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THE EXISTENCE OF COMMON FIXED POINTS FOR FAINTLY COMPATIBLE MAPPINGS IN MENGER SPACES

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Abstract: In this paper, we prove the existence of common fixed points of noncompatible faintly compatible mappings in Menger spaces. We also provide examples in support of our results.

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1. Introduction and Preliminaries

Menger [12] introduced the notion of probabilistic metric spaces (or statistical

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metric spaces), which is a generalization of metric spaces, and the study of these spaces was expanded rapidly with the pioneering works of Schweizer and Sklar [13], [14]. Especially, the theory of probabilistic metric spaces is of fundamental importance in probabilistic functional analysis.

Further, some fixed point theorems in probabilistic metric spaces have been proved by many authors; Bharucha-Reid [1], Bocsan [3], Chang [4], Ćirić [5], Hadžić [6], [7], Hicks [8], Imdad et al. [9], Kohli and Vashistha [10], Mishra [11], Sehgal and Bharucha-Reid [15], Singh and Jain [16], Singh and Pant [17]-[19], Stojaković [20]-[22] and Tan [23], etc. Since every metric space is a probabilistic metric spaces, we can use many results in probabilistic metric spaces to prove some fixed point theorems in metric spaces

In this paper, we prove the existence of common fixed points of noncompatible faintly compatible mappings in Menger spaces. We also provide examples in support of our results.

Definition 1.1. ([14]) A mapping $f : \mathbb{R} \to \mathbb{R}^+$ is called *distribution function* if it is non decreasing and left continuous with $\inf\{f(t) : t \in \mathbb{R}\} = 0$ and $\sup\{f(t) : t \in \mathbb{R}\} = 1$. We will denote $\mathcal{L}$ by the set of all distribution functions.

Definition 1.2. ([12]) A probabilistic metric space is a pair (X, F), where X is a nonempty set and $F : X \times X \to \mathcal{L}$ is a mapping defined by $F(x, y) = F_{x,y}$ satisfying for all $x, y, z \in X$ and $t, s \geq 0$,

(p1) $F_{x,y}(t) = 1$ for all $t > 0$ if and only if $x = y$,

(p2) $F_{x,y}(0) = 0$,

(p3) $F_{x,y}(t) = F_{y,x}(t)$,

(p4) If $F_{x,y}(t) = 1$ and $F_{y,z}(s) = 1$, then $F_{x,z}(t + s) = 1$.

Every metric space (X, d) can always be realized as a probabilistic metric space by considering $F : X \times X \to \mathcal{L}$ defined by $F_{x,y}(t) = H(t - d(x, y))$ for all $x, y \in X$, where H is a specific distribution function (also known as Heaviside function) defined by

$$\begin{cases} 0, & \text{if } t \leq 0, \\ 1, & \text{if } t > 0. \end{cases}$$

So probabilistic metric spaces offer a wider framework than that of the metric spaces and are general enough to cover even wider statistical situations.

Definition 1.3. ([14]). A mapping is called a *t-norm* if

(t1) $\Delta(a,1) = a, \quad \Delta(0,0) = 0,$

(t2) $\Delta(a,b) = \Delta(b,a),$

(t3) $\Delta(c,d) \geq \Delta(a,b)$ for $c \geq a$ and $d \geq b,$

(t4) $\Delta(\Delta(a,b),c) = \Delta(a,\Delta(b,c))$ for all $a,b,c \in [0,1].$

Example 1.4. The following are the four basic t-norms:

(1) The minimum t-norm $\Delta_M(a,b) = \min\{a,b\};$

(2) The product t-norm $\Delta_P(a,b) = ab;$

(3) The Lukasiewicz t-norm $\Delta_L(a,b) = \max\{a+b-1,0\};$

(4) The weakest t-norm (drastic product)

$$\Delta_D(a,b) = \begin{cases} \min\{a,b\}, & \text{if } \max\{a,b\} = 1, \\ 0, & \text{otherwise.} \end{cases}$$

In respect of above mention t-norms, we have the following ordering:

$$\Delta_D < \Delta_L < \Delta_P < \Delta_M.$$

Throughout this paper, Δ stands for an arbitrary continuous t-norm.

Definition 1.5. ([12]) A *Menger space* is a triplet (X,F,Δ), where (X,F) is a probabilistic metric space and Δ is a t-norm with the following condition for all $x,y,z \in X$ and $t,s \geq 0$,

(p5) $F_{x,z}(t+s) \geq \Delta(F_{x,y}(t), F_{y,z}(s)),$

Definition 1.6. ([11]) Let f and g be self-mappings of a Menger space (X,F,Δ). Then f and g are said to be *compatible* if

$$\lim_{n\to\infty} F(fgx_n, gfx_n, t) = 1$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = u$ for some $u \in X$ and for all $t > 0$.

Definition 1.7. Let f and g be self-mappings of a Menger space (X, F, Δ). Then f and g are said to be *noncompatible* if either $\lim_{n\to\infty} F(fgx_n, gfx_n, t)$ is non-existent or

$$\lim_{n\to\infty} F(fgx_n, gfx_n, t) \neq 1$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = u$ for some $u \in X$ and for all $t > 0$.

Definition 1.8. Let f and g be self-mappings of a Menger space (X, F, Δ). Then f and g are said to be *conditionally compatible* if whenever the sequences $\{x_n\}$ satisfying $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n$ is nonempty, there exists a sequence $\{z_n\}$ in X such that

$$\lim_{n\to\infty} fz_n = \lim_{n\to\infty} fz_n = u$$

for some $u \in X$ and for all $t > 0$ and

$$\lim_{n\to\infty} F(fgz_n, gfz_n, t) = 1$$

for all $t > 0$.

Definition 1.9. Let f and g be self-mappings of a Menger space (X, F, Δ). Then f and g are said to be *faintly compatible* if f and g are conditionally compatible and f and g commute on a nonempty subset of the set of coincidence points, whenever the set of coincidence points is nonempty.

It may be observed that compatibility is independent of conditional compatibility and compatibility implies faint compatibility, but the converse is not true in general in metric spaces ([2]).

Lemma 1.10. ([16]) *Let (X, F, Δ) be a Menger space. If there exists $k \in (0, 1)$ such that*

$$F(x, y, kt) \geq F(x, y, t)$$

for all $x, y \in X$ and $t > 0$, then $x = y$.

2. Main Results

Theorem 2.1. *Let f and g be noncompatible faintly compatible self-mappings of a Menger space (X, F, Δ) satisfying*

$$fX \subset gX; \tag{C1}$$

$$F(fx, fy, kt) \geq F(gx, gy, t) \tag{C2}$$

for all $x, y \in X$, where $0 < k < 1$.

Assume that either f or g is continuous. Then f and g have a unique common fixed point.

Proof. Let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = u$ for some $u \in X$. Since f and g are noncompatible, $\lim_{n\to\infty} F(fgx_n, gfx_n, t) \neq 1$ or non-existent. Since f and g are faintly compatible and $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = u$, there exists a sequence $\{z_n\}$ in X satisfying $\lim_{n\to\infty} fz_n = \lim_{n\to\infty} gz_n = v$ (say) such that $\lim_{n\to\infty} F(fgz_n, gfz_n, t) = 1$ for all $t > 0$. Further, since f is continuous, we get $\lim_{n\to\infty} ffz_n = fv$ and $\lim_{n\to\infty} fgz_n = fv$ and hence $\lim_{n\to\infty} gfz_n = fv$. Since $fX \subset gX$ implies that $fv = gw$ for some $w \in X$ and $\lim_{n\to\infty} ffz_n = gw$ and $\lim_{n\to\infty} gfz_n = gw$. Also using $(C2)$, we get

$$F(fw, ffz_n, kt) \geq F(gw, gfz_n, t).$$

On letting $n \to \infty$, we get $fw = gw$. Thus w is a coincidence point of f and g. Further faint compatibility implies that $fgw = gfw$ and hence $fgw = gfw = ffw = ggw$.

If $fw \neq ffw$, then using $(C2)$ we get

$$F(fw, ffw, kt) \geq F(gw, gfw, t) = F(fw, ffw, t),$$

by Lemma 1.10, which is a contradiction. Hence $fw = ffw$. Thus fw is a common fixed point of f and g.

Similarly, we can also complete the proof when g is continuous.

For uniqueness if $w_1, w_2 \in X$ such that $fw_1 = gw_1 = w_1$ and $fw_2 = gw_2 = w_2$ using $(C2)$, we get

$$\begin{aligned} F(w_1, w_2, kt) &= F(fw_1, fw_2, kt) \\ &\geq F(gw_1, gw_2, t) \\ &= F(w_1, w_2, t), \end{aligned}$$

which gives by Lemma 1.10, $w_1 = w_2$ and hence the common fixed point is unique. This completes the proof. $\square$

Example 2.2. Let (X, F, Δ) be a Menger space where $X = [0, \infty)$ with a t-norm defined by $\Delta(a, b) = \min\{a, b\}$ by

$$F(x, y, t) = \frac{t}{t + |x - y|}$$

for all $x, y \in X$ and $t > 0$. Define $f, g : X \to X$ as

$$fx = \begin{cases} 1, & \text{if } x \leq 1, \\ 2, & \text{if } x > 1, \end{cases} \qquad gx = \begin{cases} 2 - x, & \text{if } x \leq 1, \\ 4, & \text{if } x > 1. \end{cases}$$

(1) Let $\{x_n\} = \{1 + \frac{1}{n}\}$ be a sequence. Now $fx_n \to 2$, $gx_n \to 4$, $fgx_n \to 2$ and $gfx_n \to 4$ and so $F(fgx_n, gfx_n, t)$ does not converge to 1. Therefore, f and g are noncompatible.

(2) Let $\{z_n\} = \{1\}$ be a sequence. Now $fz_n \to 1$, $gz_n \to 1$, $fgz_n \to 1$ and $gfz_n \to 1$ and so $F(fgz_n, gfz_n, t) \to 1$. Therefore, f and g are conditionally compatible. Also $f1 = g1$ and $fg1 = gf1$. Hence f and g are faintly compatible.

(3) Condition $(C2)$ is satisfied with $k = \frac{1}{2}$.

Hence, all the conditions of the Theorem 2.1 are satisfied and $x = 1$ is the unique common fixed point of f and g.

The next theorem illustrates the applicability of faintly compatible mappings in finding the existence of common fixed points for mappings satisfying the strict contractive condition in Menger space.

Theorem 2.3. *Let f and g be noncompatible faintly compatible self-mappings of a Menger space (X, F, Δ) satisfying the condition $(C1)$ and*

$$(C3) \qquad F(fx, fy, t) > F(gx, gy, t)$$

for all $x, y \in X$ with $gx \neq gy$.

Assume that either f or g is continuous. Then f and g have a unique common fixed point.

Proof. Let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = u$ for some $u \in X$. Since f and g are noncompatible, $\lim_{n\to\infty} F(fgx_n, gfx_n, t) \neq 1$ or non-existent. Since f and g are faintly compatible and $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = u$, there exists a sequence $\{z_n\}$ in X satisfying $\lim_{n\to\infty} fz_n =$

$\lim_{n\to\infty} gz_n = v$ (say) such that $\lim_{n\to\infty} F(fgz_n, gfz_n, t) = 1$ for all $t > 0$. Further, since f is continuous, we have $\lim_{n\to\infty} ffz_n = fv$ and $\lim_{n\to\infty} fgz_n = fv$ and hence $\lim_{n\to\infty} gfz_n = fv$. Since $fX \subset gX$ implies that $fv = gw$ for some $w \in X$ and $\lim_{n\to\infty} ffz_n = gw$ and $\lim_{n\to\infty} gfz_n = gw$. Also using $(C3)$, we get

$$F(fw, ffz_n, t) > F(gw, gfz_n, t) = F(gw, gw, t).$$

On letting $n \to \infty$, we get $fw = gw$. Thus w is a coincidence point of f and g. Further faint compatibility implies that $fgw = gfw$ and hence $fgw = gfw = ffw = ggw$.

If $fw \neq ffw$, then using $(C3)$ we get

$$F(fw, ffw, t) > F(gw, gfw, t) = F(fw, ffw, t),$$

which is a contradiction. Hence $fw = ffw$. fw is a common fixed point of f and g.

Similarly, we can also complete the proof when g is continuous.

For uniqueness if $w_1, w_2 \in X$ such that $fw_1 = gw_1 = w_1$ and $fw_2 = gw_2 = w_2$, using $(C3)$, we get

$$\begin{aligned} F(w_1, w_2, t) &= F(fw_1, fw_2, t) \\ &> F(gw_1, gw_2, t) \\ &= F(w_1, w_2, t), \end{aligned}$$

which gives that $w_1 = w_2$ and hence the common fixed point is unique. This completes the proof. □

Example 2.4. Let (X, F, Δ) be a Menger space where $X = [4, \infty)$ with a t-norm defined by $\Delta(a, b) = \min\{a, b\}$ by

$$F(x, y, t) = \frac{t}{t + |x - y|}$$

for all $x, y \in X$ and $t > 0$. Define $f, g : X \to X$ as

$$fx = \begin{cases} 4, & \text{if } x = 4, \\ 8, & \text{if } x > 4, \end{cases} \qquad gx = \begin{cases} 4, & \text{if } x = 4, \\ x + 4, & \text{if } x > 4. \end{cases}$$

(1) Let $\{x_n\} = \{4 + \frac{1}{n}\}$ be a sequence. Now $fx_n \to 8$, $gx_n \to 8$, $fgx_n \to 8$ and $gfx_n \to 12$ and so $F(fgx_n, gfx_n, t)$ does not converge to 1. Therefore, f and g are noncompatible.

(2) Let $\{z_n\} = \{4\}$ be a sequence. Now $fz_n \to 4$, $gz_n \to 4$, $fgz_n \to 4$ and $gfz_n \to 4$ and so $F(fgz_n, gfz_n, t) \to 1$. Therefore, f and g are conditionally compatible. Also $f4 = g4$ and $fg4 = gf4$. Hence f and g are faintly compatible.

(3) Condition $(C3)$ is satisfied.

Hence, all the conditions of the Theorem 2.3 are satisfied and $x = 4$ is the unique common fixed point of f and g.

The next theorem illustrate the applicability of faintly compatible mappings in finding the existence of common fixed points for mappings satisfying Lipchitz-type condition in Menger spaces.

Theorem 2.5. *Let f and g be noncompatible faintly compatible self-mappings of a Menger space (X, F, Δ) satisfying the condition $(C1)$ and*

$$(C4) \qquad F(fx, fy, kt) \geq F(gx, gy, t)$$

for all $x, y \in X$, where $0 < k$;

$$(C5) \qquad F(fx, ffy, t) \neq \min\{F(fx, gfy, t), F(gfx, ffy, t)\}$$

for all $x, y \in X$, whenever the right side is non-one ($\neq 1$).

Assume that either f or g is continuous. Then f and g have a common fixed point.

Proof. Let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = u$ for some $u \in X$. Since f and g are noncompatible, $\lim_{n\to\infty} F(fgx_n, gfx_n, t) \neq 1$ or non-existent. Since f and g are faintly compatible, there exists a sequence $\{z_n\}$ in X satisfying $\lim_{n\to\infty} fz_n = \lim_{n\to\infty} gz_n = v$ (say) for some $v \in X$ such that $\lim_{n\to\infty} F(fgz_n, gfz_n, t) = 1$ for all $t > 0$. Further, since f is continuous, we have $\lim_{n\to\infty} ffz_n = fv$ and $\lim_{n\to\infty} fgz_n = fv$ and hence $\lim_{n\to\infty} gfz_n = fv$. Since $fX \subset gX$ implies that $fv = gw$ for some $w \in X$ and $\lim_{n\to\infty} ffz_n = gw$ and $\lim_{n\to\infty} gfz_n = gw$. Also using $(C4)$, we get

$$F(fw, ffz_n, kt) \geq F(gw, gfz_n, t).$$

On letting $n \to \infty$, we get $fw = gw$. Thus w is a coincidence point of f and g. Further faint compatibility implies that $fgw = gfw$ and hence $fgw = gfw = ffw = ggw$.

We claim that $fw = ffw$. If $fw \neq ffw$, then using $(C5)$ we get

$$\begin{aligned} F(fw, ffw, t) &\neq \min\{F(fw, gfw, t), F(gfw, ffw, t)\} \\ &= F(fw, ffw, t), \end{aligned}$$

which is a contradiction. Hence $fw = ffw$ is a common fixed point of f and g.

Similarly, we can also complete the proof when g is continuous. This completes the proof. □

Example 2.6. Let (X, F, Δ) be a Menger space where $X = [2, \infty)$ with a t-norm defined by $\Delta(a, b) = \min\{a, b\}$ by

$$F(x, y, t) = \frac{t}{t + |x - y|}$$

for all $x, y \in X$ and $t > 0$. Define $f, g : X \to X$ as

$$fx = \begin{cases} 2, & \text{if } 2 \leq x \leq 5, \\ 8, & \text{if } x > 5, \end{cases} \qquad gx = \begin{cases} 2, & \text{if } 2 \leq x \leq 5, \\ x + 3, & \text{if } x > 5. \end{cases}$$

(1) Let a sequence $\{x_n\} = \{5 + \frac{1}{n}\}$. Now $fx_n \to 8$, $gx_n \to 8$, $fgx_n \to 8$ and $gfx_n \to 11$ and so $F(fgx_n, gfx_n, t)$ does not converge to 1. Therefore, f and g are noncompatible.

(2) Let a sequence $z_n = 2 + \frac{1}{n}$ Now $fz_n \to 2$, $gz_n \to 2$, $fgz_n \to 2$ and $gfz_n \to 2$ and so $F(fgz_n, gfz_n, t) \to 1$. Therefore, f and g are conditionally compatible. Also $f2 = g2$ and $fg2 = gf2$. Hence f and g are faintly compatible.

(3) Condition $(C4)$ and $(C5)$ is satisfied.

Hence, all the conditions of the Theorem 2.5 are satisfied and $x = 2$ is the common fixed point of f and g.

Theorem 2.7. *Let f and g be noncompatible faintly compatible self-mappings of a Menger space (X, F, Δ) satisfying the condition $(C5)$. Assume that f and g are continuous. Then f and g have a common fixed point.*

Proof. Let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = u$ for some $u \in X$. Since f and g are noncompatible, $\lim_{n\to\infty} F(fgx_n, gfx_n, t) \neq 1$ or non-existent. The continuity of f and g implies that $\lim_{n\to\infty} fgx_n = fu$ and $\lim_{n\to\infty} gfx_n = gu$. In view of faint compatibility and continuity of f and g, we can easily obtain a common fixed point as has been proved in the corresponding part of Theorem 2.5. □

Remark 2.8. 1. It may be in order to point out here that our results have been proved under a noncomplete Menger space.

2. Theorem 2.7 remain trues if we replace ($C5$) by

$$(C6) \qquad F(gx, ggy, t) \neq \min\{F(gx, fgy, t), F(fgx, ggy, t)\}$$

for all $x, y \in X$, whenever the right side is non-one ($\neq 1$).

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