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GENERALIZED VAGUE SOFT EXPERT SET

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Abstract: The concept of generalized vague soft expert set and its operations are proposed. The basic operations of generalized vague soft expert set theory "AND" and "OR" operations are defined along with illustrative examples.

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1. Introduction

Molodtsov [1] mentioned a soft set as a mathematical way to represent and solve uncertainties. Roy and Maji [2] used this theory to solve some decision-making problems. Vague set theory was provided by Xu et al. [3]. Salleh et al. [4] and Alkhazaleh et al. [5], [6], [7] further extended studies on fuzzy soft sets. Alhazaymeh et al. [8] introduced the concept of soft intuitionistic fuzzy sets followed by the notions of vague soft sets [9], [10] and interval-valued vague soft sets [11], [12], [13]. Varnamkhasti and Hassan [14] and [15] later discussed and applied fuzzy sets to genetic algorithms. Adam and Hassan [16] ventured into multi Q-fuzzy parameterized soft sets. Hassan and Alhazaymeh

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[17] introduced vague soft expert set of which we are extending to generalized vague soft expert set. In this paper, we state the concept of generalized vague soft expert set and some operations with illustrative examples. Hence decision making are no longer exclusive to certainty data such as goal programming [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29] and data envelopment analysis [30], [31], [32].

2. Preliminaries

In this section, we recall some basic notions related to this work. Molodtsov [1] defined soft set in the following way. Let U be a universal set and E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$.

Definition 2.1. (see [1]) A pair (F, A) is called a soft set over U, where F is a mapping

$$F : A \to P(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A).

Definition 2.2. (see [33]) A pair (F, A) is called fuzzy soft expert set over U, where F is a mapping given by

$$F = A \to I^U$$

where I^U denotes the set of all fuzzy subsets of U.

Definition 2.3. (see [34]) A pair (F, A) is called generalized fuzzy soft expert set over U, where F is a mapping given by

$$F : A \to I^U \times I$$

where I^U denotes the set of all fuzzy subsets of U.

Definition 2.4. (see [9]) Let U be the universal set and E be the set of parameters. Let $A \subseteq E$ and $F : A \to V^U$ and α be a vague subset of A i.e. $\alpha : A \to [0, 1]$, where V^U the collection of all vague subset of U. Let $\tilde{F}_\alpha : A \to V^U \times [0, 1]$ be a function defined as follows:

$$\tilde{F}_\alpha(a) \;\; = \;\; (\tilde{F}(a) = \{x, t_{F(a)}, 1 - f_{F(a)}\}, \alpha(a)),$$

Then $\tilde{F}_\alpha$ is called the generalized vague soft set over (U, E).

3. Generalized Vague Soft Expert Set

In this section we define the concept of the generalized vague soft expert set and study some of its properties.
Let U be a universal set, E be a set of parameters, X a set of expert (agents), and $O = \{1= \text{agree}, 0= \text{disagree}\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subset Z$. Now we propose the definition of a generalized vague soft expert set and we give illustrative example on a generalized vague soft expert set.

Definition 3.1. A pair (F, A) is called generalized vague soft expert set over U, where F is a mapping given by

$$F = A \to V^U \times I$$

where V^U denotes the set of all vague subsets of U.
Here for each parameter e_i, $\tilde{F}_\mu(e_i) = (F(e_i), \mu(e_i))$ indicates not only the degree of belongingness of the elements of U in $\tilde{F}(e_i)$ but also the degree of preference of such belongingness which is represented by $\mu(e_i)$.

Example 3.1. Let $U = \{u_1, u_2, u_3\}$ be a set universe, and let $E = \{e_1, e_2, e_3\}$ a set of parameters. Let $Z = E \times X \times O$ and $A \subseteq Z$. Let μ be a fuzzy set of Z; that is, $\mu : Z \to I = [0, 1]$.
The function

$$F : A \to V^{(U)} \times I$$

is defined as follows:
$F(e_1, p, 1) = (\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.6,0.8\rangle}, \frac{u_3}{\langle 0,0\rangle}\}, 0.2)$, $F(e_1, q, 1) = (\{\frac{u_1}{\langle 0.3,0.6\rangle}, \frac{u_2}{\langle 0.9,0.9\rangle}, \frac{u_3}{\langle 1,1\rangle}\}, 0.5)$,
$F(e_1, r, 1) = (\{\frac{u_1}{\langle 0.4,0.8\rangle}, \frac{u_2}{\langle 0.2,0.2\rangle}, \frac{u_3}{\langle 0.9,0.9\rangle}\}, 0.2)$, $F(e_2, p, 1) = (\{\frac{u_1}{\langle 0.2,0.8\rangle}, \frac{u_2}{\langle 0.4,0.7\rangle}, \frac{u_3}{\langle 0.5,0.6\rangle}\}, 0.8)$,
$F(e_2, q, 1) = (\{\frac{u_1}{\langle 0.3,0.4\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.6,0.8\rangle}\}, 0.3)$, $F(e_2, r, 1) = (\{\frac{u_1}{\langle 0.1,0.1\rangle}, \frac{u_2}{\langle 0.8,0.9\rangle}, \frac{u_3}{\langle 0.3,0.3\rangle}\}, 0.6)$,
$F(e_3, p, 1) = (\{\frac{u_1}{\langle 0.3,0.6\rangle}, \frac{u_2}{\langle 0.6,0.6\rangle}, \frac{u_3}{\langle 0.5,0.5\rangle}\}, 0.9)$, $F(e_3, q, 1) = (\{\frac{u_1}{\langle 0.1,0.2\rangle}, \frac{u_2}{\langle 0.2,0.6\rangle}, \frac{u_3}{\langle 0.4,0.6\rangle}\}, 0.2)$,
$F(e_3, r, 1) = (\{\frac{u_1}{\langle 0.7,0.3\rangle}, \frac{u_2}{\langle 0.1,0.3\rangle}, \frac{u_3}{\langle 0.3,0.6\rangle}\}, 0.5)$, $F(e_1, p, 0) = (\{\frac{u_1}{\langle 0.1,0.1\rangle}, \frac{u_2}{\langle 0.8,0.9\rangle}, \frac{u_3}{\langle 0.3,0.3\rangle}\}, 0.6)$,
$F(e_1, q, 0) = (\{\frac{u_1}{\langle 0.3,0.4\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.6,0.8\rangle}\}, 0.8)$, $F(e_1, r, 0) = (\{\frac{u_1}{\langle 0.7,0.3\rangle}, \frac{u_2}{\langle 0.1,0.3\rangle}, \frac{u_3}{\langle 0.3,0.6\rangle}\}, 0.1)$
$F(e_2, p, 0) = (\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.6,0.8\rangle}, \frac{u_3}{\langle 0,0\rangle}\}, 0.3)$, $F(e_2, q, 0) = (\{\frac{u_1}{\langle 0.3,0.6\rangle}, \frac{u_2}{\langle 0.9,0.9\rangle}, \frac{u_3}{\langle 1,1\rangle}\}, 0.6)$,
$F(e_2, r, 0) = (\{\frac{u_1}{\langle 0.4,0.8\rangle}, \frac{u_2}{\langle 0.2,0.2\rangle}, \frac{u_3}{\langle 0.9,0.9\rangle}\}, 0.7)$, $F(e_3, p, 0) = (\{\frac{u_1}{\langle 0.3,0.6\rangle}, \frac{u_2}{\langle 0.6,0.6\rangle}, \frac{u_3}{\langle 0.5,0.5\rangle}\}, 0.9)$,
$F(e_3, q, 0) = (\{\frac{u_1}{\langle 0.1,0.2\rangle}, \frac{u_2}{\langle 0.2,0.6\rangle}, \frac{u_3}{\langle 0.4,0.6\rangle}\}, 0.6)$, $F(e_3, r, 0) = (\{\frac{u_1}{\langle 0.7,0.3\rangle}, \frac{u_2}{\langle 0.1,0.3\rangle}, \frac{u_3}{\langle 0.3,0.6\rangle}\}, 0.8)$.

Then we can view the generalized vague soft expert set (F, Z) as consisting of the following collection of approximations:

$(F_\mu, Z) = \{((e_1, p, 1), \{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.6,0.8\rangle}, \frac{u_3}{\langle 0,0\rangle}\}, 0.2)$,
$((e_1, q, 1), \{\frac{u_1}{\langle 0.3,0.6\rangle}, \frac{u_2}{\langle 0.9,0.9\rangle}, \frac{u_3}{\langle 1,1\rangle}\}, 0.5), ((e_1, r, 1), \{\frac{u_1}{\langle 0.4,0.8\rangle}, \frac{u_2}{\langle 0.2,0.2\rangle}, \frac{u_3}{\langle 0.9,0.9\rangle}\}, 0.2)$,
$((e_2, p, 1), \{\frac{u_1}{\langle 0.2,0.8\rangle}, \frac{u_2}{\langle 0.4,0.7\rangle}, \frac{u_3}{\langle 0.5,0.6\rangle}\}, 0.8), ((e_2, q, 1), \{\frac{u_1}{\langle 0.3,0.4\rangle}, \frac{u_2}{\langle 0.4,0.5\rangle}, \frac{u_3}{\langle 0.6,0.8\rangle}\}, 0.3)$,
$((e_2, r, 1), \{\frac{u_1}{\langle 0.1,0.1\rangle}, \frac{u_2}{\langle 0.8,0.9\rangle}, \frac{u_3}{\langle 0.3,0.4\rangle}\}, 0.6)\}, ((e_3, p, 1), \{\frac{u_1}{\langle 0.3,0.6\rangle}, \frac{u_2}{\langle 0.6,0.6\rangle}, \frac{u_3}{\langle 0.5,0.5\rangle}\}, 0.9)$,
$((e_3, q, 1), \{\frac{u_1}{\langle 0.1,0.2\rangle}, \frac{u_2}{\langle 0.2,0.6\rangle}, \frac{u_3}{\langle 0.4,0.6\rangle}\}, 0.2), ((e_3, r, 1), \{\frac{u_1}{\langle 0.7,0.3\rangle}, \frac{u_2}{\langle 0.1,0.3\rangle}, \frac{u_3}{\langle 0.3,0.6\rangle}\}, 0.5)$,

$((e_1,p,0),\{\frac{u_1}{\langle 0.1,0.1\rangle},\frac{u_2}{\langle 0.8,0.9\rangle},\frac{u_3}{\langle 0.3,0.3\rangle}\},0.6),((e_1,q,0),\{\frac{u_1}{\langle 0.3,0.4\rangle},\frac{u_2}{\langle 0.4,0.5\rangle},\frac{u_3}{\langle 0.6,0.8\rangle}\},0.8),$
$((e_1,r,0),\{\frac{u_1}{\langle 0.3,0.7\rangle},\frac{u_2}{\langle 0.1,0.3\rangle},\frac{u_3}{\langle 0.3,0.6\rangle}\},0.1),((e_2,p,0),\{\frac{u_1}{\langle 0.5,0.5\rangle},\frac{u_2}{\langle 0.6,0.8\rangle},\frac{u_3}{\langle 0,0\rangle}\},0.3),$
$((e_2,q,0),\{\frac{u_1}{\langle 0.3,0.6\rangle},\frac{u_2}{\langle 0.9,0.9\rangle},\frac{u_3}{\langle 1,1\rangle}\},0.6),((e_2,r,0),\{\frac{u_1}{\langle 0.4,0.8\rangle},\frac{u_2}{\langle 0.2,0.2\rangle},\frac{u_3}{\langle 0.9,0.9\rangle}\},0.7),$
$((e_3,p,0),\{\frac{u_1}{\langle 0.3,0.6\rangle},\frac{u_2}{\langle 0.6,0.6\rangle},\frac{u_3}{\langle 0.5,0.5\rangle}\},0.9),((e_3,q,0),\{\frac{u_1}{\langle 0.1,0.2\rangle},\frac{u_2}{\langle 0.2,0.6\rangle},\frac{u_3}{\langle 0.4,0.6\rangle}\},0.6),$
$((e_3,r,0),\{\frac{u_1}{\langle 0.7,0.3\rangle},\frac{u_2}{\langle 0.1,0.3\rangle},\frac{u_3}{\langle 0.3,0.6\rangle}\},0.8)\}.$

Definition 3.2. For two generalized vague soft expert sets (F_μ, A) and (G_δ, B) over U, (F_μ, A) is called a generalized vague soft expert subset of (G_δ, B) if

a. $B \subseteq A$, and

b. $\forall \varepsilon \in A, G_\delta(\varepsilon)$ is a generalized vague subset of $F_\mu(\varepsilon)$.

Definition 3.3. If (F_μ, A) and (G_δ, B) are two generalized vague soft expert sets over U, then "(F_μ, A) AND (G_δ, B)" denoted by $(F_\mu, A) \wedge (G_\delta, B)$ is defined by

$$(F_\mu, A) \wedge (G_\delta, B) = (H_\Omega, A \times B)$$

such that $H(\alpha,\beta) = F(\alpha)\tilde{\cap}G(\beta)$, for all $(\alpha,\beta) \in A \times B$, where $\tilde{\cap}$ is a generalized vague intersection.

Example 3.2. Consider Example 3.1. Let

$$\left(\tilde{F}, A\right) = \{(e_1,p,1),(e_3,p,1),(e_3,r,1),(e_2,p,0)\}$$

and $\left(\tilde{G}, B\right) = \{(e_1,p,1),(e_3,p,1),(e_2,p,0)\}$.

Suppose $\left(\tilde{F}_\mu, A\right)$ and $\left(\tilde{G}_\delta, B\right)$ are two generalized vague soft expert sets over U such that

$$\left(\tilde{F}, A\right) = \left\{ \left((e_1,p,1), \left\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.6,0.8\rangle}, \frac{u_3}{\langle 0,0\rangle}\right\}, 0.2\right), \right.$$
$$\left((e_3,p,1), \left\{\frac{u_1}{\langle 0.3,0.6\rangle}, \frac{u_2}{\langle 0.6,0.6\rangle}, \frac{u_3}{\langle 0.5,0.5\rangle}\right\}, 0.9\right),$$
$$\left((e_3,r,1), \left\{\frac{u_1}{\langle 0.7,0.3\rangle}, \frac{u_2}{\langle 0.1,0.3\rangle}, \frac{u_3}{\langle 0.3,0.6\rangle}\right\}, 0.5\right),$$
$$\left.\left((e_2,p,0), \left\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.6,0.8\rangle}, \frac{u_3}{\langle 0,0\rangle}\right\}, 0.3\right) \right\},$$

and

$$\left(\tilde{G},B\right)=\Bigg\{\left(\left(e_1,p,1\right),\left\{\frac{u_1}{\langle 0.4,0.6\rangle},\frac{u_2}{\langle 0.7,0.7\rangle},\frac{u_3}{\langle 1,1\rangle}\right\},0.3\right),$$
$$\left(\left(e_3,p,1\right),\left\{\frac{u_1}{\langle 0.4,0.4\rangle},\frac{u_2}{\langle 0.7,0.3\rangle},\frac{u_3}{\langle 0.4,0.7\rangle}\right\},0.7\right),$$
$$\left(\left(e_2,p,0\right),\left\{\frac{u_1}{\langle 0.4,0.7\rangle},\frac{u_2}{\langle 0.6,0.7\rangle},\frac{u_3}{\langle 0,0\rangle}\right\},1\right)\Bigg\}.$$

Then

$$\left(\tilde{F}_\mu,A\right)\wedge\left(\tilde{G}_\delta,B\right)=\left(\tilde{H}_\Omega,A\times B\right)$$
$$=\Bigg\{\left(\left(\left(e_1,p,1\right),\left(e_1,p,1\right)\right),\left\{\tfrac{u_1}{\langle 0.4,0.6\rangle},\tfrac{u_2}{\langle 0.6,0.8\rangle},\tfrac{u_3}{\langle 0,1\rangle}\right\},0.2\right),$$
$$\left(\left(\left(e_1,p,1\right),\left(e_3,p,1\right)\right),\left\{\tfrac{u_1}{\langle 0.4,0.5\rangle},\tfrac{u_2}{\langle 0.7,0.8\rangle},\tfrac{u_3}{\langle 0,0.7\rangle}\right\},0.2\right),$$
$$\left(\left(\left(e_1,p,1\right),\left(e_2,p,0\right)\right),\left\{\tfrac{u_1}{\langle 0.4,0.7\rangle},\tfrac{u_2}{\langle 0.6,0.8\rangle},\tfrac{u_3}{\langle 0,0\rangle}\right\},0.2\right),$$
$$\left(\left(\left(e_3,p,1\right),\left(e_1,p,1\right)\right),\left\{\tfrac{u_1}{\langle 0.3,0.6\rangle},\tfrac{u_2}{\langle 0.6,0.6\rangle},\tfrac{u_3}{\langle 0.5,1\rangle}\right\},0.3\right),$$
$$\left(\left(\left(e_3,p,1\right),\left(e_3,p,1\right)\right),\left\{\tfrac{u_1}{\langle 0.3,0.6\rangle},\tfrac{u_2}{\langle 0.6,0.6\rangle},\tfrac{u_3}{\langle 0.4,0.7\rangle}\right\},0.7\right),$$
$$\left(\left(\left(e_3,p,1\right),\left(e_2,p,0\right)\right),\left\{\tfrac{u_1}{\langle 0.3,0.7\rangle},\tfrac{u_2}{\langle 0.6,0.7\rangle},\tfrac{u_3}{\langle 0,0.5\rangle}\right\},0.9\right),$$
$$\left(\left(\left(e_3,r,1\right),\left(e_1,p,1\right)\right),\left\{\tfrac{u_1}{\langle 0.4,0.6\rangle},\tfrac{u_2}{\langle 0.1,0.7\rangle},\tfrac{u_3}{\langle 0.3,1\rangle}\right\},0.3\right),$$
$$\left(\left(\left(e_3,r,1\right),\left(e_3,p,1\right)\right),\left\{\tfrac{u_1}{\langle 0.4,0.4\rangle},\tfrac{u_2}{\langle 0.1,0.3\rangle},\tfrac{u_3}{\langle 0.3,0.7\rangle}\right\},0.5\right),$$
$$\left(\left(\left(e_3,r,1\right),\left(e_2,p,0\right)\right),\left\{\tfrac{u_1}{\langle 0.4,0.7\rangle},\tfrac{u_2}{\langle 0.1,0.7\rangle},\tfrac{u_3}{\langle 0,0.6\rangle}\right\},0.5\right),$$
$$\left(\left(\left(e_2,p,0\right),\left(e_1,p,1\right)\right),\left\{\tfrac{u_1}{\langle 0.4,0.6\rangle},\tfrac{u_2}{\langle 0.6,0.8\rangle},\tfrac{u_3}{\langle 0,1\rangle}\right\},0.3\right),$$
$$\left(\left(\left(e_2,p,0\right),\left(e_3,p,1\right)\right),\left\{\tfrac{u_1}{\langle 0.4,0.5\rangle},\tfrac{u_2}{\langle 0.6,0.8\rangle},\tfrac{u_3}{\langle 0,0.7\rangle}\right\},0.3\right),$$
$$\left(\left(\left(e_2,p,0\right),\left(e_3,p,1\right)\right),\left\{\tfrac{u_1}{\langle 0.4,0.7\rangle},\tfrac{u_2}{\langle 0.6,0.8\rangle},\tfrac{u_3}{\langle 0,0\rangle}\right\},0.3\right)\Bigg\}.$$

Definition 3.4. If $\left(\tilde{F}_\mu,A\right)$ and $\left(\tilde{G}_\delta,B\right)$ are two generalized vague soft expert sets over U, then "$\left(\tilde{F}_\mu,A\right)$ OR $\left(\tilde{G}_\delta,B\right)$" denoted by $\left(\tilde{F}_\mu,A\right)\bigvee\left(\tilde{G}_\delta,B\right)$ is defined by

$$\left(\tilde{F}_\mu,A\right)\bigvee\left(\tilde{G}_\delta,B\right)=\left(\tilde{H}_\Omega,A\times B\right)$$

such that $\tilde{H}\left(\alpha,\beta\right)=\tilde{F}\left(\alpha\right)\tilde{\cup}\tilde{G}\left(\beta\right)$, for all $\left(\alpha,\beta\right)\in A\times B$, where $\tilde{\cup}$ is a generalized vague union.

Example 3.3. Consider Example 3.2. Let
$\left(\tilde{F},A\right)=\{\left(e_1,p,1\right),\left(e_3,p,1\right),\left(e_3,r,1\right),\left(e_2,p,0\right)\}$
and $(\tilde{G},B)=\{(e_1,p,1),(e_3,p,1),(e_2,p,0)\}$.

Suppose $(\tilde{F}_\mu, A)$ and $(\tilde{G}_\delta, B)$ are two generalized vague soft expert sets over U such that $\left(\tilde{F}_\mu, A\right) =$

$$\left\{ \left((e_1,p,1), \left\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.6,0.8\rangle}, \frac{u_3}{\langle 0,0\rangle}\right\}, 0.2\right), \left((e_3,p,1), \left\{\frac{u_1}{\langle 0.3,0.6\rangle}, \frac{u_2}{\langle 0.6,0.6\rangle}, \frac{u_3}{\langle 0.5,0.5\rangle}\right\}, 0.9\right), \right.$$

$$\left. \left((e_3,r,1), \left\{\frac{u_1}{\langle 0.7,0.3\rangle}, \frac{u_2}{\langle 0.1,0.3\rangle}, \frac{u_3}{\langle 0.3,0.6\rangle}\right\}, 0.5\right), \left((e_2,p,0), \left\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.6,0.8\rangle}, \frac{u_3}{\langle 0,0\rangle}\right\}, 0.3\right) \right\},$$

and

$$\left(\tilde{G}_\delta, B\right) = \left\{ \left((e_1,p,1), \left\{\frac{u_1}{\langle 0.4,0.6\rangle}, \frac{u_2}{\langle 0.7,0.7\rangle}, \frac{u_3}{\langle 1,1\rangle}\right\}, 0.3\right), \right.$$

$$\left. \left((e_3,p,1), \left\{\frac{u_1}{\langle 0.4,0.4\rangle}, \frac{u_2}{\langle 0.7,0.3\rangle}, \frac{u_3}{\langle 0.4,0.7\rangle}\right\}, 0.7\right), \left((e_2,p,0), \left\{\frac{u_1}{\langle 0.4,0.7\rangle}, \frac{u_2}{\langle 0.6,0.7\rangle}, \frac{u_3}{\langle 0,0\rangle}\right\}, 1\right) \right\}.$$

Then $\left(\tilde{F}_\mu, A\right) \bigvee \left(\tilde{G}_\delta, B\right) = \left(\tilde{H}_\Omega, A \times B\right)$

$$= \left\{ \left(((e_1,p,1),(e_1,p,1)), \left\{\frac{u_1}{\langle 0.5,0.5\rangle}, \frac{u_2}{\langle 0.7,0.8\rangle}, \frac{u_3}{\langle 1,0\rangle}\right\}, 0.3\right), \right.$$

$$\left(((e_1,p,1),(e_3,p,1)), \left\{\frac{u_1}{\langle 0.5,0.4\rangle}, \frac{u_2}{\langle 0.7,0.3\rangle}, \frac{u_3}{\langle 0.4,0\rangle}\right\}, 0.7\right),$$

$$\left(((e_1,p,1),(e_2,p,0)), \left\{\frac{u_1}{\langle 0.4,0.7\rangle}, \frac{u_2}{\langle 0.6,0.7\rangle}, \frac{u_3}{\langle 0,0\rangle}\right\}, 0.3\right),$$

$$\left(((e_3,p,1),(e_1,p,1)), \left\{\frac{u_1}{\langle 0.3,0.6\rangle}, \frac{u_2}{\langle 0.6,0.6\rangle}, \frac{u_3}{\langle 0.5,0.5\rangle}\right\}, 0.9\right),$$

$$\left(((e_3,p,1),(e_3,p,1)), \left\{\frac{u_1}{\langle 0.4,0.4\rangle}, \frac{u_2}{\langle 0.7,0.6\rangle}, \frac{u_3}{\langle 1,0.5\rangle}\right\}, 0.9\right),$$

$$\left(((e_3,p,1),(e_2,p,0)), \left\{\frac{u_1}{\langle 0.4,0.4\rangle}, \frac{u_2}{\langle 0.6,0.6\rangle}, \frac{u_3}{\langle 0.5,0\rangle}\right\}, 0.9\right),$$

$$\left(((e_3,r,1),(e_1,p,1)), \left\{\frac{u_1}{\langle 0.7,0.3\rangle}, \frac{u_2}{\langle 0.7,0.3\rangle}, \frac{u_3}{\langle 1,0.6\rangle}\right\}, 0.5\right),$$

$$\left(((e_3,r,1),(e_3,p,1)), \left\{\frac{u_1}{\langle 0.7,0.3\rangle}, \frac{u_2}{\langle 0.7,0.3\rangle}, \frac{u_3}{\langle 0.4,0.6\rangle}\right\}, 0.7\right),$$

$$\left(((e_3,r,1),(e_2,p,0)), \left\{\frac{u_1}{\langle 0.7,0.3\rangle}, \frac{u_2}{\langle 0.6,0.3\rangle}, \frac{u_3}{\langle 0.3,0\rangle}\right\}, 0.5\right),$$

$$\left(((e_2,p,0),(e_1,p,1)), \left\{\frac{u_1}{\langle 0.4,0.4\rangle}, \frac{u_2}{\langle 0.7,0.3\rangle}, \frac{u_3}{\langle 1,0.7\rangle}\right\}, 0.7\right),$$

$$\left(((e_2,p,0),(e_3,p,1)), \left\{\frac{u_1}{\langle 0.4,0.4\rangle}, \frac{u_2}{\langle 0.7,0.3\rangle}, \frac{u_3}{\langle 0.4,0.7\rangle}\right\}, 0.7\right),$$

$$\left. \left(((e_2,p,0),(e_3,p,1)), \left\{\frac{u_1}{\langle 0.4,0.4\rangle}, \frac{u_2}{\langle 0.7,0.3\rangle}, \frac{u_3}{\langle 0.4,0\rangle}\right\}, 0.7\right) \right\}.$$

4. Conclusion

The concept of generalized vague soft expert set and its basic operations properties are introduced along with illustrative examples. This new extension not only provides a significant addition to existing theories for handling uncertainties, but also leads to potential areas of further research and pertinent applications.

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