

**THE INVENTORY MODELS WITH BACKORDERS AND
DEFECTIVE ITEMS DERIVED ALGEBRAICALLY AND AGM**

K. Teerapabolarn¹ §, S. Khamrod²

Department of Mathematics

Faculty of Science

Burapha University

Chonburi, 20131, THAILAND

Abstract: The algebraic arithmetic-geometric mean inequality method (the algebraic AGM method) is used to derive the optimal lot size and the optimal backorders level for the EOQ and EPQ models with backorders and defective items introduced by [6]. The method is easy to derive both the optimal lot size and optimal backorders level without derivatives.

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1. Introduction

In the context of the deterministic inventory models, the most valuable findings have concerned the economic order/production quantity (EOQ/EPQ) models with/without shortages. Especially, in previous several articles, the optimal solution for these models have been derived by differential calculus approach. However, the mathematical methodology of this approach is difficult to many

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§Correspondence author

students who lack the knowledge of differential calculus. In the past few years, some authors have tried to develop and derive the optimal solution for the EOQ and EPQ models without differential calculus, and one appropriate approach of this development is referred to as the algebraic approach. Initially, [4] considered the algebraic method to give the standard EOQ formula. After that [5] used this method to derive the EOQ model with backorders. [1] extended this method to the EPQ model with backorders, [7] tried to improve the algebraic method to solve the EOQ and EPQ models with backorders and [3] improved the method of [7], by replacing their sophisticated algebraic skill, to determine the optimal solution for these models. Recently, an optimization approach : the arithmetic-geometric mean (AGM) inequality and the Cauchy-Bunyakovsky-Schwarz (CBS) inequality, which proposed by Cárdenas-Barrón [2]. This approach is simpler than the algebraic approach, presented by [5] and [1], for deriving the EOQ and EPQ models with backorders.

We known that a common assumption of both the EOQ and EPQ models with backorders is that all units purchased or produced are of 100% good quality. For realistic system, it is difficult to purchase or produce items 100% good quality. Huang [6] assumed 100% inspection policy and the known proportion of defective items was removed prior to store or use at the end of the screening process. Additionally, He used the algebraic method of [5] and [1] to derive the optimal lot size and the optimal backorders level for the EOQ and EPQ models with backorders and defective items. However, Tu et al. [8] observed that this algebraic method had the same problem as [5] and [1]. They later used the simple method in [2], the AGM and CBS inequalities, to solve the optimal solutions for these models. Although the method is simpler than the algebraic methods in [5] and [1], but it is not easy to to set the desired variables to satisfy the CBS inequality. In this paper, we apply the algebraic method and the AGM inequality (the algebraic AGM method) to derive the optimal lot size and the optimal backorders level for the EOQ and EPQ models with backorders and defective items.

2. Method

This section presents the algebraic AGM method, which is is created by combining the algebraic method and the AGM inequality.

The algebraic method. Let a_1 and a_2 be positive real numbers and x a

decision variable, then

$$a_1x^2 - a_2x = a_1 \left(x - \frac{a_2}{2a_1} \right)^2 - \frac{a_2^2}{4a_1}.$$

The AGM inequality. Let a_1, a_2, \dots, a_n be n positive real numbers, then

$$\frac{\sum_{i=1}^n a_i}{n} \geq \sqrt[n]{\prod_{i=1}^n a_i} \text{ with equality iff } a_1 = a_2 = \dots = a_n.$$

3. Results

The assumptions and the following notation are the same in [6] that will be used in both EOQ and EPQ models with backorders and defective items.

Q = order quantity or production quantity including defective items

V = inventory level including defective items.

L = backorders level including defective items

D = demand rate for non-defective items, units per time

P = production rate for non-defective items, units per time ($P > D$)

A = ordering cost or setup cost

I = the fixed inspection cost incurred with each lot

i = unit inspection cost

h = unit stock holding cost per unit per time

v = unit backorder cost per unit per time

k = the known proportion of defective items in Q

3.1. The EOQ Model with Backorders and Defective Items

Following [8], the total inventory cost function for the EOQ model with backorders and defective items is of the form

$$\begin{aligned} TC(Q, L) &= \frac{(A + I)D}{(1 - k)Q} + \frac{Di}{1 - k} + \frac{h(1 - k)(Q - L)^2}{2Q} + \frac{v(1 - k)L^2}{2Q} \\ &= \frac{(A + I)D}{(1 - k)Q} + (1 - k) \left\{ \frac{hQ}{2} + \frac{(h + v)L^2}{2Q} - hL \right\} + \frac{Di}{1 - k}. \end{aligned} \quad (3.1)$$

Applying the algebraic method to Eq. (3.1), the total inventory cost can be written as

$$TC(Q, L) = \frac{(A + I)D}{(1 - k)Q} + (1 - k) \left\{ \frac{hvQ}{2(h + v)} + \frac{h + v}{2Q} \left(L - \frac{hQ}{h + v} \right)^2 \right\} + \frac{Di}{1 - k}, \quad (3.2)$$

which has the minimum value when

$$L = \frac{hQ}{h + v}. \quad (3.3)$$

Thus, Eq. (3.2) becomes

$$TC(Q) = \frac{(A + I)D}{(1 - k)Q} + \frac{hvQ(1 - k)}{2(h + v)} + \frac{Di}{1 - k}. \quad (3.4)$$

Applying the AGM method to Eq. (3.4), yields

$$TC(Q) \geq \sqrt{\frac{2(A + I)Dhv}{h + v}} + \frac{Di}{1 - k} \quad (3.5)$$

and $TC(Q)$ has the minimum value when $TC(Q) = \sqrt{\frac{2(A + I)Dhv}{h + v}} + \frac{Di}{1 - k}$, that is

$$\frac{(A + I)D}{(1 - k)Q} = \frac{hvQ(1 - k)}{2(h + v)}.$$

From which, it follows that

$$Q = \frac{1}{1 - k} \sqrt{\frac{2(A + I)D(h + v)}{hv}}, \quad L = \frac{hQ}{h + v} = \frac{1}{1 - k} \sqrt{\frac{2(A + I)Dh}{v(h + v)}}$$

and

$$TC(Q, L) = \sqrt{\frac{2(A + I)Dhv}{h + v}} + \frac{Di}{1 - k}$$

are the optimal lot size, the optimal backorders level and the optimal total inventory cost, respectively.

3.2. The EPQ Model with Backorders and Defective Items

Since the total inventory cost function for the EPQ model with backorders and defective items in Eq.(12) of [8] is incorrect, we so correct it as follows:

$$TC(Q, L) = \frac{(A + I)D}{(1 - k)Q} + \frac{Di}{1 - k} + \frac{h(1 - k)(Q\rho - L)^2}{2Q\rho} + \frac{v(1 - k)L^2}{2Q\rho}$$

$$= \frac{(A + I)D}{(1 - k)Q} + (1 - k) \left\{ \frac{hQ\rho}{2} + \frac{(h + v)L^2}{2Q\rho} - hL \right\} + \frac{Di}{1 - k}, \quad (3.6)$$

where $\rho = 1 - D/P$ and $Q = \frac{V+L}{\rho}$. Applying the algebraic method to Eq. (3.6), the total inventory cost can be written as

$$TC(Q, L) = \frac{(A + I)D}{(1 - k)Q} + (1 - k) \left\{ \frac{hvQ\rho}{2(h + v)} + \frac{h + v}{2Q\rho} \left(L - \frac{hQ\rho}{h + v} \right)^2 \right\} + \frac{Di}{1 - k}, \quad (3.7)$$

which has the minimum value when

$$L = \frac{hQ\rho}{h + v}. \quad (3.8)$$

Thus, Eq. (3.7) becomes

$$TC(Q) = \frac{(A + I)D}{(1 - k)Q} + \frac{hvQ\rho(1 - k)}{2(h + v)} + \frac{Di}{1 - k}. \quad (3.9)$$

Applying the AGM method to Eq. (3.9), yields

$$TC(Q) \geq \sqrt{\frac{2(A + I)D\rho hv}{h + v}} + \frac{Di}{1 - k} \quad (3.10)$$

and $TC(Q)$ has the minimum value when $TC(Q) = \sqrt{\frac{2(A+I)D\rho hv}{h+v}} + \frac{Di}{1-k}$, that is

$$\frac{(A + I)D}{(1 - k)Q} = \frac{hvQ\rho(1 - k)}{2(h + v)}.$$

From which, it follows that

$$Q = \frac{1}{1 - k} \sqrt{\frac{2(A + I)D(h + v)}{hv\rho}}, \quad L = \frac{1}{1 - k} \sqrt{\frac{2(A + I)D\rho h}{v(h + v)}}$$

and

$$TC(Q, L) = \sqrt{\frac{2(A + I)D\rho hv}{h + v}} + \frac{Di}{1 - k}$$

are the optimal lot size, the optimal backorders level and the optimal total inventory cost, respectively.

4. Conclusion

The algebraic AGM method is an optimization approach for deriving the optimal solution for the EOQ and EPQ models when backorders and defective items are allowed. It is very simple to derive both the optimal lot size and optimal backorders level. Additionally, it is simpler than the algebraic method used in [6], and is also easier than the method used in [8]. So, it could be used to introduce the basic inventory theories to students who lack the knowledge of derivatives.

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