FIXED POINTS FOR THREE CONTRACTIVE MAPPINGS
IN THREE FUZZY METRIC SPACES

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Abstract: In the present paper, we establish a common fixed point theorem in three complete fuzzy metric spaces. An analogous result for compact metric spaces is also proved. We also give example in support of our result. As an application to our results, we obtain the corresponding common fixed point theorems in metric spaces. Our results improves the results of Fisher \cite{2}, \cite{3} in two metric spaces and Jain et al. \cite{8} in three metric spaces.

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1. Introduction and Preliminaries

In 1965, Zadeh \cite{18} introduced the concept of fuzzy set as a new way to represent vagueness in our everyday life. However, when the uncertainty is due to fuzziness rather than randomness, as sometimes in the measurement of an ordinary length, it seems that the concept of a fuzzy metric space is more suitable. We can divide them into following two groups.
The first group involves those results in which a fuzzy metric on a set $X$ is treated as a mapping, where $X$ represents the totality of all fuzzy points of a set and satisfy some axioms which are analogous to the ordinary metric axioms. Thus, in such an approach numerical distances are set up between fuzzy objects.

On the other hand in second group, we keep those results in which the distance between objects is fuzzy and the objects themselves may or may not be fuzzy.

Kramosil and Michálek [9] have introduced the concept of fuzzy metric spaces in different ways. Many authors ([1], [5]-[7], [9]-[17]) have proved fixed point theorems in fuzzy metric spaces.

In this section, we recall some useful facts from the fuzzy metric spaces.

**Definition 1.1.** ([14]) A binary operation $*$ : $[0,1] \times [0,1] \to [0,1]$ is continuous $t$-norm if $*$ satisfies

(i) $*$ is commutative and associative;

(ii) $*$ is continuous;

(iii) $*(a, 1) = a$ for all $a \in [0,1]$;

(iv) $*(a,b) \leq *(c,d)$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

**Definition 1.2.** ([4]) A 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if $X$ is an arbitrary set, $*$ is a continuous $t$-norm, and $M$ is fuzzy sets on $X^2 \times [0,\infty)$ satisfying the following conditions for all $x, y, x \in X$ and $s, t > 0$

(i) $M(x, y, 0) > 0$;

(ii) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$;

(iii) $M(x, y, t) = M(y, x, t)$;

(iv) $*(M(x, y, t), M(y, z, s)) \leq M(x, z, t + s)$;

(v) $M(x, y, \cdot) : [0,\infty) \to [0,1]$ is left continuous.

The function $M(x, y, t)$ denote the degree of nearness between $x$ and $y$ with respect to $t$, respectively.

**Lemma 1.3.** ([4]) In fuzzy metric space $(X, M, *)$, $M(x, y, \cdot)$ is non-decreasing for all $x, y \in X$.

**Definition 1.4.** ([4]) Let $(X, M, T)$ be a fuzzy metric space. Then a sequence $\{x_n\}$ in $X$ is said to be

(a) convergent to a point $x \in X$ if $\lim_{n \to \infty} M(x_n, x, t) = 1$ for all $t > 0$.

(b) Cauchy sequence if $\lim_{n \to \infty} M(x_{n+p}, x_n, t) = 1$ for all $t > 0$ and $p > 0$ or if for each $\epsilon > 0$, there exist $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for each $n, m \geq n_0$. 
Definition 1.5. ([4]) A fuzzy metric space \((X, M, *)\) is said to be complete if and only if every Cauchy sequence in \(X\) is convergent.

Example 1.6. ([6]) Let \(X = \{1/n : n \in \mathbb{N}\} \cup \{0\}\) and * be the continuous \(t\)-norm defined by \(* (a, b) = ab\) (or \(* (a, b) = \min\{a, b\}\) for all \(a, b \in [0, 1]\). For each \(x, y \in X\) and \(t > 0\), define \(M(x, y, t)\) by

\[
M(x, y, t) = \begin{cases} 
\frac{t}{1 + |x - y|}, & t > 0, \\
0, & t = 0.
\end{cases}
\]

Clearly, \((X, M, *)\) is a complete fuzzy metric space.

Lemma 1.7. ([4], [9]) Let \((X, M, *)\) be a fuzzy metric space. Then \(M\) is a continuous function on \(X^2 \times (0, \infty)\).

Lemma 1.8. ([15]) Let \((X, M, *)\) be a fuzzy metric space. If a sequence \(\{x_n\}\) in \(X\) is such that for every \(n = 1, 2, 3, \ldots\),

\[
M(x_n, x_{n+1}, t) \geq 1 - c^n(1 - M(x_0, x_1, t))
\]

for every \(t > 0\), where \(c \in (0, 1)\), then \(\{x_n\}\) is a Cauchy sequence in \(X\).

2. Main Results

Now, we prove a fixed point theorem involving three complete fuzzy metric spaces.

Theorem 2.1. Let \((X, M_1, *_1)\), \((Y, M_2, *_2)\) and \((Z, M_3, *_3)\) be three complete fuzzy metric spaces. If \(T\) is a continuous mapping of \(X\) into \(Y\), \(S\) is a continuous mapping of \(Y\) into \(Z\) and \(R\) is a mapping of \(Z\) into \(X\) satisfying

\[
\begin{align*}
M_1(RST x, RST x', t) & \geq 1 - c[1 - \min\{M_1(x, RST x, t), M_1(x', RST x', t), \\
& \quad M_1(x, x', t), M_2(T x, T x', t), M_3(ST x, ST x', t)\}] \\
\end{align*}
\]  \hspace{1cm} (2.1)

for all \(x, x' \in X\),

\[
\begin{align*}
M_2(TRS y, TRS y', t) & \geq 1 - c[1 - \min\{M_2(y, TRS y, t), M_2(y', TRS y', t), \\
& \quad M_2(y, y', t), M_3(S y, S y', t), M_1(RS y, RS y', t)\}] \\
\end{align*}
\]  \hspace{1cm} (2.2)
for all \( y, y' \in Y \) and

\[
M_3(STRz, STRz', t) \\
\geq 1 - c [1 - \min\{M_3(z, RSYz, t), M_3(z', RSTz', t), \]
\]
\[M_3(z, z', t), M_1(Rz, Rz', t), M_2(TRz, TRz', t)\}] \tag{2.3}

for all \( z, z' \in Z \), where \( 0 \leq c < 1 \).

Then \( RST \) has a unique fixed point \( u \in X \), \( TRS \) has a unique fixed point \( v \in Y \) and \( STR \) has a unique fixed point \( w \in Z \). Further, \( Tu = v \), \( Sv = w \) and \( Rw = u \).

**Proof.** Let \( x_0 \) be an arbitrary point in \( X \). Define a sequence \( \{x_n\} \) in \( X \), \( \{y_n\} \) in \( Y \) and \( \{z_n\} \) in \( Z \) such that

\[
(RST)^n x_0 = x_n, \quad y_n = T x_{n-1}, \quad z_n = S y_n, \quad n = 1, 2, 3, \ldots \tag{2.4}
\]

Using inequality (2.2) we have by using (2.4)

\[
M_2(y_n, y_{n+1}, t) \\
= M_2(TRSy_{n-1}, TRSy_n, t) \\
\geq 1 - c [1 - \min\{M_2(y_{n-1}, TRSy_{n-1}, t), M_2(y_n, TRSy_n, t), \]
\]
\[M_2(y_{n-1}, y_n, t), M_3(Sy_{n-1}, Sy_n, t), M_1(RSy_{n-1}, RSy_n, t)\}] \tag{2.5}
\]

For using inequality (2.3) we have by using (2.5)

\[
M_3(z_n, z_{n+1}, t) \\
= M_2(STRz_{n-1}, STRz_n, t) \\
\geq 1 - c [1 - \min\{M_3(z_{n-1}, RSTz_{n-1}, t), M_3(z_n, RSTz_n, t), \]
\]
\[M_3(z_{n-1}, z_n, t), M_1(Rz_{n-1}, Rz_n, t), M_2(TRz_{n-1}, TRz_n, t)\}] \tag{2.6}
\]

Similarly, on using inequality (2.1) we have by using (2.5) and (2.6),

\[
M_1(x_n, x_{n+1}, t) \\
\geq 1 - c [1 - \min\{M_1(x_{n-1}, x_n, t), M_2(y_{n-1}, y_n, t), M_3(z_{n-1}, z_n, t)\}] \tag{2.7}
\]
By repeated application on inequalities (2.5)-(2.7), we get

\[ M_1(x_n, x_{n+1}, t) \geq 1 - c^{n-1}[1 - \min\{M_1(x_1, x_2, t), M_2(y_1, y_2, t), M_3(z_1, z_2, t)\}], \]

\[ M_2(y_n, y_{n+1}, t) \geq 1 - c^{n-1}[1 - \min\{M_1(x_1, x_2, t), M_2(y_1, y_2, t), M_3(z_1, z_2, t)\}], \]

\[ M_3(z_n, z_{n+1}, t) \geq 1 - c^{n-1}[1 - \min\{M_1(x_1, x_2, t), M_2(y_1, y_2, t), M_3(z_1, z_2, t)\}], \]

where \( n = 1, 2, 3, \ldots \). By Lemma 1.8, as \( 0 \leq c < 1 \), we get \( \{x_n\} \) is a Cauchy sequence in \( X \), \( \{y_n\} \) is a Cauchy sequence in \( Y \) and \( \{z_n\} \) is a Cauchy sequence in \( Z \). As \( X \), \( Y \) and \( Z \) are complete fuzzy metric space, therefore, \( \{x_n\} \) has a limit \( u \in X \), \( \{y_n\} \) has a limit \( v \in Y \) and \( \{z_n\} \) has a limit \( w \in Z \). Also, as \( T \) and \( S \) are continuous

\[ \lim_{n \to \infty} y_n = \lim_{n \to \infty} T x_n = Tu = v, \quad \lim_{n \to \infty} z_n = \lim_{n \to \infty} S x_n = Sv = w. \]

Further, by using inequality (2.1), we get

\[ M_1(RST u, x_n, t) \]
\[ = M_1(RST u, RST x_{n-1}, t) \]
\[ \geq 1 - c[1 - \min\{M_1(u, RST u, t), M_1(x_{n-1}, RST x_{n-1}, t), M_1(u, x_{n-1}, t), M_2(T u, T x_{n-1}, t), M_3(ST u, ST x_{n-1}, t)\}]. \]

On letting \( n \to \infty \), we have

\[ M_1(RST u, u, t) \]
\[ \geq 1 - c[1 - \min\{M_1(u, RST u, t), M_1(u, u, t), M_1(u, u, t), M_2(T u, v, t), M_3(ST u, ST u, t)\}]] \]
\[ \geq 1 - c[1 - M_1(u, RST u, t)], \]

therefore, we have \( RST u = u \). Hence \( RST \) has fixed point \( u \in X \).

Similarly, we can easily show by using inequality (2.2) that \( TRS \) has fixed point \( v \in Y \), that is, \( TRS v = TRST u = Tu = v \) and by using inequality (2.3) that \( STR \) has fixed point \( w \in Z \), that is, \( STR w = STRS v = Sv = w \).
Finally, for uniqueness, suppose that $RST$ has another fixed point $a \in X$, then by inequality (2.1), we have

\[
M_1(u, a, t) \\
= M_1(RSTu, RSta, t) \\
\geq 1 - c[1 - c \min\{M_1(u, RSTu, t), M_1(a, RSta, t), \]
\qquad M_1(u, a, t), M_2(Tu, Ta, t), M_3(STu, STA, t)\}] \\
= 1 - c[1 - c \min\{M_2(Tu, Ta, t), M_3(STu, STA, t)\}] .
\]

Also, by inequality (2.2), we have

\[
M_2(Tu, Ta, t) \\
= M_1(TRSTu, TRSTA, t) \\
\geq 1 - c[1 - c \min\{M_2(Tu, TRSTu, t), M_2(Ta, TRSTA, t), \]
\qquad M_2(Tu, Ta, t), M_3(STu, STA, t), M_3(RSTu, RSTA, t)\}] \\
= 1 - c[1 - c \min\{M_3(STu, STA, t), M_1(u, a, t)\}] .
\]

By using (2.9) in (2.8), we have

\[
M_1(u, a, t) \geq 1 - c[M_3(STu, STA, t)] .
\]

Further, by using inequality (2.3), we have

\[
M_2(STu, STA, t) \\
= M_3(STRSAu, STRSTA, t) \\
\geq 1 - c[1 - c \min\{M_3(STu, RSAu, t), M_3(STa, RSA, t), \]
\qquad M_3(STu, STA, t), M_1(u, a, t), M_2(STu, STA, t)\}] \\
= 1 - c[1 - c \min\{M_1(u, a, t), M_2(Tu, Ta, t)\}] .
\]

Using in (2.10), we get

\[
M_1(u, a, t) \geq 1 - c^2[M_1(u, a, t)] .
\]

Since $c < 1$, this implies $u = a$. Hence $RST$ has unique fixed point in $X$.

Similarly, we prove that $TRS$ has a unique common fixed point $v \in Y$ and $STR$ has a unique common fixed point $w \in Z$. This completes the proof. \(\square\)

In Theorem 2.1, by taking $X = Y = Z$, $M_1 = M_2 = M_3 = M$ and $*_1 = *_2 = *_3 = *$, we have following corollary.
**Corollary 2.2.** Let \((X,M,\ast)\) be a complete fuzzy metric space. If \(T\) and \(S\) are continuous mappings of \(X\) into \(X\) and \(R\) is a mapping of \(X\) into \(X\) satisfying

\[
M(RSTx, RSTx', t) \\
\geq 1 - c[1 - \min\{M(x, RSTx, t), M(x', RSTx', t), \\
M(x, x', t), M(Tx, Tx', t), M(STx, STx', t)\}]
\]

for all \(x, x' \in X\),

\[
M(TRSy, TRSy', t) \\
\geq 1 - c[1 - \min\{M(y, TRSy, t), M(y', TRSy', t), \\
M(y, y', t), M(Sy, Sy', t), M(RSy, RSy', t)\}]
\]

for all \(y, y' \in X\) and

\[
M(STRz, STRz', t) \\
\geq 1 - c[1 - \min\{M(z, RSYz, t), M(z', RSTz', t), \\
M(z, z', t), M(Rz, Rz', t), M(TRz, TRz', t)\}]
\]

for all \(z, z' \in X\), where \(0 \leq c < 1\).

Then \(RST\) has a unique fixed point \(u \in X\), \(TRS\) has a unique fixed point \(v \in Y\) and \(STR\) has a unique fixed point \(w \in Z\). Further, \(Tu = v\), \(Sv = w\) and \(Rw = u\) and if \(u = v = w\), \(u\) is a unique common fixed point of \(R\), \(S\) and \(T\).

Next, we prove a fixed point theorem involving three compact fuzzy metric spaces.

**Theorem 2.3.** Let \((X, M_1, \ast_1)\), \((Y, M_2, \ast_2)\) and \((Z, M_3, \ast_3)\) be three compact fuzzy metric spaces. If \(T\) is a continuous mapping of \(X\) into \(Y\), \(S\) is a continuous mapping of \(Y\) into \(Z\) and \(R\) is a continuous mapping of \(Z\) into \(X\) satisfying

\[
M_1(RSTx, RSTx', t) \\
> \min\{M_1(x, RSTx, t), M_1(x', RSTx', t), \\
M_1(x, x', t), M_2(Tx, Tx', t), M_3(STx, STx', t)\}
\]

for all \(x, x' \in X\),

\[
M_2(TRSy, TRSy', t) \\
> \min\{M_2(y, TRSy, t), M_2(y', TRSy', t), \\
M_2(y, y', t), M_3(Sy, Sy', t), M_1(RSy, RSy', t)\}
\]

for all \(y, y' \in Y\) and

\[
M_3(STRz, STRz', t) \\
> \min\{M_3(z, RSYz, t), M_3(z', RSTz', t), \\
M_3(z, z', t), M_3(Rz, Rz', t), M_3(TRz, TRz', t)\}
\]

for all \(z, z' \in Z\), where \(0 \leq c < 1\).
for all $y, y' \in Y$ and
\[
M_3(STRz, STRz', t) > \min \{M_3(z, RSY z, t), M_3(z', RST z', t), M_3(z, z', t), M_1(Rz, Rz', t), M_2(TRz, TRz', t)\}
\]  
(2.13)
for all $z, z' \in Z$.

Then $RST$ has a unique fixed point $u \in X$, $TRS$ has a unique fixed point $v \in Y$ and $STR$ has a unique fixed point $w \in Z$. Further, $Tu = v$, $Sv = w$ and $Rw = u$.

Proof. Suppose first of all there exists $u, u' \in X$ such that
\[
\min \{M_1(x, RST x, t), M_1(x', RST x', t), M_1(x, x', t), M_2(Tx, Tx', t), M_3(STx, STx', t)\} = 1,
\]
this gives $u = RST u$, $u' = RST u'$, $u = u'$ and $Tu = Tu'$.

Let $Tu = v$. Then we have $RSv = u$, this gives $TRSv = Tu = v$ and $STRSv = STRw = STu = Sv = w$. Then $RST$ has a fixed point $u \in X$, $TRS$ has a fixed point $v \in Y$ and $STR$ has a fixed point $w \in Z$.

Similarly, we holds if there exist $v, v' \in Y$ and $w, w' \in Z$ such that
\[
\min \{M_2(y, TRSy, t), M_2(y', TRSy', t), M_2(y, y', t), M_3(Sy, Sy', t), M_1(RSy, RSy', t)\} = 1
\]
or
\[
\min \{M_3(z, RSY z, t), M_3(z', RST z', t), M_3(z, z', t), M_1(Rz, Rz', t), M_2(TRz, TRz', t)\} = 1.
\]
Hence, this proves the results.

Now, suppose that $u, u' \in X$, $v, v' \in Y$ and $z, z'$ do not exist. Then
\[
1 - \min \{M_1(x, RST x, t), M_1(x', RST x', t), M_1(x, x', t), M_2(Tx, Tx', t), M_3(STx, STx', t)\} \neq 0
\]
for all $x, x' \in X$ and so real valued function defined by
\[
F(x, x', t) = \frac{M_1(RST x, RST x', t)}{1 - \mathcal{O}},
\]
where
\[
O = \min\{M_1(x, RST x, t), M_1(x', RST x', t),
M_1(x, x', t), M_2(T x, T x', t), M_3(ST x, ST x', t)\}
\]
is continuous. Since \( X \) is compact, \( F \) attains its maximum value, say \( c_1 \). By inequality (2.11) we have \( 0 \leq c_1 < 1 \) and so
\[
M_1(RST x, RST x', t)
\geq 1 - c_1[1 - \min\{M_1(x, RST x, t), M_1(x', RST x', t),
M_1(x, x', t), M_2(T x, T x', t), M_3(ST x, ST x', t)\}]
\]
for all \( x, x' \in X \). Similarly, By inequalities (2.12) and (2.13), there exists \( c_2 \), where \( 0 \leq c_2 < 1 \) such that
\[
M_3TRS y, TRS y', t)
\geq 1 - c_2[1 - \min\{M_2(y, TRS y, t), M_2(y', TRS y', t),
M_2(y, y', t), M_3(S y, S y', t), M_1(RSy, RS y', t)\}]
\]
for all \( y, y' \in Y \) and there exists \( c_3 \), where \( 0 \leq c_3 < 1 \) such that
\[
M_2(ST Rz, STR z', t)
\geq 1 - c_3[1 - \min\{M_3(z, RSY z, t), M_3(z', RST z', t),
M_3(z, z', t), M_1(Rz, Rz', t), M_2(TR z, TR z', t)\}]
\]
for all \( z, z' \in Z \). It follows that all conditions of Theorem 2.1 are satisfied with \( c = \min\{c_1, c_2, c_3\} \). Therefore, \( RST \) has a unique fixed point \( u \in X \), \( TRS \) has a unique fixed point \( v \in Y \) and \( STR \) has a unique fixed point \( w \in Z \). Further, \( Tu = v, Sv = w \) and \( Rw = u \). This completes the proof.

In Theorem 2.3, by taking \( X = Y = Z \), \( M_1 = M_2 = M_3 = M \) and \(*_1 = *_2 = *_3 = *\), we have following corollary.

**Corollary 2.4.** Let \((X, M, *)\) be a compact fuzzy metric space. If \( T, S \) and \( R \) are continuous mappings of \( X \) into \( X \) satisfying
\[
M(RST x, RST x', t)
> \min\{M(x, RST x, t), M(x', RST x', t), M(x, x', t),
M(T x, T x', t), M(ST x, ST x', t)\}]
\]
for all \( x, x' \in X \),
\[
M(TRSy, TRSy', t)
> \min\{M(y, TRSy, t), M(y', TRSy', t), M(y, y', t),
M(S y, S y', t), M(RSy, RS y', t)\}
\]
for all \( y, y' \in Y \) and
\[
M(ST Rz, ST Rz', t) > \min\{M(z, RSY z, t), M(z', RST z', t), M(z, z', t), M(Rz, Rz', t), M(TRz, TRz', t)\}
\]
for all \( z, z' \in X \).

Then \( RST \) has a unique fixed point \( u \in X \), \( TRS \) has a unique fixed point \( v \in Y \) and \( STR \) has a unique fixed point \( w \in Z \). Further, \( Tu = v \), \( Sv = w \) and \( Rw = u \) and if \( u = v = w \), \( u \) is a unique common fixed point of \( R \), \( S \) and \( T \).

3. Corresponding Results in Metric Spaces

Every metric space \((X, d_1)\), \((Y, d_2)\) and \((Z, d_3)\) can be realized as a fuzzy metric space \((X, M, \ast)\) by taking
\[
M(x, y, t) = \begin{cases} \frac{t}{t + d(x, y)}, & t > 0, \\ 0, & t = 0. \end{cases}
\]
Hence results in a fuzzy metric space can be used to prove existence of fixed point in a metric space.

In this section, as a sample, we utilize Theorem 2.1 to derive corresponding common fixed point theorem in metric spaces.

**Theorem 3.1.** Let \((X, d_1)\), \((Y, d_2)\) and \((Z, d_3)\) be three complete metric spaces. If \( T \) is a continuous mapping of \( X \) into \( Y \), \( S \) is a continuous mapping of \( Y \) into \( Z \) and \( R \) is a mapping of \( Z \) into \( X \) satisfying
\[
d_1(RSTx, RSTx') \\
\leq c \max\{d_1(x, RSTx), d_1(x', RSTx'), \\
d_1(x, x'), d_2(Tx, Tx'), d_3(STx, STx')\}\]
\[(2.14)\]
for all \( x, x' \in X \),
\[
d_2(TRSy, TRSy') \\
\leq c \max\{d_2(y, TRSy), d_2(y', TRSy'), \\
d_2(y, y'), d_3(Sy, Sy'), d_1(RSy, RSy')\}\]
\[(2.15)\]
for all \( y, y' \in Y \) and
\[
M_2(ST Rz, ST Rz', t) \leq c \max\{d_3(z, RSY z), d_3(z', RST z'), \\
d_3(z, z'), d_1(Rz, Rz'), d_2(TRz, TRz')\}\]
\[(2.16)\]
for all \( z, z' \in Z \), where \( 0 \leq c < 1 \).

Then \( RST \) has a unique fixed point \( u \in X \), \( TRS \) has a unique fixed point \( v \in Y \) and \( STR \) has a unique fixed point \( w \in Z \). Further, \( Tu = v \), \( Sv = w \) and \( Rw = u \).

**Proof.** Define
\[
M_1(x, y, t) = \begin{cases} \frac{t}{t + d_1(x, y)}, & t > 0, \\ 0, & t = 0 \end{cases}, \quad M_2(x, y, t) = \begin{cases} \frac{t}{t + d_2(x, y)}, & t > 0, \\ 0, & t = 0 \end{cases},
\]
and
\[
M_3(x, y, t) = \begin{cases} \frac{t}{t + d_3(x, y)}, & t > 0, \\ 0, & t = 0 \end{cases}
\]
and \(*_1(a, b) = *_2(a, b) = *_3(a, b) = \min\{a, b\} \) for all \( a, b \in [0, 1] \). Then metric spaces \((X, d_1)\), \((Y, d_2)\) and \((Z, d_3)\) can be realized as a fuzzy metric spaces \((X, M_1, *_1)\), \((Y, M_2, *_2)\) and \((Z, M_3, *_3)\), respectively. It is straightforward to notice that Theorem 3.1 satisfies all the conditions of Theorem 2.1. Also inequalities (2.14)-(2.16) of Theorem 3.1 implies inequalities (2.1)-(2.3) of Theorem 2.1. Thus, all the conditions of Theorem 2.1 are satisfied so that \( RST \) has a unique fixed point \( u \in X \), \( TRS \) has a unique fixed point \( v \in Y \) and \( STR \) has a unique fixed point \( w \in Z \). Further, \( Tu = v \), \( Sv = w \) and \( Rw = u \). \(\Box\)

**Remark 3.2.** The similar results to Theorem 3.1 can also be outlined in respects of Corollary 2.2, Theorem 2.3 and Corollary 2.4.

**Remark 3.3.** Theorem 3.1 improves the results of Jain et al. [8] in three metric spaces.

**Remark 3.4.** By taking \( Y = Z \) and \( d_3 = d_2 \) and \( R \) is the identity mapping, Theorem 3.1 improves the results of Fisher [2], [3].

**References**


