ON SUBSECTION OF A POSITIVELY SKEWED CURVE MEDIUM OF EM ON A HALF SPACE EARTH MODEL

Panyawat Haarsa¹ §, Supaporn Pothat²

¹Department of Mathematics
Srinakharinwirot University
Bangkok, 10110, THAILAND

²Wad Ban-Koh School
Bandara, Amphoe Pichai
Uttaradit, 53220, THAILAND

Abstract: The objective of this paper is to study the structure of the earth by constructing a half-space earth model with a positively skewed curve medium. The subsection of the ground is used mathematically to find the electric fields in each layer of the ground. Numerical solutions are computed to show graphically the behaviors of the electric field. The responses of electric field are obliged by a positively skewed curve medium ground profile.

AMS Subject Classification: 86A25

Key Words: electromagnetic field, subsection, positively skewed curve, Chave's algorithm, half space earth, Hankel transform

1. Introduction

There are diverse approaches used in exploration geophysics. The electromagnetic method is the most commonly used because it is much less cost than others. Besides, it responses best to a good electrical conductors at a shallow depth to as deep as many kilometers from the earth surface depending on frequency. Lee and Ignetik [3] considered the forward problem of the transient electromagnetic response of a half-space with an exponentially varying conduc-

Received: November 5, 2013

© 2014 Academic Publications, Ltd.
url: www.acadpubl.eu

§Correspondence author

tivity profiles. They indicated out that the conductivity variation of the ground may sometimes be reasonably approximated by an exponential variation since soil salinity profiles frequently show monotonically increasing or decreasing salt conductivity of the ground. Yooyuanyong and Siew [4] gave the mathematical model of electromagnetic response of a disk beneath an exponentially varying conductive overburden.

The goal of this paper is to present mathematical model and techniques for studying the structure of the earth's surface layer. We consider the ground having the conductivity which is given by $\sigma(z) = (\sigma_0 + z)e^{-bz/2}$, where σ_0 a is positive constant, b is constant. The conductivity ground profile in this paper is unlike to the model used by Yooyuanyong and Siew [4] but similar to the model used by Haarsa and Pothat [6] with different method to approach the solution of electric field.

2. Formulation and Solution of the Problem

Firstly, we consider a cylindrical polar coordinates (r, θ, z) which is introduced with $z > 0$ and taken vertically positive downward. The origin under the center of the horizontal circular loop is used. The azimuthally symmetry gives merely electric field E_θ , and magnetic fields H_r , and H_z components. These field quantities are found to satisfy Maxwell's equations Hohmann and Raiche [2] in the form of equations

$$i\omega\mu H_r = -\frac{\partial E}{\partial z}, \quad (1)$$

$$i\omega\mu H_z = \frac{1}{r} \frac{\partial(rE)}{\partial r}, \quad (2)$$

and

$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = (\sigma(z) - i\omega\varepsilon)E + J_s, \quad (3)$$

where $i = \sqrt{-1}$ is an imaginary number, $J_s = aI(\omega)\delta(r-a)\delta(z+h)/r$ is the source current density, ω is the angular frequency, δ is the delta function, $\sigma(z)$ is the electrical conductivity of medium depending on depth only, ε is the electric permittivity of medium, μ is the magnetic permeability of medium, and $I(\omega)$ is the current in a coil of a small radius a . The equations (1) - (3) can be solved to find the differential equation for electric fields as.

$$i\omega\mu J_s = -\frac{\partial^2 E}{\partial z^2} - \frac{\partial^2 E}{\partial r^2} - \frac{1}{r} \frac{\partial E}{\partial r} + \frac{E}{r^2} - (i\omega\mu\sigma(z) + \omega^2\mu\varepsilon)E. \quad (4)$$

Taking Hankel transform which is defined as

$$\tilde{E}(\lambda, z, \omega) = \int_0^{\infty} r J_1(\lambda r) E(r, z, \omega) dr,$$

where J_1 is Bessel function of the first kind of order 1, and equation (4) becomes

$$i\omega\mu a I(\omega)\delta(z+h)J_1(\lambda a) = -\frac{\partial^2 \tilde{E}}{\partial z^2} + (\lambda^2 - (i\omega\mu\sigma(z) + \omega^2\mu\varepsilon)\tilde{E}. \quad (5)$$

We next consider the ground having a positively skewed curve conductivity profile with depth denoted by $\sigma(z) = (\sigma_0 + z)e^{-bz/2}$, where σ_0 is a positive constant, b is constant. We consider a primary alternating source current carried by a coil of radius a , at $z = -h$ above the surface of the earth $z = 0$.

The electric field in air can be denoted by $\tilde{E}_{air}(\lambda, z, \omega)$ and expressed as the sum of two components,

$$\tilde{E}_{air}(\lambda, z, \omega) = \tilde{E}^p(\lambda, z, \omega) + \tilde{E}^s(\lambda, z, \omega),$$

where $\tilde{E}^p(\lambda, z, \omega)$ is the primary field and $\tilde{E}^s(\lambda, z, \omega)$ is the secondary field. Both electric fields can be obtained from equation (5). In air, $\sigma_{air}(z) \cong 0$ and the electric field is given by

$$\tilde{E}_{air}(\lambda, z, \omega) = \frac{i\omega\mu_0 a I(\omega) J_1(\lambda a) e^{-m_0|z+h|}}{2\lambda_0} + F e^{\lambda_0 z}, \quad z \leq 0 \quad (6)$$

which remains bounded as $z \rightarrow -\infty$, and F is arbitrary constant to be determined.

In ground, we can obtain the partial differential equation for the electric field as

$$\begin{aligned} & \frac{\partial^2 E_{gro}(r, z, \omega)}{\partial z^2} + \frac{\partial^2 E_{gro}(r, z, \omega)}{\partial r^2} \\ & + \frac{1}{r} \frac{\partial E_{gro}(r, z, \omega)}{\partial r} - \frac{E_{gro}(r, z, \omega)}{r^2} + k_g^2 E_{gro}(r, z, \omega) = 0. \end{aligned} \quad (7)$$

where $E_{gro}(r, z, \omega)$ is the electric field in ground, $k_g^2 = i\omega\mu_g\sigma(z) + \omega^2\mu_g\varepsilon_g$, μ_g is the magnetic permeability of ground, ε_g is the electric permittivity of ground and $\sigma(z) = (\sigma_0 + z)e^{-bz/2}$ is the conductivity of ground. Taking Hankel transform to equation (7), we obtain

$$\frac{d^2 \tilde{E}_{gro}(\lambda, z, \omega)}{dz^2} - m_g^2 \tilde{E}_{gro}(\lambda, z, \omega) = 0, \quad (8)$$

Using the exponential substitution $\xi = e^{-bz/2}$ to equation (8). We obtain

$$\xi^2 \frac{d^2 \tilde{E}_{gro}}{d\xi^2} + \xi \frac{d\tilde{E}_{gro}}{d\xi} - \frac{4}{b^2} \lambda^2 \tilde{E}_{gro} = -\frac{4}{b^2} k_g^2 \tilde{E}_{gro}, \quad (9)$$

Equation (9) is a non-homogeneous differential equation, and it can be solved for a homogeneous equation by Cauchy-Euler equation method and a particular solution by the method of variation of parameters. Hence, the solution of equation (9) is

$$\tilde{E}_{gro}(\lambda, \xi, \omega) = C_1 \xi^\psi + C_2 \xi^{-\psi} + \frac{i2\omega\mu_g}{\psi b^2} \int_0^1 G(\xi; \tau) R(\tau) d\tau, \quad (10)$$

where $\psi = 2\lambda/b$, $R(\tau) = [\sigma_0 - (2/b) \ln \tau] \tilde{E}(\lambda, \tau, \omega)$, and $G(\xi; \tau)$ is the Green's function. We calculate the integration term of equation (10) by using method of subsection. Therefore, equation (10) becomes

$$\tilde{E}_{gro}(\lambda, \xi, \omega) = C_1 \xi^\psi + C_2 \xi^{-\psi} + \frac{i\omega\mu_g d_1}{\psi b} \sum_{j=1}^n G_j(\xi; \xi_j) R_j(\xi_j)(\xi_j), \quad (11)$$

where C_1, C_2 are arbitrary constants to be determined from the boundary conditions.

Under the condition $z \rightarrow \infty$, we have $\tilde{E}_{gro}(\lambda, z, \omega) = 0$. Hence, we require $C_2 = 0$. Therefore, equation (11) becomes

$$\tilde{E}_{gro}(\lambda, \xi, \omega) = C_1 \xi^\psi + \frac{i\omega\mu_g d_1}{\psi b} \sum_{j=1}^n G_j(\xi; \xi_j) R_j(\xi_j)(\xi_j), \quad (12)$$

We can find constants F from equation (6) and C_1 from equation (12) by imposing the continuity of $\tilde{E}$ and $\frac{\partial \tilde{E}}{\partial z}$ at air-earth interface. That is,

$$\tilde{E}_{air}(\lambda, z, \omega) = \tilde{E}_{gro}(\lambda, z, \omega),$$

and

$$\frac{\partial \tilde{E}_{air}(\lambda, z, \omega)}{\partial z} = \frac{\partial \tilde{E}_{gro}(\lambda, z, \omega)}{\partial z}.$$

By using inverse Hankel transform, we obtain the electric field in air as

$$E_{air}(r, z, \omega)$$

$$\begin{aligned}
&= \frac{i\omega\mu_0}{2} \int_0^\infty aI(\omega)J_1(\lambda a)e^{-\lambda|z+h|}J_1(\lambda r)d\lambda \\
&\quad + \frac{i\omega\mu_0}{2} \int_0^\infty d_1e^{\lambda z} \sum_{j=1}^n sp(s, \xi_j, \omega)(\sigma_0 + z_j)e^{-\frac{(2\lambda z_j + bz_j)}{2}}J_1(\lambda r)d\lambda, \quad (13)
\end{aligned}$$

and the electric field in ground as

$$\begin{aligned}
&E_{gro}(r, z, \omega) \\
&= -\omega^2\mu_0^2aI(\omega) \int_0^\infty \lambda \left(\frac{e^{-\lambda|z+h|}d_1 \sum_{j=1}^n (\sigma_0 + z_j)e^{-\frac{(2\lambda z_j + bz_j)}{2}}}{\sqrt{2}\lambda^2 + [\omega\mu_g d_1 \sum_{j=1}^n sp(s, \xi_j, \omega)e^{-\frac{(2\lambda z_j + bz_j)}{2}}]^2} \right) \\
&\quad \times J_1(\lambda a)J_1(\lambda r)d\lambda \\
&\quad + i2\omega\mu_0aI(\omega) \int_0^\infty \left(\frac{\lambda^2 e^{-\lambda|z+h|}}{\sqrt{2}\lambda^2 + [\omega\mu_g d_1 \sum_{j=1}^n sp(s, \xi_j, \omega)e^{-\frac{(2\lambda z_j + bz_j)}{2}}]^2} \right) \\
&\quad \times J_1(\lambda a)J_1(\lambda r)d\lambda, \quad (14)
\end{aligned}$$

By partitioning on the ground, we can rewrite equation (14) in the matrix form, we now have

$$\left[B_{ij} + \frac{\delta_{ij}}{S_j} \right]_{m \times m} [p(s_i, \xi_j, \omega)]_{m \times m} = [R_{ij}]_{m \times m} \quad (15)$$

where $p(s_i, \xi_j, \omega) = S_j E_{jgro}(s_i, \xi_j, \omega)$, $S_j = d_1(\sigma_0 + z_j)e^{-\frac{bz_j}{2}}$, and E_{jgro} and d_1 are the electric field and the width of each ground layer, respectively. The electric field on the ground surface can be determined from equations (13) or equation (14) by taking $z = 0$. Chave [1] is used for numerical calculating the inverse Hankel transform of the electric field solutions.

3. Conclusions

In this paper, we present the approach to study the structure of the earth's surface layer. The forward problem is produced by the integral equations. We formulated the problem to get the electric fields, which could be used to find

Figure 1: Graph of real and imaginary part of electric field E versus r for a half space earth model with a positively skewed curve medium by inputting $\sigma_0 = 0.01 S m^{-1}$, $b_1 = 0.1 m^{-1}$, $d_1 = 1.5 m$, frequency = 200 Hz.

Figure 2: Graph of real and imaginary part of electric field E versus r for a half space earth model with a positively skewed curve medium by inputting $\sigma_0 = 0.01 S m^{-1}$, $b_1 = 0.1 m^{-1}$, $d_1 = 3.5 m$, frequency = 200 Hz.

Figure 3: Graph of real and imaginary part of electric field E versus r for a half space earth model with a positively skewed curve medium by inputting $\sigma_0 = 0.01 \text{ S m}^{-1}$, $b_1 = 0.1 \text{ m}^{-1}$, $d_1 = 5.5 \text{ m}$, frequency = 50 Hz.

the electric fields on the ground surface. The Cauchy-Euler equation method and the method of variation of parameters have been used to attain a solution of a non-homogeneous differential equation. We regard a half-space model having a positively skewed curve conductivity profile which is denoted by $\sigma(z) = (\sigma_0 + z)e^{-bz/2}$, where σ_0 is a positive constant, b is constant. The electric field was expressed in term of mathematical expressions and plotted. The graphs are shown the behavior of the electric field against source-receiver spacing r . The values of b and σ_0 are fixed while the value of the width of each ground layer d_1 is varied. In the experiments, we set up the frequencies which depend on d_1 . The curves of the electric fields are similar to the shape of a positively skewed curve conductivity ground profile as shown in figures 1 – 3. The curves of both parts of electric field are oscillated and tend to zero as the source receiver spacing r increases which is caused by the ground medium profile. These can be used for studying the structure of the earth's surface layers and ground exploration. It is important to study the optimal values for all the parameters in the future work. More layered earth structures will be conducted as well as other mathematical approaches may be used to study the problems.

References

- [1] A.D. Chave, " Numerical integration of related Hankel transforms by

- quadrature and continued fraction expansion", *Geophysics* **48**(1983), 1671-1686.
- [2] G. W. Hohmann and A. P. Raiche, Inversion of controlled-Source Electromagnetic Data, *Electromagnetic methods in applied geophysics* Vol. 1(1988), 469-503.(ed. Nabighian, M. N.)SEG
- [3] T.J. Lee and R. Ignatik, Transient Electromagnetic Response of a half-space with Exponential Conductivity Profile and its Applications to Salinity Mapping, *Exploration Geophysics* **25**(1994), 39-51.
- [4] S. Yooyuanyong and P. F. Siew, The electromagnetic response of a disk beneath an exponentially varying conductive overburden, *J. Australian Mathematical Society Series B.* **41**(2000), E1-E28
- [5] P. Haarsa, Electromagnetic response on a half-space earth with a positively skewed bulge conductivity, *Int. J. Applied Math. Stat.*, **42(12)**(2013), 324-329.
- [6] P. Haarsa and S. Pothat, Mathematical model of electromagnetic response from a positively skewed bulge ground structure, *Applied Math. Sci.*, **7(101)**(2013), 5013-5023.
- [7] P. Haarsa and S. Pothat, On Taylor-Series Approximation of the Response of Electromagnetic Field on a Two Layered Earth Model with a Conductive Overburden, *Int. J. Pure Appl. Math.*, **89(5)**(2013), 699-709.