(b) = B$, $F(c) = C$ where $C(y) = 0.61$, for all $y \in Y$. Here F is not f.u.θ.c. But F is f.u.q.c. Indeed, $F^+(A) = \{a\} \subseteq cl(int(F^+(A))) = cl(int(\{a\})) = cl(\{a\} = \{a,b\}$. And $F^+(B) = \{a,b\} = cl(int(\{a,b\})) = cl(\{a\}) = \{a,b\}$.

Example 28. Let $X = \{a,b,c\}$, $Y = [0,1]$, $\tau = \{\phi, X\}$, $\tau_Y = \{0_Y, 1_y, A, B\}$ where $A(y) = 0.35$, $B(y) = 0.4$, for all $y \in Y$. Let $F : (X,\tau) \to (Y,\tau_Y)$ be a fuzzy multifunction defined by $F(a) = A$, $F(b) = B$, $F(c) = C$ where $C(y) = 0.6$, for all $y \in Y$. Here it is clear that F is f.u.θ.c. But F is not f.u.q.c.

In fact, $F^+(B) = \{a, b\}$ and $cl(int(F^+(B))) = cl(int(\{a, b\})) = \phi$. Therefore, $F^+(B) \not\subseteq cl(int(F^+(B)))$ though B is fuzzy open in Y.

Example 29. Consider Example 25. We see that F is f.l.q.c. As $F^-(A) = \phi$ and hence $F^-(A) \subseteq cl(int(F^-(A)))$. Also $F^-(B) = \{c\}$ and $cl(int(F^-(B))) = cl(int(\{c\})) = cl(\{c\}) = \{b, c\}$. Therefore, $F^-(B) \subseteq cl(int(F^-(B)))$. But F is not f.l.θ.c. Since $F^-(clB) = F^-(1 \setminus B) = X \setminus F^+(B) = X \setminus \{a, b\} = \{c\}$. Therefore, $\theta - int(F^-(clB)) = \theta - int(\{c\}) \ni c$ (Indeed, $\{c\}, \{a, c\}, X$ are the only open nbds of $\{c\}$ and $cl(\{c\}) = \{b, c\} \not\subseteq \{c\}$, $cl(\{a, c\}) = X \not\subseteq \{c\}$ and $clX = X \not\subseteq \{c\}$). Hence the result.

Example 30. let $X = \{a, b, c\}$, $Y = [0, 1]$, $\tau = \{\phi, X, \{a, c\}, \{b\}\}$, $\tau_Y = \{0_Y, 1_Y, B\}$ where $B(y) = 0.4$, for all $y \in Y$. Let $F : (X, \tau) \to (Y, \tau_Y)$ be a fuzzy multifunction defined by $F(a) = A$, $F(b) = B$, $F(c) = C$ where $A(y) = 0.45$, $C(y) = 0.61$, for all $y \in Y$. Now $F^-(B) = \{x \in X : F(x)qB\} = \{c\}$ and $cl(int(F^-(B))) = cl(int(\{c\})) = \phi$. Therefore, $F^-(B) \not\subseteq cl(int(F^-(B)))$ and hence F is not f.l.q.c. But $F^-(clB) = F^-(1 \setminus B) = X \setminus F^+(B) = X \setminus \{b\} = \{a, c\}$. and $c \in \theta - int(F^-(clB))$ as $\{a, c\}$ is an open nbd of c such that $cl(\{a, c\}) = \{a, c\} = F^-(clB)$. Therefore, $F^-(B) \subseteq \theta - int(F^-(clB))$ and hence F is f.l.θ.c.

Remark 31. Fuzzy upper (lower) θ-continuity and fuzzy upper (lower) almost quasi continuity are independent notions.

Example 32. Let $X = \{a, b, c\}$, $Y = [0, 1]$, $\tau = \{\phi, X\}$, $\tau_Y = \{0_Y, 1_Y, A, B\}$ where $A(y) = 0.35$, $B(y) = 0.4$, for all $y \in Y$. Let $F : (X, \tau) \to (Y, \tau_Y)$ be a fuzzy multifunction defined by $F(a) = A$, $F(b) = B$, $F(c) = C$ where $C(y) = 0.61$, for all $y \in Y$. Here F is not f.u.a.q.c. But $F^+(A) = \{a\}$ and $\theta - int(F^+(clA)) = \theta - int(F^+(1 \setminus B)) = \theta - int(X \setminus F^-(B)) = \theta - int(X \setminus \{c\}) = \theta - int(\{a, b\}) = \phi$ and therefore $F^+(A) \not\subseteq \theta - int(F^+(clA))$. Hence F is not f.u.θ.c.

Example 33. Consider Example 24. Here F is not f.u.θ.c. Here B is the only non-null fuzzy regular open set in Y other than 1_Y. $F^+(B) = \{a, b\} \in SO(X)$ as $\{a\} \in \tau$ with $\{a\} \subseteq \{a, b\} \subseteq cl(\{a\}) = \{a, b\}$. Hence F is f.u.a.q.c.

Example 34. Consider Example 30. Here 1_Y and B are the only non-null fuzzy regular open sets in (Y, τ_Y). Now $F^-(B) = \{c\} \notin SO(X)$ and hence F is not f.l.a.q.c. But $F^-(clB) = F^-(1 \setminus B) = X \setminus F^+(B) = X \setminus \{b\} = \{a, c\}$. Now $\{a, c\}$ is an open nbd of c and $cl(\{a, c\}) = \{a, c\} = F^-(clB)$. Therefore, $c \in \theta - int(F^-(clB))$ and so $F^-(B) \subseteq \theta - int(F^-(clB))$. Hence F is f.l.θ.c.

Example 35. Let $X = \{a, b, c\}$, $Y = [0, 1]$, $\tau = \{\phi, X, \{a, c\}, \{c\}\}$, $\tau_Y = \{0_Y, 1_Y, B\}$ where $B(y) = 0.4$ for all $y \in Y$. Then B is the only regular open set in Y other than 0_Y and 1_Y. Let $F : (X, \tau) \to (Y, \tau_Y)$ be a fuzzy multifunction defined by $F(a) = A$, $F(b) = B$, $F(c) = C$ where $A(y) = 0.45$, $C(y) = 0.61$, for all $y \in Y$. Now $F^-(B) = \{x \in X : F(x)qB\} = \{c\} \in SO(X)$ and hence F is f.l.a.q.c. But $F^-(clB) = F^-(1 \setminus B) = X \setminus F^+(B) = X \setminus \{b\} = \{a, c\}$. Now $\{a, c\}$ and $\{c\}$ are the only proper open nbd of c in X with $cl(\{c\}) = X$ $= cl(\{a, c\})$ so that $X \not\subseteq \{a, c\}$ and hence $c \notin \theta - int(F^-(clB))$. Therefore, $F^-(B) \not\subseteq \theta - int(F^-(clB))$. Hence F is not f.l.θ.c.

Finally we have got the following diagram:

f.u.q.c. $\Rightarrow$ f.u.a.q.c. $\Rightarrow$ f.u.w.q.c. $\Leftarrow$ f.u.θ.c.

f.l.q.c. $\Rightarrow$ f.l.a.q.c. $\Rightarrow$ f.l.w.q.c. $\Leftarrow$ f.l.θ.c.

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