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A NOTE ON USELESS COMMUNICATION CHANNELS AND DIRECTED GRAPHS

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Abstract: The purpose of this note is to give an idea for graph theorists to explore a connection between communication channels and graph theory as indicated in our conjectures below.

1. Definitions

From this point, we define graph as follows:

Definition 1. A graph is said to be *super complete* of order n, i.e. $\widetilde{K}_n$, if and only if

1. the graph is K_n complete [1];
2. each vertex of K_n contains a loop; and

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3. any two vertices i and j would have two edges such one edge is directed from vertex i to j, and vice-versa.

Therefore, a super complete graph is a directed graph.

A super complete matrix $\widetilde{K}_n$ can be represented by an adjacency matrix in which its ij-entry would denote the weight of the edge connecting vertex i to j.

Definition 2. A *square channel matrix* or simply a *channel matrix* is a matrix describing a discrete memoryless communications channel in which its ij-entries describe probabilities $p\left(y_j|x_i\right)$ such that $x_1, x_2, \cdots, x_n$ and $y_1, y_2, \cdots,$ y_n are the input and output symbols, respectively [2].

It follows from the previous definition that the row sums of the channel matrix is 1.

Definition 3. A real non-negative square matrix A is *right stochastic* or simply *stochastic* if all its rows sum to 1, and is also considered a *Markov chain.*

Hence, a channel matrix is always a stochastic matrix.

Definition 4. The channel matrix is a *useless* (or *totally noisy*) channel and is a stochastic matrix all of whose rows are equal.

2. Results and Information-Theoretic Proofs

Conjecture 5. *A super complete graph $\widetilde{K}_n$ has $3n$ edges and vertices with degree $2n$.*

This is based from an observation with Definition 1, in which it can be seen that $\widetilde{K}_1$ has only one edge (a loop), $\widetilde{K}_2$ has two edges and two loops, and by enumerating examples for $n = [1, 10]$, $\widetilde{K}_n$ will have $3n$ edges constituting n loops.

From the information theoretic point of view, since a channel matrix is a stochastic matrix, it can be noted that such channel matrix could be represented by a super complete graph such that the ij-entries of $\widetilde{K}_n$ would represent probabilities $p(y_j|x_i)$ described in Definition 2. Thus, we consider the following statement:

Conjecture 6. *A channel matrix that is represented by a super complete graph* $\widetilde{K}_n$ *would contain vertices such that each end vertex would describe an output symbol, while a start vertex would describe an input symbol, and the edge connecting the start and end vertices would constitute the probability value.*

This can be seen by enumerating examples in which the adjacency matrix of the super complete graph would be mathematically the same with the communication channel matrix.

Conjecture 7. *Let A be a stochastic matrix representing a communication channel. Then, there is a set of channel matrices* $\mathcal{A}$ *such that* A^{∞} *will be in its steady-state, all of whose rows are equal [3].*

Combining the previous mentioed conjectures, the following result is obtained:

Conjecture 8. *A useless channel matrix can be represented by a super complete graph* $\widetilde{K}_n$ *such that:*

1. the weight of all the edges directing toward a vertex have the same values; and
2. the total weights of all edges directing outward a vertex is 1.

By inspection, since the rows of a useless channel matrix are the same, this means that the edge that would direct toward the end vertex would have the same weights. Furthermore, since the row sum of each row is 1, it also means that the total weight of all edges directing outward a start vertex is 1.

Corollary 9. *A super complete graph* $\widetilde{K}_n$ *would represent a useless channel if and only if the weight of all the edges directing toward a vertex have the same values.*

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