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A SIMPLE METHOD TO DERIVE THE EOQ AND EPQ MODELS WITH BACKORDERS

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Abstract: This paper uses the algebraic arithmetic-geometric mean inequality method (the algebraic AGM method) to derive the optimal lot size and the optimal backorders level for the EOQ and EPQ models with backorders. The method is very simple to derive both the optimal lot size and optimal backorders level without derivatives.

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1. Introduction

Since the first economic order quantity (EOQ) model was introduced by [6] and the economic production quantity (EPQ) model was presented by [8], the lot size for the EOQ and EPQ models with and without backorders have been studied

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extensively. In the past few years, many scholars have tried to develop the EOQ and EPQ models without differential calculus. An appropriate optimization approach of this development is referred to as algebraic method. Initially, [4] considered this approach to give the standard EOQ formula. After that [5] used this method to derive the EOQ model with backorders. [1] extended this method to the EPQ model with backorders, [7] tried to improve the algebraic method to solve the EOQ and EPQ models with backorders and [3] improved the method in [7] by replacing their sophisticated algebraic skill. Recently, an optimization approach: the arithmetic-geometric mean (AGM) inequality and the Cauchy-Bunyakovsky-Schwarz (CBS) inequality, which proposed by Cárdenas-Barrón [2]. He used the AGM and CBS inequalities to derive the EOQ and EPQ models with backorders, and he said that the method is simpler than the algebraic methods presented by [5] and [1]. However, we observed that it is not easy to set the desired variables to satisfy the CBS inequality. In this paper, we apply the algebraic method and the AGM inequality (the algebraic AGM method) to derive the optimal lot size and the optimal backorders level for the EOQ and EPQ models with backorders.

2. Method

This section presents the algebraic AGM method, which is used to derive the optimal solutions of EOQ and EPQ models with backorders. The method is created by combining the algebraic method and the AGM inequality.

The algebraic method. Let a_1 and a_2 be positive real numbers and x a decision variable, then

$$a_1x^2 - a_2x = a_1\left(x - \frac{a_2}{2a_1}\right)^2 - \frac{a_2^2}{4a_1}.$$

The AGM inequality. Let $a_1, a_2, ..., a_n$ be n positive real numbers, then

$$\frac{\sum_{i=1}^{n} a_i}{n} \geq \sqrt[n]{\prod_{i=1}^{n} a_i} \text{ with equality iff } a_1 = a_2 = \cdots a_n.$$

3. Results

The following notation is the same notation in [2] that will be used in both EOQ and EPQ models with backorders.

$d =$ demand rate per time unit,
$A =$ ordering cost per order,
$h =$ per unit holding cost per time unit,
$v =$ per unit backorder cost per time unit,
$p =$ production rate per time unit,
$Q =$ order quantity,
$B =$ backorders level.

3.1. The EOQ Model with Backorders

Following [2], the total inventory cost function for the EOQ model with back-oeders is of the form

$$\begin{aligned} TC(Q,B) &= \frac{Ad}{Q} + \frac{h(Q-B)^2}{2Q} + \frac{vB^2}{2Q} \\ &= \frac{Ad}{Q} + \frac{hQ}{2} + \frac{(h+v)B^2}{2Q} - hB. \end{aligned} \tag{3.1}$$

Applying the algebraic method to Eq. (3.1), the total inventory cost can be written as

$$TC(Q,B) = \frac{Ad}{Q} + \frac{hvQ}{2(h+v)} + \frac{h+v}{2Q}\left(B - \frac{hQ}{h+v}\right)^2, \tag{3.2}$$

which has the minimum value when

$$B = \frac{hQ}{h+v}. \tag{3.3}$$

Thus, Eq. (3.2) reduces to be

$$TC(Q) = \frac{Ad}{Q} + \frac{hvQ}{2(h+v)}. \tag{3.4}$$

Applying the AGM method to Eq. (3.4), yields

$$TC(Q) \geq \sqrt{\frac{2Adhv}{h+v}} \tag{3.5}$$

and $TC(Q)$ has the minimum value when $TC(Q) = \sqrt{\frac{2Adhv}{h+v}}$, that is

$$\frac{Ad}{Q} = \frac{hvQ}{2(h+v)}.$$

From which, it follows that $Q^{\star} = \sqrt{\frac{2Ad(h+v)}{hv}}$, $B^{\star} = \frac{hQ^{*}}{h+v} = \sqrt{\frac{2Adh}{v(h+v)}}$ and $TC(Q^{\star}, B^{\star}) = \sqrt{\frac{2Adhv}{h+v}}$ are the optimal lot size, the optimal backorders level and the optimal total inventory cost, respectively.

3.2. The EPQ Model with Backorders

Let $\rho = 1 - d/p$, by following [2], the total inventory cost function for the EPQ model with backoeders is of the form

$$\begin{aligned} TC(Q,B) &= \frac{Ad}{Q} + \frac{h(Q\rho - B)^2}{2Q\rho} + \frac{vB^2}{2Q\rho} \\ &= \frac{Ad}{Q} + \frac{hQ\rho}{2} + \frac{(h+v)B^2}{2Q\rho} - hB. \end{aligned} \tag{3.6}$$

Applying the algebraic method to Eq. (3.6), the total inventory cost can be written as

$$TC(Q,B) = \frac{Ad}{Q} + \frac{hvQ\rho}{2(h+v)} + \frac{h+v}{2Q\rho}\left(B - \frac{hQ\rho}{h+v}\right)^2, \tag{3.7}$$

which gives the minimum value when

$$B = \frac{hQ\rho}{h+v}. \tag{3.8}$$

Thus, Eq. (3.7) becomes

$$TC(Q) = \frac{Ad}{Q} + \frac{hvQ\rho}{2(h+v)}. \tag{3.9}$$

Applying the AGM method to Eq. (3.9), yields

$$TC(Q) \geq \sqrt{\frac{2Ad\rho hv}{h+v}} \tag{3.10}$$

and $TC(Q)$ has the minimum value when $TC(Q) = \sqrt{\frac{2Ad\rho hv}{h+v}}$, that is

$$\frac{Ad}{Q} = \frac{hvQ\rho}{2(h+v)}.$$

From which, it follows that $Q^{\star} = \sqrt{\frac{2Ad(h+v)}{hv\rho}}$, $B^{\star} = \frac{hQ^{*}\rho}{h+v} = \sqrt{\frac{2Ad\rho h}{v(h+v)}}$ and $TC(Q^{\star}, B^{\star}) = \sqrt{\frac{2Ad\rho hv}{h+v}}$ are the optimal lot size, the optimal backorders level and the optimal total inventory cost, respectively.

4. Conclusion

The algebraic AGM method is an optimization approach for deriving the optimal solution for the EOQ and EPQ models when backorders are allowed. It is very simple to derive both the optimal lot size and optimal backorders level. Additionally, it is simpler than the algebraic methods proposed by [5], [1] and [7], and is also easier than the method proposed by [2]. So, it could be used to introduce the basic inventory theories to students who lack the knowledge of differential calculus.

References

[1] L.E. Cárdenas-Barrón, The economic production quantity (EPQ) with shortage derived algebraically. *Int. J. Product. Econom.*, **70** (2001), 289–292.

[2] L.E. Cárdenas-Barrón, An easy method to derive EOQ and EPQ models with backorders. *Comp. math. Appl.*, **59** (2010), 948–952.

[3] S.K.J. Chang, J.P.C. Chuang, H.J. Chen, Short comments on technical note-The EOQ and EPQ models with shortages derived without derivatives. *Int. J. Product. Econom.*, **97** (2005), 241–243.

[4] R.W. Grubbström, *Material requirements planning and manufacturing resource planning*. International Encyclopedia of business and Management. Routledge, London, 1996.

[5] R.W. Grubbström, A. Erdem, The EOQ with backlogging derived without derivatives. *Int. J. Product. Econom.*, **59** (1999), 529–530.

[6] F.W. Harris, How many parts to make at once factory. *The Magazine of Management*, **10** (1913), 135–136.

[7] R. Ronald, G.K. Yang, P. Chu, The EOQ and EPQ models with shortages derived without derivatives. *Int. J. Product. Econom.*, **92** (2004), 197–200.

[8] E.W. Taft, The most economical production lot. *The Iron Age*, **101** (1918), 1410–1412.