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THE MAPPING METHOD FOR SOLVING A NEW MODEL OF NONLINEAR PARTIAL DIFFERENTIAL EQUATION

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Abstract: In this paper we present a new model of Benjamin-Bona-Mahony (BBM) equation. Then we apply the mapping method to solve the new model. Exact travelling wave solutions are obtained and expressed in terms of hyperbolic functions, trigonometric functions, rational functions and elliptic functions.

Key Words: mapping method, exact solutions, combined Benjamin-Bona-Mahony (BBM), modified Benjamin-Bona-Mahony (mBBM) equation

1. Introduction

In recent years, directly searching for exact solutions of nonlinear partial differential equations (PDEs) has become more and more attractive field in different branches of physics and applied mathematics. These equations appear in con-

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densed matter, solid state physics, fluid mechanics, chemical kinetics, plasma physics, nonlinear optics, propagation of fluxions in Josephson junctions, theory of turbulence, ocean dynamics, biophysics and star formation and many others.

In order to get exact solutions directly, many powerful methods have been introduced such as the $\left(\frac{G'}{G}\right)$-expansion method [1], inverse scattering method [2,3], Hirota's bilinear method [4,5], the tanh method [6,7], the sine-cosine method [8,9], Backlund transformation method [10,11], the homogeneous balance [12,13], Darboux transformation [14], the Jacobi elliptic function expansion method [15].

Recently, Yan-Ze Peng [16] introduced a new approach, namely, the mapping method for a reliable treatment of the nonlinear wave equations. The useful mapping method is then widely used by many authors [17,18,19].

Benjamin-Bona-Mahony (BBM) equation [20], is nonlinear wave equations modeling unidirectional propagation of long wave in dispersive media. It is originally derived by using a Pade$^{'}$ $(2,2)$ approximation of the phase velocity that arises in linear wave theory [21].

2. Mapping Method

Consider the general nonlinear partial differential equations (PDEs), say, in two variables,

$$P(u, u_x, u_t, u_{xx}, u_{xt}, ...) = 0. \tag{1}$$

Let $u(x,t) = u(\xi)$, $\xi = \mu\,(x - ct)$ then equation (1) reduces to a nonlinear ordinary differential equation (ODE)

$$Q(u, u', u'', ...) = 0. \tag{2}$$

Assume the solution of equation (2) takes the form

$$u(x,t) = u(\xi) = a_0 + \sum_{i=1}^{m} a_i\,(f(\xi))^i + b_i\,(f(\xi))^{-i}\,, \tag{3}$$

where the coefficients $a_i(i = 0, 1, 2, ..., m\ ), \mu, c$ are constants to be determined, and $f = f(\xi)$ satisfies a nonlinear ordinary differential equation

$$\frac{df(\xi)}{d\xi} = \sqrt{pf^2(\xi) + \frac{1}{2}qf^4(\xi) + r},\ \ p, q, r \in R, \tag{4}$$

where the coefficients $a_0, a_i, b_i (i = 1, 2, ...m\ ), \mu$ and c are constants to be determined and $f = f(\xi)$ satisfies (4), the parameter m will be found by balancing the highest-order nonlinear terms with the highest-order partial derivative term in the given equation. Substituting (3) into (2), using (4) repeatedly, and setting the coefficients of the each order of $f^i(\xi)$, $f^i(\xi)\sqrt{pf^2(\xi) + \frac{1}{2}qf^4(\xi) + r}$ to zero, we obtain a set of nonlinear algebraic equations for $a_0, a_i, b_i (i = 1, 2, ...n\ ), \mu, c$. With the aid of the computer program Maple, we can solve the set of nonlinear algebraic equations and obtain all the constants $a_0, a_i, b_i (i = 1, 2, ...n\ ), \mu$ and c. The ODE (4) has the following solutions

1. $f(\xi) = \text{sech}(\xi)$, $[p = 1, q = -2, r = 0]$,
2. $f(\xi) = \tanh(\xi)$, $[p = -2, q = 2, r = 1]$,
3. $f(\xi) = \frac{1}{2}\tanh(2\xi)$ or $\frac{1}{2}\coth(2\xi)$, $[p = -8, q = 32, r = 1]$,
4. $f(\xi) = \frac{1}{2}\tan(2\xi)$, or $-\frac{1}{2}\cot(2\xi)$, $[p = 8, q = 32, r = 1]$,
5. $f(\xi) = \text{sn}\xi$, $\left[p = -\left(k^2 + 1\right), q = 2k^2, r = 1\right]$,
6. $f(\xi) = \text{ns}\xi$, $\left[p = -\left(k^2 + 1\right), q = 2, r = k^2\right]$,
7. $f(\xi) = \text{cd}\xi$, $\left[p = -\left(k^2 + 1\right), q = 2k^2, r = 1\right]$,
8. $f(\xi) = \text{dc}\xi$, $\left[p = -\left(k^2 + 1\right), q = 2, r = k^2\right]$,
9. $f(\xi) = \text{cn}\xi$, $\left[p = 2k^2 - 1, q = -2k^2, r = 1 - k^2\right]$,
10. $f(\xi) = \text{nc}\xi$, $\left[p = 2k^2 - 1, q = 2\left(1 - k^2\right),\ r = -k^2\right]$,
11. $f(\xi) = \text{dn}\xi$, $\left[p = 2 - k^2, q = -2, r = -\left(1 - k^2\right)\right]$,
12. $f(\xi) = \text{nd}\xi$, $\left[p = 2 - k^2, q = 2\left(k^2 - 1\right), r = -1\right]$,
13. $f(\xi) = \text{cs}\xi$, $\left[p = 2 - k^2, q = 2, r = 1 - k^2\right]$,
14. $f(\xi) = \text{sc}\xi$, $\left[p = 2 - k^2, q = 2\left(1 - k^2\right), r = 1\right]$,
15. $f(\xi) = \text{ds}\xi$, $\left[p = -1 + 2k^2, q = 2, r = -k^2\left(1 - k^2\right)\right]$,
16. $f(\xi) = \text{sd}\xi$, $\left[p = -1 + 2k^2, q = 2k^2\left(k^2 - 1\right), r = 1\right]$,
17. $f(\xi) = \text{sc}\xi \pm \text{nc}\xi$, $\left[p = \frac{1+k^2}{2}, q = \frac{1-k^2}{2}, r = \frac{1-k^2}{4}\right]$,

18. $f(\xi) = \frac{\mathrm{sn}\xi}{1 \pm \mathrm{dn}\xi}$, $\left[p = \frac{k^2-2}{2}, q = \frac{k^2}{2}, r = \frac{1}{4}\right]$,

19. $f(\xi) = \frac{\mathrm{dn}\xi}{1 \pm k\mathrm{sn}\xi}$, $\left[p = \frac{k^2+1}{2}, q = \frac{k^2-1}{2}, r = \frac{1-k^2}{4}\right]$,

20. $f(\xi) = k\mathrm{cn}\xi \pm \mathrm{dn}\xi$, $\left[p = \frac{k^2+1}{2}, q = \frac{-1}{2}, r = \frac{-\left(1-k^2\right)^2}{4}\right]$,

21. $f(\xi) = \frac{\mathrm{cn}\xi}{1 \pm \mathrm{sn}\xi}$, $\left[p = \frac{k^2+1}{2}, q = \frac{1-k^2}{2}, r = \frac{1-k^2}{4}\right]$,

22. $f(\xi) = k\mathrm{sn}\xi \pm i\mathrm{dn}\xi$, $\left[p = \frac{1-2k^2}{2}, q = \frac{1}{2}, r = \frac{k^2}{4}\right]$,

23. $f(\xi) = k\mathrm{sn}\xi \pm i\mathrm{cn}\xi$, $\left[p = \frac{k^2-2}{2}, q = \frac{k^2}{2}, r = \frac{k^2}{4}\right]$,

24. $f(\xi) = \mathrm{ns}\xi \pm \mathrm{ds}\xi$, $\left[p = \frac{k^2-2}{2}, q = \frac{1}{2}, r = \frac{k^4}{4}\right]$,

25. $f(\xi) = \mathrm{ns}\xi - \mathrm{cs}\xi$, $\left[p = \frac{1-2k^2}{2}, q = \frac{1}{2}, r = \frac{1}{4}\right]$,

26. $f(\xi) = \frac{\mathrm{cn}\xi}{\sqrt{1-k^2}\mathrm{sn}\xi \pm \mathrm{dn}\xi}$, $\left[p = \frac{1-2k^2}{2}, q = \frac{1}{2}, r = \frac{1}{4}\right]$,

27. $f(\xi) = \frac{\mathrm{sn}\xi}{\mathrm{cn}\xi \pm \mathrm{dn}\xi}$, $\left[p = \frac{1+k^2}{2}, q = \frac{\left(1-k^2\right)^2}{2}, r = \frac{1}{4}\right]$,

28. $f(\xi) = \frac{\mathrm{cn}\xi}{\sqrt{1-k^2} \pm \mathrm{dn}\xi}$, $\left[p = \frac{k^2-2}{2}, q = \frac{k^2}{2}, r = \frac{1}{4}\right]$,

29. $f(\xi) = \frac{-1}{\sqrt{\frac{c}{2}}\xi}$, $[p = 0, q = c, r = 0]$,

30. $f(\xi) = e^{\xi}$, $[p = 1, q = 0, r = 0]$.

The multiple exact special solutions of nonlinear partial differential equation (1) are obtained by making use of (3) and the solutions of ODE(4).

3. Application

In this section, we present our proposed equation, namely, a combined Benjamin-Bona-Mahony (BBM) and modified Benjamin-Bona-Mahony (mBBM) equation as the form

$$u_t(x,t) + u_x(x,t) + \beta u(x,t)\, u_x(x,t) + \beta u^2(x,t)\, u_x(x,t) - \alpha u_{xxt}(x,t) = 0 \quad (5)$$

Where

$$u_t\left(x,t\right)+u_x\left(x,t\right)+\beta u\left(x,t\right)u_x\left(x,t\right)-\alpha u_{xxt}\left(x,t\right)=0,\quad \beta>0,\alpha\neq 0 \qquad (6)$$

is Benjamin-Bona-Mahony (BBM) equation

and

$$u_t\left(x,t\right)+u_x\left(x,t\right)+\beta u^2\left(x,t\right)u_x\left(x,t\right)-\alpha u_{xxt}\left(x,t\right)=0 \qquad (7)$$

is modified Benjamin-Bona-Mahony (mBBM) equation.

Substituting $u(x,t)=u(\xi)$, $\xi=\lambda\left(x-ct\right)$ in (5) and integrating once yields

$$\left(1-c\right)u\left(\xi\right)+\frac{\beta}{2}\left(u\left(\xi\right)\right)^2+\frac{\beta}{3}\left(u\left(\xi\right)\right)^3+c\lambda^2\alpha\frac{d^2}{d\xi^2}u\left(\xi\right)=0. \qquad (8)$$

Balancing the order of the nonlinear term u^3 with the highest derivative u'' gives $3m=m+2$ that gives $m=1$. Now, we apply the mapping method to solve our equation. consequently we get the original solutions for our new equation, as the following:
Assume, the solution of (8) has the form

$$u(\xi)=a_0+a_1f\left(\xi\right)+b_1f\left(\xi\right)^{-1} \qquad (9)$$

Where

$$\frac{df(\xi)}{d\xi}=\sqrt{pf^2(\xi)+\frac{1}{2}qf^4(\xi)+r},\ p,q,r\in R. \qquad (10)$$

Substituting (9) in (8) and using (10), collecting the coefficients of each power of $f^i, 0\leq i\leq 6$, setting each coefficient to zero, and solving the resulting system, obtain the following sets of solutions.

1.

$$a_0=0,a_1=b_1=0,c=c,\lambda=\lambda,$$

2.

$$a_0=a_0,a_1=b_1=0,c=c,\lambda=\lambda,$$

3.

$$a_0=\frac{-1}{2},a_1=\sqrt{-\frac{q}{4p}},b_1=0,c=1-\frac{\beta}{6},\lambda=\pm\sqrt{\frac{\beta}{-12\alpha p+2\alpha\beta p}},$$

4.

$$a_0 = \frac{-1}{2}, a_1 = -\sqrt{-\frac{q}{4p}}, b_1 = 0, c = 1 - \frac{\beta}{6}, \lambda = \pm\sqrt{\frac{\beta}{-12\alpha p + 2\alpha\beta p}},$$

5.

$$a_0 = \frac{-1}{2}, a_1 = 0, b_1 = \sqrt{-\frac{r}{2p}}, c = 1 - \frac{\beta}{6}, \lambda = \pm\sqrt{\frac{\beta}{-12\alpha p + 2\alpha\beta p}},$$

6.

$$a_0 = \frac{-1}{2}, a_1 = 0, b_1 = -\sqrt{-\frac{r}{2p}}, c = 1 - \frac{\beta}{6}, \lambda = \pm\sqrt{\frac{\beta}{-12\alpha p + 2\alpha\beta p}},$$

7.

$$a_0 = \frac{-1}{2}, a_1 = \frac{1}{2}\sqrt{\frac{q\left(-p + 3\sqrt{2rq}\right)}{p^2 - 18rq}}, b_1 = \frac{1}{2}\frac{-6rq + p\sqrt{2rq}}{\sqrt{\frac{q\left(-p+3\sqrt{2rq}\right)}{p^2-18rq}}\left(p^2 - 18rq\right)},$$

$$c = 1 - \frac{\beta}{6}, \lambda = \pm\frac{\sqrt{\alpha\beta\left(p^2\beta - 6p^2 + 108rq - 18rq\beta\right)\left(-p + 3\sqrt{2rq}\right)}}{\alpha\left(p^2\beta - 6p^2 + 108rq - 18rq\beta\right)},$$

8.

$$a_0 = \frac{-1}{2}, a_1 = -\frac{1}{2}\sqrt{\frac{q\left(-p + 3\sqrt{2rq}\right)}{p^2 - 18rq}}, b_1 = \frac{1}{2}\frac{6rq - p\sqrt{2rq}}{\sqrt{\frac{q\left(-p+3\sqrt{2rq}\right)}{p^2-18rq}}\left(p^2 - 18rq\right)},$$

$$c = 1 - \frac{\beta}{6}, \lambda = \pm\frac{\sqrt{\alpha\beta\left(p^2\beta - 6p^2 + 108rq - 18rq\beta\right)\left(-p + 3\sqrt{2rq}\right)}}{\alpha\left(p^2\beta - 6p^2 + 108rq - 18rq\beta\right)},$$

9.

$$a_0 = \frac{-1}{2}, a_1 = \frac{1}{2}\sqrt{\frac{q\left(p + 3\sqrt{2rq}\right)}{-p^2 + 18rq}}, b_1 = -\frac{1}{2}\frac{6rq + p\sqrt{2rq}}{\sqrt{\frac{q\left(p+3\sqrt{2rq}\right)}{18rq-p^2}}\left(p^2 - 18rq\right)},$$

$$c=1-\frac{\beta}{6},\lambda=\pm\frac{\sqrt{-\alpha\beta\left(p^2\beta-6p^2+108rq-18rq\beta\right)\left(p+3\sqrt{2rq}\right)}}{\alpha\left(p^2\beta-6p^2+108rq-18rq\beta\right)},$$

10.

$$a_0=\frac{-1}{2},a_1=-\frac{1}{2}\sqrt{\frac{q\left(p+3\sqrt{2rq}\right)}{-p^2+18rq}},b_1=\frac{1}{2}\frac{6rq+p\sqrt{2rq}}{\sqrt{\frac{q\left(p+3\sqrt{2rq}\right)}{18rq-p^2}}\left(p^2-18rq\right)},$$

$$c=1-\frac{\beta}{6},\lambda=\pm\frac{\sqrt{-\alpha\beta\left(p^2\beta-6p^2+108rq-18rq\beta\right)\left(p+3\sqrt{2rq}\right)}}{\alpha\left(p^2\beta-6p^2+108rq-18rq\beta\right)}.$$

Using (9), the solution of (10) when $p=1,q=-2,r=0$, and the sets of solutions (1)-(10), we get

$u_1\left(x,t\right)=0,$

$u_2\left(x,t\right)=a_0 \qquad \forall a_0\in R,$

for $\frac{-\beta}{\alpha(\beta-6)}>0$, we obtain

$u_{3,4}\left(x,t\right)=-\frac{1}{2}\pm\frac{1}{\sqrt{2}}\text{sech}\left(\sqrt{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right),$

for $\frac{-\beta}{\alpha(\beta-6)}<0$, we get

$u_{5,6}\left(x,t\right)=-\frac{1}{2}\pm\frac{1}{\sqrt{2}}\sec\left(\sqrt{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right).$

Using (9), the solution of (10) when $p=-2,q=2,r=1$, and the sets of solutions (3)-(10), we get

for $\frac{\beta}{\alpha(\beta-6)}>0$, we obtain

$u_{7,8}\left(x,t\right)=-\frac{1}{2}\pm\frac{1}{2}\tanh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right),$

$u_{9,10}\left(x,t\right)=-\frac{1}{2}\pm\frac{1}{2}\coth\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right),$

$u_{11,12}\left(x,t\right)=-\frac{1}{2}\pm\frac{1}{2\sqrt{2}}\tan\left(\frac{1}{2\sqrt{2}}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)$
$\mp\frac{1}{2\sqrt{2}}\cot\left(\frac{1}{2\sqrt{2}}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right),$

$u_{13,14}\left(x,t\right)=-\frac{1}{2}\pm\frac{1}{4}\tanh\left(\frac{1}{4}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)$
$\pm\frac{1}{4}\coth\left(\frac{1}{4}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right).$

for $\frac{\beta}{\alpha(\beta-6)}<0$, we get
$u_{15,16}(x,t)=-\frac{1}{2}\pm\frac{i}{2}\tan\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right),$
$u_{17,18}(x,t)=-\frac{1}{2}\pm\frac{i}{2}\cot\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right),$
$u_{19,20}(x,t)=-\frac{1}{2}\pm\frac{i}{2\sqrt{2}}\tanh\left(\frac{1}{2\sqrt{2}}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)$
$\mp\frac{i}{2\sqrt{2}}\coth\left(\frac{1}{2\sqrt{2}}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right),$
$u_{21,22}(x,t)=-\frac{1}{2}\pm\frac{i}{4}\tan\left(\frac{1}{4}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)$
$\pm\frac{i}{4}\cot\left(\frac{1}{4}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right).$

Using (9), the solution of (10) when $p=8, q=32, r=1$, and the sets of solutions (3)-(10), we get, $[\,u_{7,8}(x,t), u_{9,10}(x,t), ..., u_{21,22}(x,t)]$

Using (9), the solution of (10) when $p=-8, q=32, r=1$, and the sets of solutions (3)-(10), we get, $[\,u_{7,8}(x,t), u_{9,10}(x,t), ..., u_{21,22}(x,t)]$.

Using (9), the solution of (10) when $p=-\left(k^2+1\right), q=2k^2, r=1$, and the sets of solutions (3)-(10), we get

$u_{23,24,...,30}(x,t)=a_0+a_1\text{sn}\xi+b_1\text{ns}\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k\to 1$ we obtain, $[\,u_{7,8}(x,t), u_{9,10}(x,t), ..., u_{21,22}(x,t)]$, when $k\to 0$ we obtain
for $\frac{\beta}{\alpha(\beta-6)}>0$,
$u_{31,32}(x,t)=-\frac{1}{2}\pm\frac{1}{\sqrt{2}}\csc\left(\frac{1}{\sqrt{2}}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right),$

for $\frac{\beta}{\alpha(\beta-6)}<0$,

$u_{33,34}(x,t)=-\frac{1}{2}\pm\frac{i}{\sqrt{2}}\text{csch}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right).$

Using (9), the solution of (10) when $p=-\left(k^2+1\right), q=2k^2, r=1$, and the sets of solutions (3)-(10), we get

$u_{35,36,...,42}(x,t)=a_0+a_1\text{cd}\xi+b_1\text{dc}\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k\to 1$ we obtain constant solutions, when $k\to 0$ we obtain, $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$.

Using (9), the solution of (10) when $p=-\left(k^2+1\right), q=2, r=k^2$, and the sets of solutions (3)-(10), we get

$u_{43,44,\ldots,50}(x,t) = a_0 + a_1 \text{ns}\xi + b_1 \text{sn}\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k \to 1$ we obtain, $[\ u_{7,8}(x,t), u_{9,10}(x,t), ..., u_{21,22}(x,t)]$, when $k \to 0$ we obtain, $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$.

Using (9), the solution of (10) when $p = -\left(k^2+1\right), q = 2, r = k^2$, and the sets of solutions (3)-(10), we get

$u_{51,52,\ldots,57}(x,t) = a_0 + a_1 \text{dc}\xi + b_1 \text{cd}\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions 3-10.

Note that, when $k \to 1$ we obtain constant solution, when $k \to 0$ we obtain $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$.

Using (9), the solution of (10) when $p = 2k^2 - 1, q = -2k^2$, $r = 1 - k^2$, and the sets of solutions (3)-(10), we get

$u_{58,59,\ldots,65}(x,t) = a_0 + a_1 \text{cn}\xi + b_1 \text{nc}\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k \to 1$ we obtain, $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$, when $k \to 0$ we obtain, $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$.

Using (9), the solution of (10) when $p = 2k^2 - 1, q = 2\left(1-k^2\right),\ \ r = -k^2$, and the sets of solutions (3)-(10), we get

$u_{66,67,\ldots,73}(x,t) = a_0 + a_1 \text{nc}\xi + b_1 \text{cn}\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k \to 1$ we obtain, $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$, when $k \to 0$ we obtain, $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$.

Using (9), the solution of (10) when $p = 2 - k^2, q = -2$, $r = -\left(1-k^2\right)$, and the sets of solutions (3)-(10), we get

$u_{74,75,\ldots,81}(x,t) = a_0 + a_1 \text{dn}\xi + b_1 \text{nd}\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k \to 1$ we obtain, $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$, when $k \to 0$ we obtain constant solutions.

Using (9), the solution of (10) when $p = 2 - k^2, q = 2\left(k^2-1\right), r = -1$, and the sets of solutions (3)-(10), we get

$u_{82,83,\ldots,89}(x,t) = a_0 + a_1 \text{nd}\xi + b_1 \text{dn}\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k \to 1$ we obtain, $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$, when $k \to 0$ we obtain constant solutions .

Using (9), the solution of (10) when $p = 2 - k^2, q = 2, r = 1 - k^2$, and the sets of solutions (3)-(10), we get

$u_{90,91,\ldots,97}(x,t) = a_0 + a_1 \mathrm{cs}\xi + b_1 \mathrm{sc}\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k \to 1$ we obtain, $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$, when $k \to 0$ we obtain, $[\ u_{7,8}(x,t), u_{9,10}(x,t), ..., u_{21,22}(x,t)]$.

Using (9), the solution of (10) when $p = 2 - k^2, q = 2\left(1 - k^2\right), r = 1$, and the sets of solutions (3)-(10), we get

$u_{98,99,\ldots,105}(x,t) = a_0 + a_1 \mathrm{sc}\xi + b_1 \mathrm{cs}\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k \to 1$ we obtain, $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$, when $k \to 0$ we obtain, $[\ u_{7,8}(x,t), u_{9,10}(x,t), ..., u_{21,22}(x,t)]$.

Using (9), the solution of (10) when $p = -1 + 2k^2, q = 2, r = -k^2\left(1 - k^2\right)$, and the sets of solutions (3)-(10), we get

$u_{106,107,\ldots,113}(x,t) = a_0 + a_1 \mathrm{ds}\xi + b_1 \mathrm{sd}\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k \to 1$ we obtain, $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$, when $k \to 0$ we obtain also, $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$.

Using (9), the solution of (10) when $p = -1 + 2k^2, q = 2k^2\left(k^2 - 1\right), r = 1$, and the sets of solutions (3)-(10), we get

$u_{114,115,\ldots,121}(x,t) = a_0 + a_1 \mathrm{sd}\xi + b_1 \mathrm{ds}\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k \to 1$ we obtain, $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$, when $k \to 0$ we obtain also, $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$.

Using (9), the solution of (10) when $p = \frac{1+k^2}{2}, q = \frac{1-k^2}{2}, r = \frac{1-k^2}{4}$, and the sets of solutions (3)-(10), we get

$u_{122,123,\ldots,129}(x,t) = a_0 + a_1(\mathrm{sc}\xi \pm \mathrm{nc}\xi) + b_1 \frac{1}{\mathrm{sc}\xi \pm \mathrm{nc}\xi}$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k \to 1$ we obtain constant solutions, when $k \to 0$ we obtain, for $\frac{\beta}{\alpha(\beta-6)} < 0$,

$$u_{130,131}(x,t) = -\frac{1}{2} + \frac{i}{2}\left(\tan\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x - \left(1 - \frac{\beta}{6}\right)t\right)\right) \pm \sec\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x - \left(1 - \frac{\beta}{6}\right)t\right)\right)\right),$$

$$u_{132,133}(x,t) = -\frac{1}{2} - \frac{i}{2}\left(\tan\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x - \left(1 - \frac{\beta}{6}\right)t\right)\right) \pm \sec\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x - \left(1 - \frac{\beta}{6}\right)t\right)\right)\right),$$

$$u_{134,135}(x,t) = -\frac{1}{2} + \frac{i}{2\left(\tan\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x - \left(1 - \frac{\beta}{6}\right)t\right)\right) \pm \sec\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x - \left(1 - \frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{136,137}(x,t)=-\frac{1}{2}-\frac{i}{2\left(\tan\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\sec\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{138,139}(x,t)=-\frac{1}{2}+\frac{i}{4}\left(\tan\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\sec\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)$$
$$-\frac{\frac{i}{4}}{\tan\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\sec\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{140,141}(x,t)=-\frac{1}{2}-\frac{i}{4}\left(\tan\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\sec\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)$$
$$+\frac{\frac{i}{4}}{\tan\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\sec\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{142,143}(x,t)=-\frac{1}{2}+\frac{\sqrt{2}}{4}\left(i\tanh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\operatorname{sech}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)$$
$$+\frac{\frac{\sqrt{2}}{4}}{i\tanh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\operatorname{sech}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{144,145}(x,t)=-\frac{1}{2}-\frac{\sqrt{2}}{4}\left(i\tanh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\operatorname{sech}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)$$
$$-\frac{\frac{\sqrt{2}}{4}}{i\tanh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\operatorname{sech}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{146,147}(x,t)=-\frac{1}{2}+\frac{\sqrt{2}}{4}\left(i\tanh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\operatorname{sech}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)$$
$$+\frac{\frac{\sqrt{2}}{4}}{i\tanh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\mp\operatorname{sech}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{148,149}(x,t) = -\tfrac{1}{2} - \tfrac{\sqrt{2}}{4}\left(i\tanh\left(\tfrac{\sqrt{2}}{2}\sqrt{\tfrac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\tfrac{\beta}{6}\right)t\right)\right) \pm \operatorname{sech}\left(\tfrac{\sqrt{2}}{2}\sqrt{\tfrac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\tfrac{\beta}{6}\right)t\right)\right)\right)$$

$$-\frac{\frac{\sqrt{2}}{4}}{i\tanh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \mp \operatorname{sech}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

for $\frac{\beta}{\alpha(\beta-6)} > 0$,

$$u_{150,151}(x,t) = -\tfrac{1}{2} + \tfrac{1}{2}\tanh\left(\sqrt{\tfrac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\tfrac{\beta}{6}\right)t\right)\right) \pm \tfrac{i}{2}\operatorname{sech}\left(\sqrt{\tfrac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\tfrac{\beta}{6}\right)t\right)\right),$$

$$u_{152,153}(x,t) = -\tfrac{1}{2} - \tfrac{1}{2}\tanh\left(\sqrt{\tfrac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\tfrac{\beta}{6}\right)t\right)\right) \pm \tfrac{i}{2}\operatorname{sech}\left(\sqrt{\tfrac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\tfrac{\beta}{6}\right)t\right)\right),$$

$$u_{154,155}(x,t) = -\tfrac{1}{2} + \frac{1}{2\left(\tanh\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm i\operatorname{sech}\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{156,157}(x,t) = -\tfrac{1}{2} - \frac{1}{2\left(\tanh\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm i\operatorname{sech}\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{158,159}(x,t) = -\tfrac{1}{2} - \tfrac{1}{4}\left(\tanh\left(\tfrac{1}{2}\sqrt{\tfrac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\tfrac{\beta}{6}\right)t\right)\right) \pm i\operatorname{sech}\left(\tfrac{1}{2}\sqrt{\tfrac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\tfrac{\beta}{6}\right)t\right)\right)\right)$$

$$+\frac{\frac{1}{4}}{\tanh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm i\operatorname{sech}\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{160,161}(x,t) = -\tfrac{1}{2} + \tfrac{1}{4}\left(\tanh\left(\tfrac{1}{2}\sqrt{\tfrac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\tfrac{\beta}{6}\right)t\right)\right) \pm i\operatorname{sech}\left(\tfrac{1}{2}\sqrt{\tfrac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\tfrac{\beta}{6}\right)t\right)\right)\right)$$

$$-\frac{\frac{1}{4}}{\tanh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm i\operatorname{sech}\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$u_{162,163}(x,t) = -\frac{1}{2} +$

$$\frac{\sqrt{2}}{4}\left(\tan\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm \sec\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)$$

$$+\frac{\frac{\sqrt{2}}{4}}{\tan\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm \sec\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$u_{164,165}(x,t) = -\frac{1}{2} -$

$$\frac{\sqrt{2}}{4}\left(\tan\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm \sec\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)$$

$$-\frac{\frac{\sqrt{2}}{4}}{\tan\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm \sec\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$u_{166,167}(x,t) = -\frac{1}{2}$

$$+\frac{\sqrt{2}}{4}\left(\tan\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm \sec\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)$$

$$+\frac{\frac{\sqrt{2}}{4}}{\tan\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \mp \sec\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$u_{168,169}(x,t) = -\frac{1}{2} -$

$$\frac{\sqrt{2}}{4}\left(\tan\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm \sec\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)$$

$$-\frac{\frac{\sqrt{2}}{4}}{\tan\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \mp \sec\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}.$$

Using (9), the solution of (10) when $p = \frac{k^2-2}{2}, q = \frac{k^2}{2}\ , r = \frac{1}{4}$, and the sets of solutions (3)-(10), we get

$u_{170,171,\ldots,177}(x,t) = a_0 + a_1\frac{\mathrm{sn}\xi}{1\pm\mathrm{dn}\xi} + b_1\frac{1\pm\mathrm{dn}\xi}{\mathrm{sn}\xi}$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k \to 1$ we obtain for $\frac{\beta}{\alpha(\beta-6)} > 0$,

$$u_{178,179}(x,t) = -\frac{1}{2} + \frac{\frac{1}{2}\tanh\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1 \pm \mathrm{sech}\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{180,181}(x,t) = -\frac{1}{2} - \frac{\frac{1}{2}\tanh\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm\operatorname{sech}\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{182,183}(x,t) = -\frac{1}{2} + \frac{\frac{1}{2}\left(1\pm\operatorname{sech}\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{\tanh\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{184,185}(x,t) = -\frac{1}{2} - \frac{\frac{1}{2}\left(1\pm\operatorname{sech}\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{\tanh\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$\begin{aligned} u_{186,187}(x,t) &= -\frac{1}{2} + \frac{\frac{\sqrt{2}}{4}\left(\tan\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{1\pm\sec\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)} \\ &\quad + \frac{\frac{\sqrt{2}}{4}\left(1\pm\sec\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{\tan\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}, \end{aligned}$$

$$\begin{aligned} u_{188,189}(x,t) &= -\frac{1}{2} + \frac{\frac{\sqrt{2}}{4}\left(\tan\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{1\pm\sec\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)} \\ &\quad + \frac{\frac{\sqrt{2}}{4}\left(1\pm\sec\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{\tan\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}, \end{aligned}$$

$$\begin{aligned} u_{190,191}(x,t) &= -\frac{1}{2} + \frac{\frac{1}{4}\left(\tanh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{1\pm\operatorname{sech}\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)} \\ &\quad + \frac{\frac{1}{4}\left(1\pm\operatorname{sech}\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{\tanh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}, \end{aligned}$$

$$u_{192,193}(x,t) = -\frac{1}{2} - \frac{\frac{1}{4}\left(\tanh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{1\pm\operatorname{sech}\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}$$

$$-\frac{\frac{1}{4}\left(1\pm\operatorname{sech}\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{\tanh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}.$$

for $\frac{\beta}{\alpha(\beta-6)}<0$,

$$u_{194,195}(x,t)=-\frac{1}{2}+i\frac{\frac{1}{2}\tan\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm\sec\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{196,197}(x,t)=-\frac{1}{2}-\frac{\frac{1}{2}\tan\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm\sec\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{198,199}(x,t)=-\frac{1}{2}+\frac{\frac{1}{2}\left(1\pm\sec\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{i\tan\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{200,201}(x,t)=-\frac{1}{2}-\frac{\frac{1}{2}\left(1\pm\sec\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{i\tan\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$\begin{aligned}u_{202,203}(x,t) &= -\frac{1}{2}+i\frac{\frac{\sqrt{2}}{4}\left(\tanh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{1\pm\operatorname{sech}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}\\ &\quad -i\frac{\frac{\sqrt{2}}{4}\left(1\pm\operatorname{sech}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{\tanh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},\end{aligned}$$

$$\begin{aligned}u_{204,205}(x,t) &= -\frac{1}{2}-i\frac{\frac{\sqrt{2}}{4}\left(\tanh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{1\pm\operatorname{sech}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}\\ &\quad +i\frac{\frac{\sqrt{2}}{4}\left(1\pm\operatorname{sech}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{\tanh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},\end{aligned}$$

$$u_{206,207}(x,t) = -\frac{1}{2}+i\frac{\frac{1}{4}\left(\tan\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{1\pm\sec\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}$$

$$+i\frac{\frac{1}{4}\left(1+\sec\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{\tan\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{208,209}(x,t) = -\frac{1}{2}-i\frac{\frac{1}{4}\left(\tan\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{1\pm\sec\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)} -i\frac{\frac{1}{4}\left(1+\sec\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}{\tan\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}.$$

When $k \to 0$ we obtain, $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$.

Using (9), the solution of (10) when $p=\frac{k^2+1}{2}, q=\frac{k^2-1}{2}$ $, r=\frac{1-k^2}{4}$, and the sets of solutions (3)-(10), we get

$u_{210,211,\ldots,218}(x,t)=a_0+a_1\frac{\text{dn}\xi}{1\pm k\text{sn}\xi}+b_1\frac{1\pm k\text{sn}\xi}{\text{dn}\xi}$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k \to 1$ we obtain constant solutions, also when $k \to 0$ we obtain constant solutions.

Using (9), the solution of (10) when $p=\frac{k^2+1}{2}, q=\frac{-1}{2}$ $, r=\frac{-\left(1-k^2\right)^2}{4}$, and the sets of solutions (3)-(10), we get

$u_{219,220,\ldots,226}(x,t)=a_0+a_1(k\text{cn}\xi\pm\text{dn}\xi)+\frac{b_1}{(k\text{cn}\xi\pm\text{dn}\xi)}$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k \to 1$ we obtain, $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$, when $k \to 0$ we obtain constant solution.

Using (9), the solution of (10) when $p=\frac{k^2+1}{2}, q=\frac{1-k^2}{2}$ $, r=\frac{1-k^2}{4}$, and the sets of solutions (3)-(10), we get

$u_{227,228,\ldots,234}(x,t)=a_0+a_1\left(\frac{\text{cn}\xi}{1\pm\text{sn}\xi}\right)+b_1\frac{1\pm\text{sn}\xi}{\text{cn}\xi}$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k \to 1$ we obtain constant solution, when $k \to 0$ we obtain

for $\frac{\beta}{\alpha(\beta-6)}<0$,

$$u_{235,236}(x,t)=-\frac{1}{2}+\frac{i}{2}\frac{\cos\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm\sin\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{237,238}(x,t)=-\frac{1}{2}-\frac{i}{2}\frac{\cos\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm\sin\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{239,240}(x,t)=-\frac{1}{2}+\frac{i}{2}\frac{1\pm\sin\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cos\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{241,242}(x,t)=-\frac{1}{2}-\frac{i}{2}\frac{1\pm\sin\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cos\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$\begin{aligned}u_{243,244}(x,t) &= -\frac{1}{2}+\frac{i}{4}\frac{\cos\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm\sin\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}\\ &\quad -\frac{i}{4}\frac{1\pm\sin\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cos\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},\end{aligned}$$

$$\begin{aligned}u_{245,246}(x,t) &= -\frac{1}{2}-\frac{i}{4}\frac{\cos\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm\sin\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}\\ &\quad +\frac{i}{4}\frac{1\pm\sin\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cos\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},\end{aligned}$$

$$\begin{aligned}u_{247,248}(x,t) &= -\frac{1}{2}+\frac{i}{4}\frac{\cos\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm\sin\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}\\ &\quad -\frac{i}{4}\frac{1\mp\sin\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cos\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},\end{aligned}$$

$$u_{249,250}(x,t) = -\frac{1}{2}-\frac{i}{4}\frac{\cos\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm\sin\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}$$

$$+\frac{i}{4}\frac{1\mp\sin\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cos\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$\begin{aligned}u_{251,252}(x,t) &= -\frac{1}{2}+\frac{\sqrt{2}}{4}\frac{\cosh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm i\sinh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}\\ &\quad -\frac{\sqrt{2}}{4}\frac{1\mp i\sinh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cosh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},\end{aligned}$$

$$\begin{aligned}u_{253,254}(x,t) &= -\frac{1}{2}-\frac{\sqrt{2}}{4}\frac{\cosh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm i\sinh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}\\ &\quad +\frac{\sqrt{2}}{4}\frac{1\mp i\sinh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cosh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}.\end{aligned}$$

for $\frac{\beta}{\alpha(\beta-6)}>0$,

$$u_{255,256}(x,t)=-\frac{1}{2}+\frac{1}{2}\frac{\cosh\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{i\pm\sinh\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{257,258}(x,t)=-\frac{1}{2}-\frac{1}{2}\frac{\cosh\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{i\pm\sinh\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{259,260}(x,t)=-\frac{1}{2}+\frac{1}{2}\frac{i\pm\sinh\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cosh\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{261,262}(x,t)=-\frac{1}{2}-\frac{1}{2}\frac{i\pm\sinh\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cosh\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{263,264}(x,t) = -\frac{1}{2}+\frac{i}{4}\frac{\cosh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm i\sinh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}$$

$$-\frac{i}{4}\frac{1\pm i\sinh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cosh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{265,266}\left(x,t\right) = -\frac{1}{2}-\frac{i}{4}\frac{\cosh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm i\sinh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)} +\frac{i}{4}\frac{1\pm i\sinh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cosh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{267,268}\left(x,t\right) = -\frac{1}{2}+\frac{i}{4}\frac{\cosh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm i\sinh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)} -\frac{i}{4}\frac{1\mp i\sinh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cosh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{269,270}\left(x,t\right) = -\frac{1}{2}-\frac{i}{4}\frac{\cosh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm i\sinh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)} +\frac{i}{4}\frac{1\mp i\sinh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cosh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{271,272}\left(x,t\right) = -\frac{1}{2}+\frac{\sqrt{2}}{4}\frac{\cos\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm\sin\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)} -\frac{\sqrt{2}}{4}\frac{1\mp\sin\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cos\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)},$$

$$u_{273,274}\left(x,t\right) = -\frac{1}{2}-\frac{\sqrt{2}}{4}\frac{\cos\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{1\pm\sin\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}$$

$$+\frac{\sqrt{2}}{4}\frac{1\mp\sin\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cos\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}.$$

Using (9), the solution of (10) when $p=\frac{1-2k^2}{2}, q=\frac{1}{2}$, $r=\frac{k^2}{4}$, and the sets of solutions (3)-(10), we get

$u_{275,276,...282}(x,t)=a_0+a_1(k\text{sn}\xi\pm i\text{dn}\xi)+b_1\frac{1}{k\text{sn}\xi\pm i\text{dn}\xi}$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k\to 1$ we obtain, $[u_{130,131}(x,t), u_{132,133}(x,t), ..., u_{168,169}(x,t)]$, when $k\to 0$, we obtain constant solutions.

Using (9), the solution of (10) when $p=\frac{k^2-2}{2}, q=\frac{k^2}{2}$, $r=\frac{k^2}{4}$, and the sets of solutions (3)-(10), we get

$u_{283,284,...,290}(x,t)=a_0+a_1(k\text{sn}\xi\pm i\text{cn}\xi)+b_1\frac{1}{k\text{sn}\xi\pm i\text{cn}\xi}$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k\to 1$ we obtain, $[u_{130,131}(x,t), u_{132,133}(x,t), ..., u_{168,169}(x,t)]$, when $k\to 0$, we obtain constant solutions.

Using (9), the solution of (10) when $p=\frac{k^2-2}{2}, q=\frac{1}{2}$, $r=\frac{k^4}{4}$, and the sets of solutions (3)-(10), we get

$u_{291,292,...,298}(x,t)=a_0+a_1(\text{ns}\xi\pm\text{ds}\xi)+b_1\frac{1}{\text{ns}\xi\pm\text{ds}\xi}$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k\to 1$ we obtain,
for $\frac{\beta}{\alpha(\beta-6)}>0$,

$$u_{299,300}(x,t)=-\frac{1}{2}+\frac{1}{2}\left(\coth\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\text{csch}\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right),$$

$$u_{301,302}(x,t)=-\frac{1}{2}-\frac{1}{2}\left(\coth\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\text{csch}\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right),$$

$$u_{303,304}(x,t)=-\frac{1}{2}+\frac{1}{2\left(\coth\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\text{csch}\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{305,306}(x,t)=-\frac{1}{2}-\frac{1}{2\left(\coth\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\text{csch}\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{307,308}(x,t) = -\frac{1}{2} + \frac{+\sqrt{2}}{\frac{\sqrt{2}}{4}\left(\cot\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm \csc\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)} \over 4\left(\cot\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm \csc\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)$$

$$u_{309,310}(x,t) = -\frac{1}{2} + \frac{\sqrt{2}}{4}\left(\cot\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm \csc\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)$$
$$+\frac{\sqrt{2}}{4\left(\cot\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \mp \csc\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{311,312}(x,t) = -\frac{1}{2} - \frac{\sqrt{2}}{4}\left(\cot\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm \csc\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)$$
$$-\frac{\sqrt{2}}{4\left(\cot\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm \csc\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{313,314}(x,t) = -\frac{1}{2} - \frac{\sqrt{2}}{4}\left(\cot\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm \csc\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)$$
$$-\frac{\sqrt{2}}{4\left(\cot\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \mp \csc\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{315,316}(x,t) = -\frac{1}{2} + \frac{1}{4}\left(\coth\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm \operatorname{csch}\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)$$
$$-\frac{1}{4\left(\coth\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \mp \operatorname{csch}\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{317,318}(x,t) = -\frac{1}{2} -$$

$$\frac{\frac{1}{4}\left(\coth\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\operatorname{csch}\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)+1}{4\left(\coth\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\mp\operatorname{csch}\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

for $\frac{\beta}{\alpha(\beta-6)}<0$,

$$u_{319,320}(x,t)=-\frac{1}{2}+\frac{i}{2}\left(\cot\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\csc\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right),$$

$$u_{321,322}(x,t)=-\frac{1}{2}-\frac{i}{2}\left(\cot\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\csc\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right),$$

$$u_{323,324}(x,t)=-\frac{1}{2}+\frac{i}{2\left(\cot\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\csc\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{325,326}(x,t)=-\frac{1}{2}-\frac{i}{2\left(\cot\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\csc\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{327,328}(x,t)=-\frac{1}{2}+\frac{i\sqrt{2}}{4}\left(\coth\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\operatorname{csch}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)$$
$$\frac{-i\sqrt{2}}{4\left(\coth\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\operatorname{csch}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{329,330}(x,t)=-\frac{1}{2}+\frac{i\sqrt{2}}{4}\left(\coth\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\operatorname{csch}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)$$
$$\frac{-i\sqrt{2}}{4\left(\coth\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\mp\operatorname{csch}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{331,332}(x,t)=-\frac{1}{2}$$

$$\frac{-\frac{i\sqrt{2}}{4}\left(\coth\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\operatorname{csch}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)+i\sqrt{2}}{4\left(\coth\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\operatorname{csch}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{333,334}(x,t)=-\frac{1}{2}$$

$$\frac{-\frac{i\sqrt{2}}{4}\left(\coth\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\operatorname{csch}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)+i\sqrt{2}}{4\left(\coth\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\mp\operatorname{csch}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{335,336}(x,t)=-\frac{1}{2}+$$

$$\frac{\frac{i}{4}\left(\cot\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\csc\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)+i}{4\left(\cot\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\mp\csc\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)},$$

$$u_{337,338}(x,t)=-\frac{1}{2}-$$

$$\frac{\frac{i}{4}\left(\cot\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm\csc\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)-i}{4\left(\cot\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\mp\csc\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\right)}.$$

When $k\to 0$, we obtain, $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$.

Using (9), the solution of (10) when $p=\frac{1-2k^2}{2}, q=\frac{1}{2}$, $r=\frac{1}{4}$, and the sets of solutions (3)-(10), we get

$u_{339,340,\dots,346}(x,t)=a_0+a_1(\text{ns}\xi-\text{cs}\xi)+b_1\frac{1}{\text{ns}\xi-\text{cs}\xi}$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k\to 1$ we obtain, $[u_{299,300}(x,t), u_{301,302}(x,t), ..., u_{337,338}(x,t)]$, also when $k\to 0$ we obtain, $[u_{299,300}(x,t), u_{301,302}(x,t), ..., u_{337,338}(x,t)]$.

Using (9), the solution of (10) when $p=\frac{1-2k^2}{2}, q=\frac{1}{2}$, $r=\frac{1}{4}$, and the sets of solutions (3)-(10), we get

$u_{347,348,\dots,354}(x,t)=a_0+a_1\left(\frac{\text{cn}\xi}{\sqrt{1-k^2}\text{sn}\xi\pm\text{dn}\xi}\right)+b_1\frac{\sqrt{1-k^2}\text{sn}\xi\pm\text{dn}\xi}{\text{cn}\xi}$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k \to 1$ constant solutions, when $k \to 0$ we obtain, $[u_{235,236}(x,t)$, $u_{237,238}(x,t), ..., u_{273,274}(x,t)]$.

Using (9), the solution of (10) when $p = \frac{1+k^2}{2}, q = \frac{\left(1-k^2\right)^2}{2}, r = \frac{1}{4}$, and the sets of solutions (3)-(10), we get

$u_{355,356,\ldots,362}(x,t) = a_0 + a_1\left(\frac{\text{sn}\xi}{\text{cn}\xi \pm \text{dn}\xi}\right) + b_1\frac{\text{cn}\xi \pm \text{dn}\xi}{\text{sn}\xi}$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k \to 1$ we get $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$, when $k \to 0$ we obtain
for $\frac{\beta}{\alpha(\beta-6)} > 0$,

$$u_{363,364}(x,t) = -\frac{1}{2} + \frac{i}{2}\left(\frac{\sin\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cos\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm 1}\right),$$

$$u_{365,366}(x,t) = -\frac{1}{2} - \frac{i}{2}\left(\frac{\sin\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cos\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm 1}\right),$$

$$u_{367,368}(x,t) = -\frac{1}{2} + \frac{i}{2}\left(\frac{\cos\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm 1}{\sin\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}\right),$$

$$u_{369,370}(x,t) = -\frac{1}{2} - \frac{i}{2}\left(\frac{\cos\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm 1}{\sin\left(\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}\right),$$

$$u_{371,372}(x,t) = -\frac{1}{2} + \frac{1}{4}\frac{\cosh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm 1}{\sinh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}$$

$$+\frac{1}{4}\frac{\sinh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cosh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm 1},$$

$$u_{373,374}(x,t) = -\frac{1}{2} - \frac{1}{4}\frac{\cosh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm 1}{\sinh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}$$

$$-\frac{1}{4}\frac{\sinh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cosh\left(\frac{1}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right) \pm 1},$$

$$u_{375,376}(x,t)=-\frac{1}{2}+\frac{\sqrt{2}}{4}\frac{\cos\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm 1}{\sin\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}$$

$$+\frac{\sqrt{2}}{4}\frac{\sin\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cos\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm 1},$$

$$u_{377,378}(x,t)=-\frac{1}{2}+\frac{\sqrt{2}}{4}\frac{\cos\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm 1}{\sin\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}$$

$$+\frac{\sqrt{2}}{4}\frac{\sin\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cos\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\mp 1},$$

$$u_{379,380}(x,t)=-\frac{1}{2}-\frac{\sqrt{2}}{4}\frac{\cos\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm 1}{\sin\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}$$

$$-\frac{\sqrt{2}}{4}\frac{\sin\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cos\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm 1},$$

$$u_{381,382}(x,t)=-\frac{1}{2}-\frac{\sqrt{2}}{4}\frac{\cos\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm 1}{\sin\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}$$

$$-\frac{\sqrt{2}}{4}\frac{\sin\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cos\left(\frac{\sqrt{2}}{2}\sqrt{\frac{\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\mp 1}.$$

for $\frac{\beta}{\alpha(\beta-6)}<0$

$$u_{383,384}(x,t)=-\frac{1}{2}+\frac{1}{2}\left(\frac{\sinh\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cosh\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm 1}\right),$$

$$u_{385,386}(x,t)=-\frac{1}{2}-\frac{1}{2}\left(\frac{\sinh\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cosh\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm 1}\right),$$

$$u_{387,388}(x,t)=-\frac{1}{2}+\frac{1}{2}\left(\frac{\cosh\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm 1}{\sinh\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}\right),$$

$$u_{389,390}(x,t)=-\frac{1}{2}-\frac{1}{2}\left(\frac{\cosh\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm 1}{\sinh\left(\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}\right),$$

$$u_{391,392}(x,t)=-\frac{1}{2}+\frac{i}{4}\frac{\cos\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm 1}{\sin\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}$$

$$-\frac{i}{4}\frac{\sin\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cos\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm 1},$$

$$u_{393,394}(x,t)=-\frac{1}{2}-\frac{i}{4}\frac{\cos\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm 1}{\sin\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}$$

$$+\frac{i}{4}\frac{\sin\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cos\left(\frac{1}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm 1},$$

$$u_{395,396}(x,t)=-\frac{1}{2}+\frac{\sqrt{2}}{4}\frac{\cosh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm 1}{\sinh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}$$

$$-\frac{\sqrt{2}}{4}\frac{\sinh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cosh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\mp 1},$$

$$u_{397,398}(x,t)=-\frac{1}{2}-\frac{\sqrt{2}}{4}\frac{\cosh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\pm 1}{\sinh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}$$

$$+\frac{\sqrt{2}}{4}\frac{\sinh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)}{\cosh\left(\frac{\sqrt{2}}{2}\sqrt{\frac{-\beta}{\alpha(\beta-6)}}\left(x-\left(1-\frac{\beta}{6}\right)t\right)\right)\mp 1}.$$

Using (9), the solution of (10) when $p=\frac{k^2-2}{2}, q=\frac{k^2}{2}\ , r=\frac{1}{4}$, and the sets of solutions (3)-(10), we get

$u_{399,399,\ldots,406}(x,t)=a_0+a_1\left(\frac{\mathrm{cn}\xi}{\sqrt{1-k^2}\pm\mathrm{dn}\xi}\right)+b_1\frac{\sqrt{1-k^2}\pm\mathrm{dn}\xi}{\mathrm{cn}\xi}$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)-(10).

Note that, when $k\to 1$ we get constant solutions, when $k\to 0$ we obtain, $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$.

4. Conclusion

In this paper, the mapping method has been successfully implemented to find new traveling wave solutions for our new proposed equation namely, a combined Benjamin -Bona-Mahony (BBM) and modified Benjamin-Bona-Mahony (mBBM) equation. The results show that this method is a powerful Mathematical tool for obtaining exact solutions for our equation. It is also a promising method to solve other nonlinear partial differential equations.

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