International Journal of Pure and Applied Mathematics
Volume 95 No. 4 2014, 503-514
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v95i4.2

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DELAY-DEPENDENT DECENTRALIZED EXPONENTIAL STABILITY OF NONLINEAR LARGE-SCALE SYSTEMS WITH NON-DIFFERENTIABLE AND INTERVAL TIME-VARYING DELAY IN INTERCONNECTION

Grienggrai Rajchakit

Department of Mathematics
Maejo University
Chiangmai, 50290, THAILAND

Abstract: This paper addresses decentralized exponential stability problem for a class of nonlinear large-scale systems with time-varying delay in interconnection is considered. The time delay is any continuous function belonging to a given interval, but not necessary to be differentiable. By constructing a suitable augmented Lyapunov-Krasovskii functional combined with Leibniz-Newton's formula, new delay-dependent sufficient conditions for the existence of decentralized exponential stability is established in terms of LMIs.

AMS Subject Classification: 93C30, 93D20, 37C75, 34G20, 93A15
Key Words: decentralized exponential stability, nonlinear large-scale systems, interval time-varying delay, Lyapunov function, linear matrix inequalities

1. Introduction

In view of reliability and practical implementation, the decentralized stabilization of large scale interconnected systems becomes a very important problem

Received: January 16, 2014

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and has been studied extensively for more than two decades. However, the problem of decentralized stabilization of large scale systems with delays has generally been overlooked. The majority of the works on decentralized stabilization are based on interconnection pattens with no delays. In fact, this is generally not the case for many physical systems due to transport and computation time. Large scale interconnected systems can be found in such diverse fields as electrical power systems, space structures, manufacturing process, transportation, and communication.An important motivation for the design of decentralized schemes is that the information exchange between subsystems of a large-scale system is not needed;thus, the individual subsystems controllers are simple and use only locally available information. To the best of our knowledge, interval time-varying delay and exponential stability of nonlinear large-scale systems with time-varying delays interacted between subsystems, non-differentiable time-varying delays have not been fully studied yet (see, e.g., [1–31] and the references therein), which are important in both theories and applications. This motivates our research. In fact, this problem is difficult to solve; particularly, when the time-varying delays are interval, non-differentiable and the output is subjected to such time-varying delay functions.

The objective of this paper is to extend the previous works to nonlinear large-scale systems with time-varying delays interacted between subsystems and develop a new and simple approach for the design of delay-dependent decentralized exponential stability of nonlinear large-scale systems with non-differentiable and interval time-varying delay in interconnection.

The paper is organized as follows. Section 2 presents definitions and some well-known technical propositions needed for the proof of the main results. Main result for decentralized exponential stability of nonlinear large-scale systems is presented in Section 3.

2. Preliminaries

The following noted in this paper. R^+ denotes the set of all real non-negative numbers; R^n denotes the $n-$dimensional space with the scalar product $\langle .,. \rangle$ and the vector norm $\| \, . \, \|$; $M^{n\times r}$ denotes the space of all matrices of $(n \times r)-$dimensions; A^T denotes the transpose of matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\min/\max}(A) = \min/\max\{\mathrm{Re}\lambda; \lambda \in \lambda(A)\}; C^1([a,b]\,, R^n)$ denotes the set of all R^n-valued differentiable functions on $[a,b]\,; L_2\,([0,\infty]\,, R^r)$ stands for the set of all square-integrable R^r-valued functions on $[0,\infty]$. $x_t := \{x(t+s) :$

$s \in [-h, 0]\}, \|x_t\| = \sup_{s\in[-h,0]} \| x(t+s) \|$; $C([0,t], R^n)$ denotes the set of all R^n-valued continuous functions on $[0,t]$; Matrix A is called semi-positive definite ($A \geq 0$) if $\langle Ax, x\rangle \geq 0$, for all $x \in R^n; A$ is positive definite ($A > 0$) if $\langle Ax, x\rangle > 0$ for all $x \neq 0; A > B$ means $A - B > 0$. $*$ denotes the symmetric term in a matrix.

Consider a class of nonlinear large-scale systems with interval time-varying delays composed of N interconnected subsystems $i = \overline{1, N}$ of the form

$$\dot{x}_i(t) = A_i x_i(t) + \sum_{j\neq i, j=1}^{N} D_{ij} x_j(t) + \sum_{j\neq i, j=1}^{N} f_i(t, x_i(t), x_j(t - h_{ij}(t))), \quad t \in R^+, \tag{1}$$

$$x_i(t) = \varphi_i(t), \quad \forall t \in [-h_2, 0],$$

where $x^T(t) = [x_1^T(t), \ldots, x_N^T(t)], x_i(t) \in R^{n_i}$, is the state vector, the systems matrices A_i, D_{ij} are of appropriate dimensions.

The nonlinear perturbations $f_i(t, x_i(t), x_j(t - h_{ij}(t))$ satisfies the following condition

$$\sum_{j\neq i, j=1}^{N} f_i(t, x_i(t), x_j(t - h_{ij}(t)) \leq \beta_{i1} x_i(t) + \sum_{j\neq i, j=1}^{N} \beta_{i2} x_j(t - h_{ij}(t),$$

$$\sum_{j\neq i, j=1}^{N} f_i^T(t, x_i(t), x_j(t - h_{ij}(t)) f_i(t, x_i(t), x_j(t - h_{ij}(t))$$

$$\leq \beta_{i1}^2 x_i^T(t) x_i(t) + \sum_{j\neq i, j=1}^{N} \beta_{i2}^2 x_j^T(t - h_{ij}(t) x_j(t - h_{ij}(t),$$

where $\beta_{i1}, \beta_{i2}, i = \overline{1, N}$ are positive constants. Let us denote that

$$f_i = f_i(t, x_i(t), x_j(t - h_{ij}(t)).$$

The time delays $h_{ij}(.)$ are continuous and satisfy the following condition:

$$0 \leq h_1 \leq h_{ij}(t) < h_2, \quad t \geq 0, \quad \forall i, j = \overline{1, N},$$

and the initial function $\varphi(t) = \left[\varphi_1(t), \ldots \varphi_N(t)^T\right], \varphi_i(t) \in C^1([-h_2, 0], R^{n_i})$, with the norm

$$\|\varphi_i\| = \sup_{-h\leq t\leq 0} \{\|\varphi_i(t)\|, \|\dot{\varphi}_i(t)\|\}, \quad \|\varphi\| = \sqrt{\sum_{i=1}^{N} \|\varphi_i\|^2}.$$

Definition 1. Given $\alpha > 0$. The zero solution of system (1) is α-exponentially stable if there exist a positive number $N > 0$ such that every solution $x(t,\varphi)$ satisfies the following condition:

$$\| x(t,\varphi) \| \leq Ne^{-\alpha t} \| \varphi \|, \quad \forall t \in R^+.$$

Proposition 1. *For any $x, y \in R^n$ and positive definite matrix $P \in R^{n\times n}$, we have*

$$2x^T y \leq y^T Py + x^T P^{-1}x.$$

Proposition 2. (Schur complement lemma [30]). *Given constant matrices X, Y, Z with appropriate dimensions satisfying $X = X^T, Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0 \quad \text{or} \quad \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} < 0.$$

Proposition 3. [32] *For any constant matrix $Z = Z^T > 0$ and scalar $h, \bar{h}, 0 < h < \bar{h}$ such that the following integrations are well defined, then*

$$-\int_{t-h}^{t} x(s)^T Zx(s)ds \leq -\frac{1}{h}\left(\int_{t-h}^{t} x(s)ds\right)^T Z\left(\int_{t-h}^{t} x(s)ds\right),$$

$$-\int_{-\bar{h}}^{-h}\int_{t+\theta}^{t} x(s)^T Zx(s)dsd\theta \leq -\frac{2}{\bar{h}^2 - h^2}\left(\int_{-\bar{h}}^{-h}\int_{t+\theta}^{t} x(s)dsd\theta\right)^T Z\left(\int_{-\bar{h}}^{-h}\int_{t+\theta}^{t} x(s)dsd\theta\right).$$

3. Main Results

In this section, we investigate the decentralized exponential stability of nonlinear large-scale control system (1) with interval time-varying delays. It will be seen from the following theorem that neither free-weighting matrices nor any transformation are employed in our derivation. Before introducing main result, the following notations of several matrix variables are defined for simplicity.

$$M_{11}^i = A_i^T P_i + A_i P_i + 2\alpha P_i + 2Q_i - 2S_{i1}A_i + \sum_{j\neq i, j=1}^{N} P_i D_{ij} D_{ij}^T P_i$$

$$+\sum_{j\neq i,j=1}^{N} S_{i1}D_{ij}D_{ij}^TS_{i1}^T + S_{i4}A_iA_i^TS_{i4}^T + 2\beta_{i1}P_i,$$

$$M_{1k}^i = -S_{i4}A_i, \quad \forall k=\overline{2,N}, M_{1(N+1)}^i = -S_{i2}A_i, M_{1(N+2)}^i = S_{i3}A_i,$$

$$M_{1(N+3)}^i = S_{i1} - S_{i5}A_i, M_{kk}^i = \sum_{j\neq i,j=1}^{N} S_{i4}D_{ij}D_{ij}^TS_{i4}^T + 7I + 2\beta_{i2}P_i, \forall k=\overline{2,N},$$

$$M_{k(N+1)}^i = 0, M_{k(N+2)}^i = 0, M_{k(N+3)}^i = S_{i4},$$

$$M_{(N+1)(N+1)}^i = -e^{2\alpha h_1}Q_i + \sum_{j\neq i,j=1}^{N} S_{i2}D_{ij}D_{ij}^TS_{i2}^T, M_{(N+1)(N+2)}^i = 0,$$

$$M_{(N+1)(N+3)}^i = S_{i2}, M_{(N+2)(N+2)}^i = -e^{2\alpha h_2}Q_i + \sum_{j\neq i,j=1}^{N} S_{i3}D_{ij}D_{ij}^TS_{i3}^T,$$

$$M_{(N+2)(N+3)}^i = S_{i3}, M_{(N+3)(N+3)}^i = 2S_{i5} + \sum_{j\neq i,j=1}^{N} S_{i5}D_{ij}D_{ij}^TS_{i5}^T,$$

$$\lambda_{i1} = \lambda_{min}(P_i), \lambda_1 = \min_{i=\overline{1,N}} \lambda_{i1}, \lambda_2 = \max_{i=\overline{1,N}} \lambda_{i2}, \lambda_{i2} = \lambda_{max}(P_i) + \alpha^{-1}\lambda_{max}(Q_i).$$

The following is the main result of the paper, which gives sufficient conditions for the decentralized exponential stability of nonlinear large-scale system (1) with interval time-varying delays. Essentially, the proof is based on the construction of Lyapunov Krasovskii functions satisfying Lyapunov stability theorem for time-delay system [30].

Theorem 1. *Given $\alpha > 0$. System (1) is $\alpha-$exponentially stable if there exist symmetric positive definite matrices $P_i, Q_i, i=\overline{1,N}$, and matrices $S_{ij}, i=\overline{1,N}, j=1,2,...,5$ such that the following LMI holds*

$$\mathcal{M}^i = \begin{bmatrix} M_{11}^i & M_{12}^i & \dots & M_{1(N+3)}^i \\ * & M_{22}^i & \dots & M_{2(N+3)}^i \\ . & . & \dots & . \\ * & * & \dots & M_{(N+3)(N+3)}^i \end{bmatrix} < 0, \qquad i=\overline{1,N}. \tag{2}$$

Moreover, the solution $x(t,\varphi)$ of the system satisfies

$$\| x(t,\varphi) \| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t} \| \varphi \|, \quad \forall t \in R^+.$$

Proof. We consider the following Lyapunov-Krasovskii functional for the system

(1)

$$V(t,x_t)=\sum_{i=1}^{N}\sum_{j=1}^{3}V_{ij}(t,x_t),$$

where

$$\begin{aligned}
V_{i1}&=x_i^T(t)P_ix_i(t),\\
V_{i2}&=\int_{t-h_1}^{t}e^{2\alpha(s-t)}x_i^T(s)Q_ix_i(s)\,ds,\\
V_{i3}&=\int_{t-h_2}^{t}e^{2\alpha(s-t)}x_i^T(s)Q_ix_i(s)\,ds.
\end{aligned}$$

It easy to verify that

$$\sum_{i=1}^{N}\lambda_{i1}\|x_i(t)\|^2\le V(t,x_t),\quad V(0,x_0)\le\sum_{i=1}^{N}\lambda_{i2}\|\varphi_i\|^2. \tag{3}$$

Taking the derivative of V in t along the solution of system (1), we have

$$\begin{aligned}
\dot{V_{i1}}&=2x_i^T(t)P_i\dot{x}_i(t)=2x_i^T(t)[A_i^TP_i+A_iP_i]x_i(t)+2x_i^T(t)P_iD_{ij}x_j(t-h_{ij}(t))\\
&\quad+2x_i^T(t)P_i\sum_{j\neq i,j=1}^{N}f_i(t,x_i(t),x_j(t-h_{ij}(t));\\
\dot{V_{i2}}&=x_i^T(t)Q_ix_i(t)-e^{-2\alpha h_1}x_i^T(t-h_1)Q_ix_i(t-h_1)-2\alpha V_{i2};\\
\dot{V_{i3}}&=x_i^T(t)Q_ix_i(t)-e^{-2\alpha h_2}x_i^T(t-h_2)Q_ix_i(t-h_2)-2\alpha V_{i3}.
\end{aligned}$$

Note that

$$\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}x_j(t-h_{ij}(t))^Tx_j(t-h_{ij}(t))\\
&\qquad=\sum_{i=1}^{N}\left[\sum_{j=1,i\neq j}^{N}x_i(t-h_{ji}(t))^Tx_i(t-h_{ji}(t))\right],
\end{aligned}$$

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}a_{ij}x_j(t-h_{ij}(t))^Tx_j(t-h_{ij}(t))$$

$$= \sum_{i=1}^{N} \left[\sum_{j=1, i\neq j}^{N} a_{ji} x_i(t-h_{ji}(t))^T x_i(t-h_{ji}(t)) \right].$$

Therefore, we have

$$\begin{aligned}
&\dot{V}(.) + 2\alpha V(.) \\
&\leq \sum_{i=1}^{N} [x_i^T(t)[A_i^T P_i + A_i P_i + 2\alpha P_i + 2Q_i - 2S_{i1}A_i \\
&+ \sum_{j\neq i, j=1}^{N} P_i D_{ij} D_{ij}^T P_i + \sum_{j\neq i, j=1}^{N} S_{i1} D_{ij} D_{ij}^T S_{i1}^T + S_{i4} A_i A_i^T S_{i4}^T + 2\beta_{i1} P_i] x_i(t) \\
&+ x_i^T(t-h_1)[-e^{2\alpha h_1} Q_i + \sum_{j\neq i, j=1}^{N} S_{i2} D_{ij} D_{ij}^T S_{i2}^T] x_i(t-h_1) \\
&+ x_i^T(t-h_2)[-e^{2\alpha h_2} Q_i + \sum_{j\neq i, j=1}^{N} S_{i3} D_{ij} D_{ij}^T S_{i3}^T] x_i(t-h_2) \\
&+ \dot{x}_i^T(t)[2S_{i5} + \sum_{j\neq i, j=1}^{N} S_{i5} D_{ij} D_{ij}^T S_{i5}^T] \dot{x}_i(t) \\
&+ \sum_{j\neq i, j=1}^{N} x_i^T(t-h_{ji}(t))[\sum_{j\neq i, j=1}^{N} S_{i4} D_{ij} D_{ij}^T S_{i4}^T + 7I + 2\beta_{i2} P_i] x_i(t-h_{ji}(t)) \\
&+ 2x_i^T(t)[-S_{i2}A_i] x_i(t-h_1)] + \sum_{i=1}^{N} [2x_i^T(t)[S_{i3}A_i] x_i(t-h_2) \\
&+ 2x_i^T(t)[S_{i1} - S_{i5}A_i]\dot{x}_i(t) + 2x_i^T(t-h_1) S_{i2} \dot{x}_i(t) + 2x_i^T(t-h_1) S_{i3} \dot{x}_i(t) \\
&+ 2\sum_{j\neq i, j=1}^{N} x_i^T(t-h_{ji}(t)) S_{i4} \dot{x}_i(t) - 2\sum_{j\neq i, j=1}^{N} x_i^T(t-h_{ji}(t)) S_{i4} A_i x_i(t) \\
&= \sum_{i=1}^{N} \zeta_i^T(t) \mathcal{M}^i \zeta_i(t),
\end{aligned}$$

where $\zeta_i^T(t) = \left[x_i^T(t), x_i^T(t-h_1), x_i^T(t-h_2), \left(x_i^T(t-h_{ji}) \right)_{j\neq i, j=1}^{N}, \dot{x}_i^T(t) \right].$

By condition (2), we obtain

$$\dot{V}(t, x_t) \leq -2\alpha V(t, x_t), \quad \forall t \in R^+. \tag{4}$$

Integrating both sides of (4) from 0 to t, we obtain

$$V(t, x_t) \leq V(\varphi)e^{-2\alpha t}, \quad \forall t \in R^+.$$

Furthermore, taking condition (3) into account, we have

$$\lambda_1 \parallel x(t,\varphi) \parallel^2 \leq V(x_t) \leq V(\varphi)e^{-2\alpha t} \leq \lambda_2 e^{-2\alpha t} \parallel \varphi \parallel^2,$$

then

$$\parallel x(t,\varphi) \parallel \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t} \parallel \varphi \parallel, \quad t \in R^+.$$

This completes the proof of the theorem.

Remark 2. Theorem 1 provides sufficient conditions for nonlinear large-scale system (1) in terms of the solutions of LMIs, which guarantees the closed-loop system to be exponentially stable with a prescribed decay rate α. The developed method using new inequalities for lower bounding cross terms eliminate the need for over bounding and provide larger values of the admissible delay bound. Note that the time-varying delays are non-differentiable, therefore, the methods proposed in [1–31] are not applicable to system (1). The LMI condition (2) depends on parameters of the system under consideration as well as the delay bounds. The feasibility of the LMIs can be tested by the reliable and efficient Matlab LMI Control Toolbox [30].

4. Conclusion

In this paper, the problem of the decentralized exponential stability for nonlinear large-scale time-varying delay systems has been studied. The time delay is assumed to be a function belonging to a given interval, but not necessary to be differentiable. The developed method using new inequalities for lower bounding cross terms eliminate the need for over-bounding and provide larger values of the delay bound.

Acknowledgments

This work was supported by the Thailand Research Fund Grant, the Commission for Higher Education and Faculty of Science, Maejo University, Thailand. The authors thank anonymous reviewers for valuable comments and suggestions, which allowed us to improve the paper.

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