IMPROVING IZPM FOR UNBALANCED FUZZY TRANSPORTATION PROBLEMS

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Abstract: This note points out how IZPM [5] for unbalanced fuzzy transportation problems can be improved by breaking the ties not by selecting arbitrary.

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1. Introduction

In today’s highly competitive market, the pressure on organizations is to find better ways to create and deliver value added service to the customers in order to become stronger. Transportation models provide a powerful framework to meet the challenge. They ensure the efficient movement and timely availability of raw materials and finished goods.

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The basic transportation problem was originally developed by Hitchcock [8]. The transportation problems can be modeled as a standard linear programming problem, which can then be solved by the simplex method. Charnes and Cooper [2] developed a stepping stone method which provides an alternative way of determining the simplex method information. In general, the transportation problems are solved with the assumptions that the coefficients or cost parameters are specified in a precise way i.e., in crisp environment.

In real life, there are many diverse situations due to uncertainty in judgments, lack of evidence etc. Sometimes it is not possible to get relevant precise data for the cost parameter. Zimmermann [9] showed that solutions obtained by fuzzy linear programming method and are always efficient. Subsequently, Zimmermann’s fuzzy linear programming has developed into several fuzzy optimization methods for solving the transportation problems. Chanas and Kuchta [1] proposed the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers, and developed an algorithm for obtaining the optimal solution.

In this note a simple rule is given for improving IZPM for the unbalanced fuzzy transportation problems. This note points out how IZPM for an unbalanced fuzzy transportation problem can be improved by breaking the ties not by selecting arbitrary. In the proposed algorithm transportation costs are represented by triangular fuzzy numbers. To illustrate the proposed algorithm a numerical example is solved and the obtained result is compared with the result of an arbitrary selection procedure.

2. Preliminaries

In this section, some basic preliminaries are given below

Definition 2.1. A fuzzy number $\tilde{A}$ is denoted as a triangular fuzzy number by $(a_1, a_2, a_3)$ and its membership function $\mu_{\tilde{A}}(x)$ is given as:

$$
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\
\frac{x-a_2}{a_3-a_2} & \text{if } a_2 \leq x \leq a_3 \\
0 & \text{otherwise}
\end{cases}
$$

Arithmetic Operations: In this section, arithmetic operations between two triangular fuzzy numbers, defined on the universal set of real numbers $\mathbb{R}$, are presented.

If $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers, then the following is obtained.
1. \( \tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \)

2. \( \tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1) \)

3. \( \tilde{A} \times \tilde{B} = (a, b, c) \) where \( T = \{a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3\} \), \( a = \min\{T\} \), \( b = a_2 b_2 \) and \( c = \max\{T\} \)

4. If \( k \in \mathbb{R} \) then \( k\tilde{A} = k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3) \) for \( k \geq 0 \).

### 3. Proposed Method

In this section, a new method is proposed for finding a fuzzy optimal solution using ranking function, in which transportation costs are represented as triangular fuzzy numbers instead of normal fuzzy trapezoidal numbers.

**Step 1. Develop the cost table from the given problem**

Construct the fuzzy transportation table for the given fuzzy transportation problem, then convert it into a balanced one, if it is not.

**Step 2. Find the opportunity cost table**

1. Locate the smallest element in each row of the given cost table and then subtract that from each element of that row, and

2. In the reduced matrix obtained from 2(a), locate the smallest element in each column and then subtract that from each element of that column. Each row and column have at least one zero value.

**Step 3. Optimality criterion**

1. Verify each supply is less than or equal to the total demand, whose reduced costs are zero.

2. Verify each demand is less than or equal to the total supply, whose reduced costs are zero.

3. If 3(a) and 3(b) are satisfied, then go to Step 6 else go to Step 4.
3.1. Step 4. Revise the opportunity cost table

Draw a minimal set of horizontal and vertical lines to cover all the zeros in the opportunity cost table obtained from Step 2. (Omitting the unsatisfied supply and demand of 3(a) and 3(b)).

Step 5. Develop the new revised opportunity cost table

1. From among the cells not covered by any line, choose the smallest element. Call this value k.

2. Subtract k from every element in the cell not covered by a line.

3. Add k to every element in the cell covered by the two lines, i.e., intersection of two lines.

4. Elements in cells covered by one line remain unchanged.

5. Go to Step 3.

Step 6. Make assignments in the opportunity cost matrix

The procedure of making assignments is as follows

1. Identify the largest unit fuzzy transportation cost cell in the cost matrix obtained from either Step 2 or Step 5. If there is a tie, select the largest unit fuzzy transportation cost cell from the given fuzzy transportation problem. Call this cell as (i,j).

2. Make the assignment, by selecting i\textsuperscript{th} row and/or j\textsuperscript{th} column with exactly one zero cell.

3. Adjust the supply and demand and cross out the satisfied row or column.

4. Repeat this procedure until the entire available supply at various sources and demand at various destinations are satisfied.

4. Numerical Example

Let us consider the following problems.
Problem 1.

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
<th>D5</th>
<th>Availability(ai)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>(4,7,10)</td>
<td>(8,10,12)</td>
<td>(10,14,18)</td>
<td>(6,8,10)</td>
<td>(0,0,0)</td>
<td>(25,30,35)</td>
</tr>
<tr>
<td>S2</td>
<td>(4,7,10)</td>
<td>(4,12,20)</td>
<td>(4,12,20)</td>
<td>(5,6,7)</td>
<td>(0,0,0)</td>
<td>(25,40,55)</td>
</tr>
<tr>
<td>S3</td>
<td>(1,5,9)</td>
<td>(6,8,10)</td>
<td>(10,15,20)</td>
<td>(6,9,12)</td>
<td>(0,0,0)</td>
<td>(40,50,60)</td>
</tr>
<tr>
<td>Demand(bj)</td>
<td>(10,20,30)</td>
<td>(15,20,25)</td>
<td>(15,25,35)</td>
<td>(10,30,50)</td>
<td>(10,25,40)</td>
<td></td>
</tr>
</tbody>
</table>

Problem 2.

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
<th>Availability(ai)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>(1,2,3)</td>
<td>(4,7,10)</td>
<td>(10,14,18)</td>
<td>(0,0,0)</td>
<td>(3,5,7)</td>
</tr>
<tr>
<td>S2</td>
<td>(2,3,4)</td>
<td>(2,3,4)</td>
<td>(0,1,2)</td>
<td>(0,0,0)</td>
<td>(5,8,11)</td>
</tr>
<tr>
<td>S3</td>
<td>(1,5,9)</td>
<td>(3,4,5)</td>
<td>(4,7,10)</td>
<td>(0,0,0)</td>
<td>(4,7,10)</td>
</tr>
<tr>
<td>S4</td>
<td>(0,1,2)</td>
<td>(5,6,7)</td>
<td>(1,2,3)</td>
<td>(0,0,0)</td>
<td>(10,15,20)</td>
</tr>
<tr>
<td>Demand(bj)</td>
<td>(4,7,10)</td>
<td>(5,9,13)</td>
<td>(13,18,23)</td>
<td>(0,1,2)</td>
<td></td>
</tr>
</tbody>
</table>

Optimal solution by the suggested approach

Total cost of supplying the requirement is:

<table>
<thead>
<tr>
<th>Problem</th>
<th>(-210,770,2100)</th>
<th>(-490,770,2380)</th>
<th>(-1290,770,3250)</th>
<th>(-1620,770,3600)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Defuzzified value</td>
<td>828.333</td>
<td>828.333</td>
<td>840</td>
<td>843.333</td>
</tr>
<tr>
<td>2</td>
<td>(-30,75,238)</td>
<td>(-58,75,260)</td>
<td>(-116,75,326)</td>
<td>(-116,75,326)</td>
</tr>
<tr>
<td>Defuzzified value</td>
<td>84.667</td>
<td>84.667</td>
<td>85</td>
<td>85</td>
</tr>
</tbody>
</table>

In the tabulated values furnished above, the first two columns and corresponding rows displays the optimum solution extracted by breaking the ties not by selecting arbitrary, and the successive column values displays the arbitrary selection.

5. Concluding Remarks

As a result of using the suggestion given in this note, it is possible to get an optimal solution of an unbalanced fuzzy transportation problem occurring in real life situations.
References


