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s^*-CLOSURE OPERATOR AND s^*-REGULARITY IN FUZZY SETTING

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Abstract: In this paper a new type of fuzzy regularity, viz. fuzzy s^*-regularity has been introduced and studied by introducing a newly defined closure operator, viz.,s^*-closure operator. Also we have found the mutual relationship among other closure operators defined earlier. In Section 4, it has been shown that s^*-closure operator is an idempotent operator in fuzzy s^*-regular space and in the last section some characterizations of s^*-closure operator have been done via fuzzy net.

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1. Introduction

Throughout the paper, by (X, τ) or simply by X we mean a fuzzy topological space (fts, for short) in the sense of Chang [5]. The support of a fuzzy set [9] A in X will be denoted by $suppA$ [8] and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. A fuzzy point [8] with the singleton support $x \in X$ and the value t $(0 < t \leq 1)$ at x will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking values

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0 and 1 in X respectively. The complement [9] of a fuzzy set A in X will be denoted by $1_X \setminus A$ [9] and is defined by $(1_X \setminus A)(x) = 1 - A(x)$, for all $x \in X$. For two fuzzy sets A and B in X, we write $A \leq B$ if and only if $A(x) \leq B(x)$, for each $x \in X$, and AqB means A is quasi-coincident (q-coincident, for short) with B [8] if $A(x) + B(x) > 1$, for some $x \in X$. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not{q} B$ respectively. clA and $intA$ of a fuzzy set [9] A in X respectively stand for the fuzzy closure [5] and fuzzy interior [5] of A in X. A fuzzy set A in X is called fuzzy semiopen [1] (resp. β-open [6], α-open [4]) if $A \leq clintA$ (resp., $A \leq clintclA$, $A \leq intclintA$). The complement of a fuzzy semiopen (resp. β-open, α-open) set is called a fuzzy semiclosed [1] (resp. β-closed [6], α-closed [4]) set. The smallest fuzzy β-closed (resp., α-closed) set containing a fuzzy set A is called fuzzy β-closure (resp. α-closure) of A and is denoted by βclA [6] (resp., αclA [4]), i.e., $\beta clA = \bigwedge\{U : A \leq U$ and U is fuzzy β-closed$\}$ (resp., $\alpha clA = \bigwedge\{U : A \leq U$ and U is fuzzy α-closed$\}$. A fuzzy set A in X is fuzzy β-closed (resp., α-closed) if $A = \beta clA$ [6] (resp., $A = \alpha clA$ [4]). A fuzzy set B is called a quasi neighbourhood (q-nbd, for short) of a fuzzy set A in an fts X if there is a fuzzy open set U in X such that $AqU \leq B$. If, in addition, B is fuzzy open (resp. semiopen, β-open, α-open) then B is called a fuzzy open (resp., semi-, β-open, α-open) q-nbd of A. In particular, a fuzzy set B in X is a fuzzy open (resp. semi-, β-open, α-open) q-nbd of a fuzzy point x_t in X if $x_t qU \leq B$, for some fuzzy open (resp., semiopen, β-open, α-open) set U in X.

2. Fuzzy s^*-Closure Operator: Some Properties

In this section fuzzy s^*-closure operator has been introduced and studied.

Let us recall a definition from [1] for ready reference.

Definition 1.1. Let A be a fuzzy set and x_t, a fuzzy point in an fts (X, τ). Then x_t is called a fuzzy semi-cluster point of A if every semi-q-nbd of x_t is q-coincident with A.

The union of all fuzzy semi-cluster points of A is called the fuzzy semiclosure of A, to be denoted by $sclA$.

A fuzzy set A is semiclosed if $A = sclA$.

The union of all fuzzy semiopen sets contained in a fuzzy set A in X is called fuzzy semi interior of A, to be denoted by $sintA$.

A fuzzy set A is semiopen if $A = sintA$.

Definition 1.2. A fuzzy point x_t in an fts X is called a fuzzy s^*-cluster point of a fuzzy set A in X if $sclUqA$ for every fuzzy semi-q-nbd U of x_t.

The union of all fuzzy s^*-cluster points of A is called fuzzy s^*-closure of A, to be denoted by $[A]_s$. A fuzzy set A is called fuzzy s^*-closed if $A = [A]_s$ and the complement of fuzzy s^*-closed set is called fuzzy s^*-open.

Theorem 1.3. *For any two fuzzy semiopen sets A and B in an fts X, $A \not q B \Rightarrow sclA \not q B$ and $A \not q sclB$.*

Proof. If possible, let $sclAqB$. Then there exists $x \in X$ such that $sclA(x) + B(x) > 1$. Let $sclA(x) = t$. Then $B(x) + t > 1 \Rightarrow x_t qB$ and $x_t \in sclA$. By Definition 1.1, BqA, a contradiction.

Similarly, we can prove that $A \not q sclB$.

Note 1.4. Definition 1.1 and Definition 1.2 together imply that $sclA \leq [A]_s$, for any fuzzy set A in an fts X. The sign of equality may not be true, in general, can be shown from the following example.

Example 1.5. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5$, $A(b) = 0.45$; $B(a) = 0.7$, $B(b) = 0.5$. Then (X, τ) is an fts. The collection of all fuzzy semiopen sets is of the form $\{0_X, 1_X, U, V\}$ where $U(a) = 0.5, 0.45 \leq U(b) \leq 0.55, V \geq B$. Consider the fuzzy point $b_{0.41}$ and the fuzzy set C where $C(a) = 0.3, C(b) = 0.4$. Then the fuzzy set W where $W(a) = 0.7, W(b) = 0.6$ is a fuzzy semi-q-nbd of $b_{0.41}$ and $W \not q C$. But any fuzzy semi-q-nbd of $b_{0.41}$ is of the form V and $sclV = 1_X qC$. Hence $b_{0.41} \in [C]_s$ but $b_{0.41} \notin sclC$.

The following theorem shows that under which condition, the two closure operators scl and s^* coincide.

Theorem 1.6. *For a fuzzy semiopen set A in an fts (X, τ), $[A]_s = sclA$.*

Proof. By Note 1.4, it suffices to show that $[A]_s \leq sclA$, for any fuzzy semiopen set A in X.

Let x_t be a fuzzy point in X such that $x_t \notin sclA$. Then there exists a fuzzy semi-q-nbd V of x_t such that $V \not q A$. Then $V(y) + A(y) \leq 1$, for all $y \in X \Rightarrow V(y) \leq 1 - A(y)$, for all $y \in X \Rightarrow sclV \leq scl(1_X \setminus A) = 1_X \setminus A$ (since $1_X \setminus A$ is fuzzy semiclosed in X). Thus $sclV \not q A$ and consequently, $x_t \notin [A]_s$. Hence $[A]_s \leq sclA$ for a fuzzy semiopen set A in X.

We now characterize fuzzy s^*-closure operator of a fuzzy set A in an fts X.

Theorem 1.7. *For any fuzzy set A in an fts (X, τ), $[A]_s = \bigcap\{[U]_s$: U is fuzzy semiopen in X and $A \leq U\}$.*

Proof. Clearly, L.H.S. $\leq$ R.H.S.

If possible, let $x_t \in$ R.H.S, but $x_t \notin$ L.H.S. Then there exists a fuzzy semi-q-nbd V of x_t such that $sclV \not{q} A$ and so $A \leq 1_X \setminus sclV$ and $1_X \setminus sclV$ being fuzzy semiopen set in X containing A, by our assumption, $x_t \in [1_X \setminus sclV]_s$. But $sclV \not{q}(1_X \setminus sclV)$ and so $x_t \notin [1_X \setminus sclV]_s$, a contradiction. This completes the proof.

Remark 1.8. Theorem 1.6 and Theorem 1.7 together imply that $[A]_s$ is fuzzy semiclosed in X for a fuzzy set A in X.

Theorem 1.9. *In an fts (X, τ), the following hold:*

(a) the fuzzy sets 0_X and 1_X are fuzzy s^*-closed sets in X,

(b) for two fuzzy sets A and B in X, if $A \leq B$, then $[A]_s \leq [B]_s$,

(c) the intersection of any two fuzzy s^*-closed sets in X is fuzzy s^*-closed in X.

Proof. (a) and (b) are obvious.

(c) Let A and B be any two fuzzy s^*-closed sets in X. Then $A = [A]_s$ and $B = [B]_s$. Now $A \bigwedge B \leq A$, $A \bigwedge B \leq B$. Then by (b), $[A \bigwedge B]_s \leq [A]_s$ and $[A \bigwedge B]_s \leq [B]_s$. Therefore, $[A \bigwedge B]_s \leq [A]_s \bigwedge [B]_s = A \bigwedge B$.

Conversely, let $x_t \in A \bigwedge B$. Then $x_t \in A = [A]_s$ and $x_t \in B = [B]_s$. Then $A(x) \geq t, B(x) \geq t$, i.e., $(A \bigwedge B)(x) = min\{A(x), B(x)\} \geq t$. Now for any fuzzy semi-q-nbd V of x_t, $sclVqA, sclVqB$. Then $V(x) + t > 1$. Therefore, $sclV(x) + (A \bigwedge B)(x) > 1 - t + t = 1$. Therefore, $sclVq(A \bigwedge B)$ for any fuzzy semi-q-nbd V of x_t and hence $x_t \in [A \bigwedge B]_s$. Consequently, $[A]_s \bigwedge [B]_s \leq [A \bigwedge B]_s$.

Remark 1.10. In fact, the intersection of any collection of fuzzy s^*-closed sets is fuzzy s^*-closed. Since the intersection of two fuzzy semiopen sets is known to be fuzzy semiopen, so the union of two fuzzy s^*-closed sets is fuzzy s^*-closed.

Corollary 1.11. *The fuzzy s^*-open sets in an fts (X, τ) form a base for a fuzzy topology τ_{s^*} (say) which is coarser than fuzzy topology τ of (X, τ).*

Result 1.12. *We conclude that $x_t \in [y_{t'}]_s$ does not imply $y_{t'} \in [x_t]_s$ where $x_t, y_{t'} (0 < t, t' < 1)$ are fuzzy points in X as shown from the following example.*

Example 1.13. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.7, B(b) = 0.5$. Then (X, τ) is an fts. Here fuzzy semiopen sets are $0_X, 1_X, U, V$ where $U(a) = 0.5, 0.4 \leq U(b) \leq 0.6$ and $V \geq B$. Then fuzzy semiclosed sets are $0_X, 1_X, 1_X \setminus U, 1_X \setminus V$ where $(1_X \setminus U)(a) =$

$0.5, 0.4 \le (1_X \setminus U)(b) \le 0.6$ and $1_X \setminus V \le 1_X \setminus B$. Consider the fuzzy points $a_{0.1}$ and $b_{0.61}$. We claim that $a_{0.1} \in [b_{0.61}]_s$. Indeed, any fuzzy semi-q-nbd of $a_{0.1}$ is of the form $V(a) > 0.9, V(b) \ge 0.5$. So $sclV = 1_X q b_{0.61}$. But $b_{0.61} \notin [a_{0.1}]_s$, as A is a fuzzy semi-q-nbd of $b_{0.61}$ but $sclA = A \not q a_{0.1}$.

3. s^*-Closure Operator: Mutual Relationship with other Closure Operators

In this section we have found some mutual relationship of s^*-closure operator among other closure operators, viz., α^*-closure operator, β^*-closure operator, θ-closure operator.

We recall some definitions for ready references.

Definition 2.1. (see [2]) A fuzzy point x_t in an fts X is called a fuzzy β^*-cluster point of a fuzzy set A in X if $\beta clU q A$ for every fuzzy β-open q-nbd U of x_t.

The union of all fuzzy β^*-cluster points of A is called fuzzy β^*-closure of A, to be denoted by $[A]_\beta$. A is called fuzzy β^*-closed if $A = [A]_\beta$ and the complement of fuzzy β^*-closed set is called fuzzy β^*-open.

Result 2.2. $[A]_\beta \le [A]_s$, *for any fuzzy set A in an fts X.*

Proof. Let $x_t \in [A]_\beta$. Then for any fuzzy β-open q-nbd V of x_t, $\beta clV q A$. Let W be a fuzzy semi-q-nbd of x_t. Since fuzzy semiopen sets are fuzzy β-open, W is a fuzzy β-open q-nbd of x_t. As a result, $\beta clW q A$. Again, $\beta clW \le sclW, sclW q A$. Consequently, $x_t \in [A]_s$.

Remark 2.3. It is clear from the following example that $[A]_\beta \neq [A]_s$, for any fuzzy set A in an fts X, in general.

Example 2.4. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.4, A(b) = 0.7$. Then (X, τ) is an fts. The collection of fuzzy β-open sets is $\{0_X, 1_X, A, U\}$ where $U \not\le 1_X \setminus A$ and the collection of fuzzy semiopen sets is $\{0_X, 1_X, V\}$ where $V \ge A$. The collection of all fuzzy β-closed sets is $\{0_X, 1_X, 1_X \setminus A, 1_X \setminus U\}$ and that of fuzzy semiclosed sets is $\{0_X, 1_X, 1_X \setminus V\}$ where $1_X \setminus V \le 1_X \setminus A$.

Consider the fuzzy set C given by $C(a) = 0.5, C(b) = 0.6$ and the fuzzy point $a_{0.6}$. Clearly $a_{0.6} \notin [C]_\beta$ as $a_{0.6} q U$, where $U(a) = 0.41, U(b) = 0.31$, but $\beta clU = U \not q C$. Now fuzzy semi-q-nbds of $a_{0.6}$ are of the form W where $W(a) > 0.4, W(b) \ge 0.7$ and $sclW = 1_X q C$ and so $a_{0.6} \in [C]_s$.

Definition 2.5. (see [3]) A fuzzy point x_t in an fts X is called a fuzzy α^*-cluster point of a fuzzy set A in X if $\alpha clU q A$ for every fuzzy α-open q-nbd U of x_t.

The union of all fuzzy α^*-cluster points of A is called fuzzy α^*-closure of A, to be denoted by $[A]_\alpha$. A is called fuzzy α^* -closed if $A = [A]_\alpha$ and the complement of fuzzy α^*-closed set is called fuzzy α^* -open.

Remark 2.6. It is clear from definitions that fuzzy α-open sets are fuzzy semiopen and as a result, $[A]_s \leq [A]_\alpha$. But the converse is not true as seen from the following example.

Example 2.7. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A, B\}$ where $A(a) = 0.45, A(b) = 0.4$, $B(a) = 0.6, B(b) = 0.5$. Then (X, τ) is an fts. Let V be a fuzzy set in X defined by $V(a) = V(b) = 0.4$. Then $a_{0.5} \in [V]_\alpha$. In fact, fuzzy α-open sets are $0_X, 1_X, A, U$ where $U \geq B$ and fuzzy α-closed sets are $0_X, 1_X, 1_X \setminus A, 1_X \setminus U$ where $1_X \setminus U \leq 1_X \setminus B$. Any fuzzy α-open q-nbd of $a_{0.5}$ is of the form U and $\alpha clU = 1_X qV$. Now fuzzy semiopen sets are $0_X, 1_X, C, D$ where $0.45 \leq C(a) \leq 0.55$, $0.4 \leq C(b) \leq 0.6$, $D \geq B$ and fuzzy semiclosed sets are $0_X, 1_X, 1_X \setminus C, 1_X \setminus D$ where $1_X \setminus D \leq 1_X \setminus B$. Now $E(a) = 0.51, E(b) = 0.4$ is a fuzzy semi-q-nbd of $a_{0.5}$, but $sclE = E \not{q} V$ and so $a_{0.5} \notin [V]_s$.

The following two examples show that s^*-closure operator and closure operator are independent notions.

Example 2.8. Consider Example 2.7 and consider a fuzzy set V where $V(a) = 0.4, V(b) = 0.6$. Then $clV = 1_X \setminus A$ and so $a_{0.5} \in clV$, but $a_{0.5} \notin [V]_s$ as $E(a) = 0.51, E(b) = 0.4$ is a fuzzy semi-q-nbd of $a_{0.5}$, but $sclE = E \not{q} V$.

Example 2.9. Consider Example 1.13. Consider the fuzzy point $a_{0.4}$ and the fuzzy set W given by $W(a) = 0.1, W(b) = 0.4$. Here B is a fuzzy open q-nbd of $a_{0.4}$ but $B \not{q} W$ and so $a_{0.4} \in clW$. Again any fuzzy semi-q-nbd of $a_{0.4}$ is of the form V where $V \geq B$. Then $sclV = 1_X qW$ and so $a_{0.4} \in [W]_s$.

Definition 2.10. (see [7]) Let A be a fuzzy set and x_t, a fuzzy point in an fts X. x_t is said to be a fuzzy θ-cluster point of A if fuzzy closure of every fuzzy open q-nbd of x_t is q-coincident with A.

The union of all fuzzy θ-cluster points of A is called the fuzzy θ-closure of A, to be denoted by $[A]_\theta$. A is called fuzzy θ-closed if $A = [A]_\theta$ and the complement of a fuzzy θ-closed set is called fuzzy θ-open.

Note 2.11. It is clear from definitions that $[A]_s \leq [A]_\theta$, for any fuzzy set A in X. But the converse may not be true, as seen from the following example.

Example 2.12. Consider Example 2.7. Consider the fuzzy point $a_{0.5}$ and the fuzzy set V given by $V(a) = V(b) = 0.4$. It has been shown that $a_{0.5} \notin [V]_s$. We claim that $a_{0.5} \in [V]_\theta$. Indeed, other than 1_X, B is the only fuzzy open q-nbd of $a_{0.5}$ such that $clB = 1_X qV$.

4. Fuzzy s^*-Regular Space: Some Characterizations

In this section a new type of fuzzy regularity has been introduced and studied and shown that in this space s^*-closure operator and semi-closure operator coincide.

Definition 3.1. An fts (X, τ) is said to be fuzzy s^*-regular if for each fuzzy point x_t and each fuzzy semi-q-nbd U of x_t, there exists a fuzzy semiopen set V in X such that $x_t qV \leq sclV \leq U$.

Theorem 3.2. *For an fts (X, τ), the following conditions are equivalent:*

(a) X is fuzzy s^*-regular space.

(b) For any fuzzy set A in X, $[A]_s = sclA$,

(c) For each fuzzy point x_t and each fuzzy semiclosed set F with $x_t \notin F$, there exists a fuzzy semiopen set U such that $x_t \notin sclU$ and $F \leq U$.

(d) For each fuzzy point x_t and each fuzzy semiclosed set F such that $x_t \notin F$, there exist fuzzy semiopen sets U and V in X such that $x_t qU, F \leq V$ and $U \not{q} V$.

(e) For any fuzzy set A and any fuzzy semiclosed set F with $A \not\leq F$, there exist fuzzy semiopen sets U and V such that $AqU, F \leq V$ and $U \not{q} V$.

(f) For any fuzzy set A and any fuzzy semiopen set U with AqU, there exists a fuzzy semiopen set V such that $AqV \leq sclV \leq U$.

Proof. (a) $\Rightarrow$ (b): By Note 1.4, it suffices to show that $[A]_s \leq sclA$, for any fuzzy set A in X.

Let $x_t \in [A]_s$ and V be any fuzzy semi-q-nbd of x_t. By (a), there exists a fuzzy semiopen set W such that $x_t qW \leq sclW \leq V$. Since $x_t \in [A]_s$, $sclWqA$ and so VqA. Consequently, $x_t \in sclA \Rightarrow [A]_s \leq sclA$.

(b) $\Rightarrow$ (a): Let x_t be a fuzzy point in X and U be any fuzzy semi-q-nbd of x_t. Then $U(x) + t > 1 \Rightarrow x_t \notin (1_X \setminus U) = scl(1_X \setminus U) = [1_X \setminus U]_s$ (by (b)). Then there exists a fuzzy semi-q-nbd V of x_t such that $sclV \not{q}(1_X \setminus U) \Rightarrow sclV \leq U$. Then $x_t qV \leq sclV \leq U \Rightarrow X$ is fuzzy s^*-regular.

(a) $\Rightarrow$ (c): Let x_t be a fuzzy point in X and F, a fuzzy semiclosed set in X with $x_t \notin F$. Then $F(x) < t \Rightarrow 1 - F(x) + t > 1 \Rightarrow x_t q(1_X \setminus F)$. By (a), there exists a fuzzy semiopen set W such that $x_t qW \leq sclW \leq 1_X \setminus F$. Therefore, $F \leq 1_X \setminus sclW = U$ (say) which is fuzzy semiopen. Now $x_t qW \Rightarrow x_t qsintW \leq W \leq sint(sclW) \Rightarrow x_t qsint(sclW) \Rightarrow (sint(sclW))(x) + t > 1 \Rightarrow$

$1-(sint(sclW))(x)<t \Rightarrow x_t \notin 1_X \setminus (sint(sclW)) \Rightarrow x_t \notin scl(1_X \setminus sclW) \Rightarrow x_t \notin sclU$.

(c) $\Rightarrow$ (d): Let x_t be a fuzzy point in X and F, a fuzzy semiclosed set in X with $x_t \notin F$. By (c), there exists a fuzzy semiopen set U such that $x_t \notin sclU$ and $F \leq U$, Now $x_t \notin sclU \Rightarrow$ there exists a fuzzy semi-q-nbd W of x_t such that $W \not{q} U$.

(d) $\Rightarrow$ (e): Let A be any fuzzy set and F, any fuzzy semiclosed set in X with $A \not\leq F$. Then there exists $x \in X$ such that $A(x) > F(x)$. Let $A(x) = t$. Then $x_t \notin F$. By (d), there exist fuzzy semiopen sets U and V such that $x_t q U, F \leq V$ and $U \not{q} V$. Again, $U(x) + A(x) = U(x) + t > 1 \Rightarrow AqU$.

(e) $\Rightarrow$ (f): Let A be any fuzzy set and U, any fuzzy semiopen set in X with AqU. Then $A \not\leq 1_X \setminus U$ which is fuzzy semiclosed. By (e), there exist fuzzy semiopen sets V and W such that $AqV, 1_X \setminus U \leq W$ and $V \not{q} W$. Then by Theorem 1.3, $sclV \not{q} W$. Thus $AqV \leq sclV \leq 1_X \setminus W \leq U$.

(f) $\Rightarrow$ (a): Obvious.

Corollary 3.3. *An fts (X, τ) is fuzzy s^*-regular if and only if every fuzzy semiclosed set in X is fuzzy s^*-closed in X.*

Proof. Let (X, τ) be fuzzy s^*-regular space and A, a fuzzy semiclosed set in X. Then by Theorem 3.2 (a) $\Rightarrow$ (b), $A = sclA = [A]_s$ and hence A is fuzzy s^*-closed in X.

Conversely, let $A = [A]_s$ for any fuzzy semiclosed set in X. Let B be any fuzzy set in X. Then $sclB = [sclB]_s$. Then $[B]_s \leq [sclB]_s = sclB$. Again from Note 1.4, $sclB \leq [B]_s$ and so $[B]_s = sclB$ for any fuzzy set B in X. Hence by Theorem 3.2 (b) $\Rightarrow$ (a), X is fuzzy s^*-regular space.

Remark 3.4. In a fuzzy s^*-regular space (X, τ), $[[A]_s]_s = [A]_s$.

Proof. By Theorem 3.2 (a) $\Rightarrow$ (b), $[[A]_s]_s = [sclA]_s = scl(sclA) = sclA = [A]_s$ (by Theorem 3.2 (a) $\Rightarrow$ (b)).

5. Fuzzy s^*-Closure Operator: Some Characterizations

In this section fuzzy s^*-closure operator of a fuzzy set is characterized in terms of fuzzy s^*-cluster point of a fuzzy net and its fuzzy s^*-convergence.

Definition 4.1. A fuzzy point x_t in an fts (X, τ) is called a fuzzy s^*-cluster point of a fuzzy net $\{S_n : n \in (D, \geq)\}$ if for every fuzzy semi-q-nbd U of x_t and for any $n \in D$, there exists $m \in D$ with $m \geq n$ such that $S_m q sclU$.

Definition 4.2. A fuzzy net $\{S_n : n \in (D, \geq)\}$ in an fts (X, τ) is said to s^*-converge to a fuzzy point x_t if for any fuzzy semi-q-nbd U of x_t, there exists $m \in D$ such that $S_n qsclU$ for all $n \geq m$ $(n \in D)$. This is denoted by $S_n \overrightarrow{s^*} x_t$.

Theorem 4.3. *A fuzzy point x_t is a fuzzy s^*-cluster point of a fuzzy net $\{S_n : n \in (D, \geq)\}$ in an fts (X, τ) if and only if there exists a fuzzy subnet of $\{S_n : n \in (D, \geq)\}$ which s^*-converges to x_t.*

Proof. Let x_t be a fuzzy s^*-cluster point of the fuzzy net $\{S_n : n \in (D, \geq)\}$. Let $s(Q_{x_t})$ denote the set of fuzzy semi-closures of all fuzzy semi-q-nbds of x_t. Then for any $A \in s(Q_{x_t})$, there exists $n \in D$ such that $S_n qA$. Let E denote the set of all ordered pairs (n, A) such that $n \in D$, $A \in s(Q_{x_t})$ and $S_n qA$. Then $(E, \gg)$ is a directed set, where $(m, A) \gg (n, B)$ $((m, A), (n, B) \in E)$ iff $m \geq n$ in D and $A \leq B$. Then $T : (E, \gg) \to (X, \tau)$ given by $T(m, A) = S_m$ is clearly a fuzzy subnet of $\{S_n : n \in (D, \geq)\}$.

We claim that $T \overrightarrow{s^*} x_t$. Let V be any fuzzy semi-q-nbd of x_t. Then there exists $n \in D$ such that $(n, sclV) \in E$ and so $S_n qsclV$. Now for any $(m, A) \gg (n, sclV)$, $T(m, A) = S_m qA \leq sclV \Rightarrow T(m, A) qsclV$. Consequently, $T \overrightarrow{s^*} x_t$.

Conversely, if x_t is not a fuzzy s^*-cluster point of the fuzzy net $\{S_n : n \in (D, \geq)\}$, then there exists a fuzzy semi-q-nbd U of x_t and an $n \in D$ such that $S_m \ /qsclU$, for all $m \geq n$. Then clearly, no fuzzy subnet of the net $\{S_n : n \in (D, \geq)\}$ can s^*-converge to x_t.

Theorem 4.4. *Let A be a fuzzy set in an fts (X, τ). A fuzzy point $x_t \in [A]_s$ iff there exists a fuzzy net $\{S_n : n \in (D, \geq)\}$ in A, which s^*-converges to x_t.*

Proof. Let $x_t \in [A]_s$. Then for any fuzzy semi-q-nbd U of x_t, $sclU qA$, .e., there exists $y^U \in suppA$ and a real number s_U with $0 < s_U \leq A(y^U)$ such that the fuzzy point $y^U_{s_U}$ with support y^U and value s_U belong to A and $y^U_{s_U} qsclU$. We choose and fix one such $y^U_{s_U}$ for each U. Let $\mathcal{D}$ denote the set of all fuzzy semi-q-nbds of x_t. Then $(\mathcal{D}, \succeq)$ is a directed set under inclusion relation, i.e., B, $C \in \mathcal{D}$, $B \succeq C$ iff $B \leq C$. Then $\{y^U_{s_U} \in A : y^U_{s_U} qsclU$ and $U \in \mathcal{D}\}$ is a fuzzy net in A such hat it s^*-converges to x_t. Indeed, for any fuzzy semi-q-nbd U of x_t, if $V \in \mathcal{D}$ and $V \succeq U$ (i.e., $V \leq U$), then $y^V_{s_V} qsclV \leq sclU \Rightarrow y^V_{s_V} qsclU$.

Conversely, let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net in A such that $S_n \overrightarrow{s^*} x_t$. Then for any fuzzy semi-q-nbd U of x_t, there exists $m \in D$ such that $n \geq m \Rightarrow S_n qsclU \Rightarrow AqsclU$ (since $S_n \in A$). Hence $x_t \in [A]_s$.

Remark 3.5. It is clear that an improved version of he converse of the last theorem can be written as "$x_t \in A]_s$ if there exists a fuzzy net in A with x_t as a fuzzy s^*-cluster point".

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