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ALMOST CONTRA REGULAR GENERALIZED *b*-CONTINUOUS FUNCTIONS

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Abstract: In this paper, the authors introduce a new class of functions called almost contra regular generalized b-continuous function (briefly almost contra rgb-continuous) in topological spaces. Some characterizations and several properties concerning almost contra rgb-continuous functions are obtained.

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1. Introduction

In 2002, Jafari and Noiri introduced and studied a new form of functions called contra-precontinuous functions. The purpose of this paper is to introduce and study almost contra rgb-continuous functions via the concept of rgb-closed sets. Also, properties of almost contra rgb-continuity are discussed. Moreover, we obtain basic properties and preservation theorems of almost contra rgb-continuous functions and relationships between almost contra rgb-continuity and rgb-regular graphs.

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Throughout this paper (X, τ) and (Y, σ) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. Let $A \subseteq X$, the closure of A and interior of A will be denoted by $cl(A)$ and $int(A)$ respectively, union of all rgb-open sets X contained in A is called rgb-interior of A and it is denoted by $rgb - int(A)$, the intersection of all rgb-closed sets of X containing A is called rgb-closure of A and it is denoted by $rgb - cl(A)$.

2. Preliminaries

Definition 1. Let A subset A of a topological space (X, τ), is called

(1) a **pre-open set** [24] if $A \subseteq int(cl(A))$.

(2) a **semi-open set**[18] if $A \subseteq cl(int(A))$.

(3) a ***b*-open set** [5] if $A \subseteq cl(int(A)) \cup int(cl(A))$.

(4) a **generalized *b*- closed set** (briefly gb- closed) [2] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(5) a **generalized *αb*- closed set** (briefly $g\alpha b$- closed) [33] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X.

(6) a **regular generalized *b*- closed set** (briefly rgb-closed set)[21] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

Definition 2. A function $f : (X, \tau) \to (Y, \sigma)$ is called

(1) almost contra continuous[31] if $f^{-1}(V)$ is closed in (X, τ) for every regular-open set V of (Y, σ).

(2) almost contra b-continuous [3] if $f^{-1}(V)$ is b-closed in (X, τ) for every regular open set V of (Y, σ).

(3) almost contra pre-continuous [15] if $f^{-1}(V)$ is pre-closed in (X, τ) for every regular open set V of (Y, σ).

(4) a contra semi-continuous [14] if $f^{-1}(V)$ is semi-closed in (X, τ) for every regular open set V of (Y, σ).

(5) a contra gb-continuous [2] if $f^{-1}(V)$ is gb-closed in (X, τ) for every regular open set V of (Y, σ).

3. Almost Contra Regular Generalized b-ConTinuOus Functions

In this section, we introduce Almost contra regular generalized b-continuous functions and investigate some of their properties.

Definition 3. A function $f : (X, \tau) \to (Y, \sigma)$ is called almost contra regular generalized b-continuous if $f^{-1}(V)$ is rgb-closed in (X, τ) for every regular open set V in (Y, σ).

Example 4. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{b\}, \{a, b\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = b$, $f(c) = a$. Clearly f is almost contra rgb-continuous.

Theorem 5. *If $f : X \to Y$ is contra rgb-continuous then it is almost contra rgb-continuous.*

Proof. Obvious, because every regular open set is open set. □

Remark 6. Converse of the above theorem need be true in general as seen from the following example.

Example 7. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \varphi, \{b\}, \{a, b\}, \{b, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. Then f is almost contra rgb-continuous function but not contra rgb-continuous, because for the open set $\{b\}$ in Y and $f^{-1}(b) = \{a\}$ is not rgb-closed in X.

Theorem 8. *(i) Every almost contra pre-continuous function is almost contra rgb-continuous function.*

(ii) Every almost contra semi-continuous function is almost contra rgb-continuous function.

(iii) Every almost contra rgb-continuous function is almost contra gb-continuous function.

(iv) Every almost contra rgb-continuous function is almost contra $g\alpha b$-continuous function.

(v) *Every almost contra b-continuous function is almost contra rgb-continuous function.*

Remark 9. Converse of the above statements is not true as shown in the following example.

Example 10. (i) Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{b\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{Y, \varphi, \{b\}, \{c\}, \{b, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$. Clearly f is almost contra rgb-continuous but f is not contra pre-continuous. Because $f^{-1}(\{c\}) = \{b\}$ is not pre-closed in (X, τ) where $\{c\}$ is regular-open in (Y, σ).

(ii) Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{c\}, \{a, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = a$, $f(c) = b$. Clearly f is almost contra rgb-continuous but f is not almost contra semi-continuous. Because $f^{-1}(\{c\}) = \{a\}$ is not semi-closed in (X, τ) where $\{c\}$ is regular-open in (Y, σ).

(iii) Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{b\}, \{c\}, \{b, c\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{b\}, \{a, b\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$. Clearly f is almost contra gb-continuous but f is not contra rgb-continuous. Because $f^{-1}(\{b\}) = \{b\}$ is not rgb-closed in (X, τ) where $\{b\}$ is regular-open in (Y, σ).

(iv) Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{c\}, \{a, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = b$, $f(c) = a$. Clearly f is almost contra $g\alpha b$ -continuous but f is not contra rgb-continuous. Because $f^{-1}(\{b\}) = \{b\}$ is not rgb-closed in (X, τ) where $\{b\}$ is regular-open in (Y, σ).

(v) Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \varphi, \{b\}, \{b, c\}\}$ and $\sigma = \{Y, \varphi, \{a\}, \{c\}, \{a, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. Clearly f is almost contra rgb-continuous but f is not contra b-continuous. Because $f^{-1}(\{a\}) = \{b\}$ is not b-closed in (X, τ) where $\{a\}$ is regular-open in (Y, σ).

Theorem 11. *The following are equivalent for a function $f : X \to Y$,*

(1) *f is almost contra rgb-continuous.*

(2) *for every regular closed set F of Y, $f^{-1}(F)$ is rgb-open set of X.*

(3) for each $x \in X$ and each regular closed set F of Y containing $f(x)$, there exists rgb-open U containing x such that $f(U) \subset F$.

(4) for each $x \in X$ and each regular open set V of Y not containing $f(x)$, there exists rgb-closed set K not containing x such that $f^{-1}(V) \subset K$.

Proof. (1)⇒(2): Let F be a regular closed set in Y, then YF is a regular open set in Y. By (1), $f^{-1}(Y - F) = X - f^{-1}(F)$ is rgb-closed set in X. This implies $f^{-1}(F)$ is rgb-open set in X. Therefore, (2) holds.

(2)⇒(1): Let G be a regular open set of Y. Then YG is a regular closed set in Y. By (2), $f^{-1}(Y - G)$ is rgb-open set in X. This implies $X - f^{-1}(G)$ is rgb- open set in X, which implies $f^{-1}(G)$ is rgb-closed set in X. Therefore, (1) hold.

(2)⇒(3): Let F be a regular closed set in Y containing $f(x)$, which implies $x \in f^{-1}(F)$. By (2), $f^{-1}(F)$ is rgb-open in X containing x. Set $U = f^{-1}(F)$, which implies U is rgb-open in X containing x and $f(U) = f(f^{-1}(F)) \subset F$. Therefore (3) holds.

(3)⇒(2): Let F be a regular closed set in Y containing $f(x)$, which implies $x \in f^{-1}(F)$. From (3), there exists rgb-open Ux in X containing x such that $f(Ux) \subset F$. That is $Ux \subset f^{-1}(F)$. Thus $f^{-1}(F) = \cup\{Ux : x \in f^{-1}(F)$, which is union of rgb-open sets. Therefore, $f^{-1}(F)$ is rgb-open set of X.

(3)⇒(4): Let V be a regular open set in Y not containing $f(x)$. Then YV is a regular closed set in Y containing $f(x)$. From (3), there exists a rgb-open set U in X containing x such that $f(U) \subset YV$ .This implies $U \subset f^{-1}(Y - V) = Xf^{-1}(V)$. Hence, $f^{-1}(V) \subset XU$. Set $K = XU$, then K is rgb-closed set not containing x in X such that $f^{-1}(V) \subset K$.

(4)⇒(3): Let F be a regular closed set in Y containing $f(x)$. Then YF is a regular open set in Y not containing $f(x)$. From (4), there exists rgb-closed set K in X not containing x such that $f^{-1}(Y - F) \subset K$. This implies $X - f^{-1}(F) \subset K$. Hence, $X - K \subset f^{-1}(F)$, that is $f(X - K) \subset F$. Set $U = XK$, then U is rgb-open set containing x in X such that $f(U) \subset F$. □

Theorem 12. *The following are equivalent for a function $f : X \to Y$,*

(1) f is almost contra rgb-continuous.

(2) $f^{-1}(Int(Cl(G)))$ is rgb-closed set in X for every open subset G of Y.

(3) $f^{-1}(Cl(Int(F)))$ is rgb-open set in X for every closed subset F of Y.

Proof. (1)⇒(2).Let G be an open set in Y. Then $Int(Cl(G))$ is regular open set in Y. By (1), $f^{-1}(Int(Cl(G)) \in rgb - C(X)$.

(2)⇒(1): Proof is obvious.

(1)⇒(3): Let F be a closed set in Y. Then $Cl(Int(G))$ is regular closed set in Y. By (1), $f^{-1}(Cl(Int(G)) \in rgb - O(X)$.

(3)⇒(1): Proof is obvious. □

Definition 13. A function $f : X \to Y$ is said to be R-map if $f^{-1}(V)$ is regular open in X for each regular open set V of Y.

Definition 14. A function $f : X \to Y$ is said to be perfectly continuous if $f^{-1}(V)$ is clopen in X for each open set V of Y.

Theorem 15. *For two functions $f : X \to Y$ and $g : Y \to Z$, let $g \circ f : X \to Z$ is a composition function. Then, the following properties hold.*

(1) If f is almost contra rgb-continuous and g is an R-map, then $g \circ f$ is almost contra rgb-continuous.

(2) If f is almost contra rgb-continuous and g is perfectly continuous, then $g \circ f$ is contra rgb-continuous.

(3) If f is contra rgb-continuous and g is almost continuous, then $g \circ f$ is almost contra rgb-continuous.

Proof. (1) Let V be any regular open set in Z. Since g is an R-map, $g^{-1}(V)$ is regular open in Y. Since f is almost contra rgb-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is rgb-closed set in X. Therefore $g \circ f$ is almost contra rgb-continuous.

(2) Let V be any regular open set in Z. Since g is perfectly continuous, $g^{-1}(V)$ is clopen in Y. Since f is almost contra rgb-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is rgb-open and rgb-closed set in X. Therefore $g \circ f$ is continuous and contra rgb-continuous.

(3) Let V be any regular open set in Z. Since g is almost continuous, $g^{-1}(V)$ is open in Y. Since f is almost contra rgb-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is rgb- closed set in X. Therefore $g \circ f$ is almost contra rgb-continuous.

□

Theorem 16. *Let $f : X \to Y$ is a contra rgb-continuous and $g : Y \to Z$ is rgb-continuous. If Y is $Trgb$-space, then $g \circ f : X \to Z$ is an almost contra rgb-continuous.*

Proof. Let V be any regular open and hence open set in Z. Since g is rgb-continuous $g^{-1}(V)$ is rgb-open in Y and Y is $Trgb$-space implies $g^{-1}(V)$ open in Y. Since f is contra rgb-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is rgb-closed set in X. Therefore, $g \circ f$ is an almost contra rgb-continuous. □

Theorem 17. *If $f : X \to Y$ is surjective strongly rgb-open (or strongly rgb-closed) and $g : Y \to Z$ is a function such that $g \circ f : X \to Z$ is an almost contra rgb-continuous, then g is an almost contra rgb-continuous.*

Proof. Let V be any regular closed (resp. regular open) set in Z. Since $g \circ f$ is an almost contra rgb-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is rgb-open (resp. rgb-closed) in X. Since f is surjective and strongly rgb-open (or strongly rgb-closed), $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is rgb-open(or rgb-closed). Therefore g is an almost contra rgb-continuous. □

Definition 18. A function $f : X \to Y$ is called weakly rgb-continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in rgb - O(X; x)$ such that $f(U) \subset cl(V)$.

Theorem 19. *If a function $f : X \to Y$ is an almost contra rgb-continuous, then f is weakly rgb-continuous function.*

Proof. Let $x \in X$ and V be an open set in Y containing $f(x)$. Then $cl(V)$ is regular closed in Y containing $f(x)$. Since f is an almost contra rgb-continuous function by Theorem 3.10 (2), $f^{-1}(cl(V))$ is rgb-open set in X containing x. Set $U = f^{-1}(cl(V))$, then $f(U) \subset f(f^{-1}(Cl(V))) \subset cl(V)$. This shows that f is almost weakly rgb-continuous function. □

Definition 20. A space X is called locally rgb-indiscrete if every rgb-open set is closed in X.

Theorem 21. *If a function $f : X \to Y$ is almost contra rgb-continuous and X is locally rgb-indiscrete space, then f is almost continuous.*

Proof. Let U be a regular open set in Y. Since f is almost contra rgb-continuous $f^{-1}(U)$ is rgb-closed set in X and X is locally rgb-indiscrete space, which implies $f^{-1}(U)$ is an open set in X. Therefore f is almost continuous. □

Lemma 22. *Let A and X_0 be subsets of a space X. If $A \in rgb - O(X)$ and $X_0 \in \tau^\alpha$, then $A \cap X_0 \in rgb - O(X_0)$.*

Theorem 23. *If $f : X \to Y$ is almost contra rgb-continuous and $X_0 \in \tau^\alpha$ then the restriction $f/X_0 : X_0 \to Y$ is almost contra rgb-continuous.*

Proof. Let V be any regular open set of Y. By theorem , we have $f^{-1}(V) \in rgb - O(X)$ and hence $(f/X_0)^{-1}(V) = f^{-1}(V) \cap X_0 \in rgb - O(X_0)$. By lemma 3.20, it follows that f/X_0 is almost contra rgb-continuous. □

Theorem 24. *If $f : X \to \prod Y_\lambda$ is almost contra rgb-continuous, then $P_\lambda \circ f : X \to Y_\lambda$ is almost contra rgb-continuous for each $\lambda \in \nabla$, where P_λ is the projection of $\prod Y_\lambda$ onto Y_λ.*

Proof. Let Y_λ be any regular open set of Y. Since P_λ is continuous open, it is an R-map and hence $(P_\lambda)^{-1} \in RO(\prod Y_\lambda)$. By theorem, $f^{-1}(P_\lambda^{-1}(V)) = (P_\lambda \circ f)^{-1} \in rgbO(X)$. Hence $P_\lambda \circ f$ is almost contra rgb-continuous. □

4. *rgb*-Regular Graphs and Strongly Contra *rgb*-Closed Graphs

Definition 25. A graph G_f of a function $f : X \to Y$ is said to be rgb-regular (strongly contra rgb-closed) if for each $(x, y) \in (X \times Y)\backslash G_f$, there exist a rgb-closed set U in X containing x and $V \in R - O(Y)$ such that $(U \times V) \cap G_f = \varphi$.

Theorem 26. *If $f : X \to Y$ is almost contra rgb-continuous and Y is T_2, then G_f is rgb-regular in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y)\backslash G_f)$. It is obvious that $f(x) \neq y$. Since Y is T_2, there exists $V, W \in RO(Y)$ such that $f(x)) \in V$, $Y \in W$ and $V \cap W = \varphi$. Since f is almost contra rgb-continuous, $f^{-1}(V)$ is a rgb-closed set in X containing x. If we take $U = f^{-1}(V)$, we have $f(U) \subset V$. Hence, $f(U) \cap W = \varphi$ and G_f is rgb-regular. □

Theorem 27. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function and $g : (X, \tau) \to (X \times Y, \tau \times \sigma)$ the graph function defined by $g(x) = (x, f(x))$ for every $x \in X$. Then f is almost rgb-continuous if and only if g is almost rgb-continuous.*

Proof. Necessary: Let $x \in X$ and $V \in rgb - O(Y)$ containing $f(x)$. Then, we have $g(x) = (x, f(x)) \in R - O(X \times Y)$. Since f is almost rgb-continuous, there exists a rgb-open set U of X containing x such that $g(U) \subset X \times Y$. Therefore, we obtain $f(U) \subset V$. Hence f is almost rgb continuous.

Sufficiency: Let $x \in X$ and w be a regular open set of $X \times Y$ containing $g(x)$. There exists $U_1 \in RO(X, \tau)$ and $V \in RO(Y, \sigma)$ such that $(x, f(x)) \in (U_1 \times V) \subset W$. Since f is almost rgb-continuous, there exists $U_2 \in rgb - O(X, \tau)$ such that $x \in U_2$ and $f(U_2) \subset V$. Set $U = U_1 \cap U_2$. We have $x \in Ux \in rgb - O(X, \tau)$ and $g(U) \subset (U_1 \times V) \subset W$. This shows that g is almost rgb-continuous. □

Theorem 28. *If a function $f : X \to Y$ be a almost rgb-continuous and almost continuous, then f is regular set-connected.*

Proof. Let $V \in RO(Y)$. Since f is almost contra rgb-continuous and almost continuous, $f^{-1}(V)$ is rgb-closed and open. So $f^{-1}(V)$ is clopen. It turns out that f is regular set-connected. □

5. Connectedness

Definition 29. A Space X is called rgb-connected if X cannot be written as a disjoint union of two non-empty rgb-open sets.

Theorem 30. *If $f : X \to Y$ is an almost contra rgb-continuous surjection and X is rgb-connected, then Y is connected.*

Proof. Suppose that Y is not a connected space. Then Y can be written as $Y = U_0 \cup V_0$ such that U_0 and V_0 are disjoint non-empty open sets. Let $U = int(cl(U_0))$ and $V = int(cl(V_0))$. Then U and V are disjoint nonempty regular open sets such that $Y = U \cup V$. Since f is almost contra rgb-continuous, then $f^{-1}(U)$ and $f^{-1}(V)$ are rgb-open sets of X. We have $X = f^{-1}(U) \cup f^{-1}(V)$ such that $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint. Since f is surjective, this shows that X is not rgb-connected. Hence Y is connected. □

Theorem 31. *The almost contra rgb-continuous image of rgb-connected space is connected.*

Proof. Let $f : X \to Y$ be an almost contra rgb-continuous function of a rgb-connected space X onto a topological space Y. Suppose that Y is not a connected space. There exist non-empty disjoint open sets V_1 and V_2 such that $Y = V_1 \cup V_2$. Therefore, V_1 and V_2 are clopen in Y. Since f is almost contra rgb-continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are rgb-open in X. Moreover, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are non-empty disjoint and $X = f^{-1}(V_1) \cup f^{-1}(V_2)$. This shows that X is not rgb-connected. This is a contradiction and hence Y is connected. □

Definition 32. A topological space X is said to be rgb-ultra connected if every two non-empty rgb-closed subsets of X intersect.

We recall that a topological space X is said to be hyper connected if every open set is dense.

Theorem 33. *If X is rgb-ultra connected and $f : X \to Y$ is an almost contra rgb-continuous surjection, then Y is hyper connected.*

Proof. Suppose that Y is not hyperconnected. Then, there exists an open set V such that V is not dense in Y. So, there exist non-empty regular open subsets $B_1 = int(cl(V))$ and $B_2 = Y - cl(V)$ in Y. Since f is almost contra rgb-continuous, $f^{-1}(B_1)$ and $f^{-1}(B_2)$ are disjoint rgb-closed. This is contrary to the rgb-ultra-connectedness of X. Therefore, Y is hyperconnected. □

6. Separation Axioms

Definition 34. A topological space X is said to be $rgb - T_1$ space if for any pair of distinct points x and y, there exist a rgb-open sets G and H such that $x \in G$, $y \notin G$ and $x \notin H$, $y \notin H$.

Theorem 35. *If $f : X \to Y$ is an almost contra rgb-continuous injection and Y is weakly Hausdorff, then X is $rgb - T_1$.*

Proof. Suppose Y is weakly Hausdorff. For any distinct points x and y in X, there exist V and W regular closed sets in Y such that $f(x) \in V$, $f(y) \notin V$

, $f(y) \in W$ and $f(x) \notin W$. Since f is almost contra rgb-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are rgb-open subsets of X such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $y \in f^{-1}(W)$ and $x \notin f^{-1}(W)$. This shows that X is $rgb - T_1$. □

Corollary 36. *If $f : X \to Y$ is a contra rgb-continuous injection and Y is weakly Hausdorff, then X is $rgb - T_1$.*

Definition 37. A topological space X is called Ultra Hausdorff space, if for every pair of distinct points x and y in X, there exist disjoint clopen sets U and V in X containing x and y, respectively.

Definition 38. A topological space X is said to be $rgb - T_2$ space if for any pair of distinct points x and y, there exist disjoint rgb-open sets G and H such that $x \in G$ and $y \in H$.

Theorem 39. *If $f : X \to Y$ is an almost contra rgb-continuous injective function from space X into a Ultra Hausdorff space Y, then X is $rgb - T_2$.*

Proof. Let x and y be any two distinct points in X. Since f is an injective $f(x) \neq f(y)$ and Y is Ultra Hausdorff space, there exist disjoint clopen sets U and V of Y containing $f(x)$ and $f(y)$ respectively. Then $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$, where $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint rgb-open sets in X. Therefore X is $rgb - T_2$. □

Definition 40. A topological space X is called Ultra normal space, if each pair of disjoint closed sets can be separated by disjoint clopen sets.

Definition 41. A topological space X is said to be rgb-normal if each pair of disjoint closed sets can be separated by disjoint rgb-open sets.

Theorem 42. *If $f : X \to Y$ is an almost contra rgb-continuous closed injection and Y is ultra normal, then X is rgb-normal.*

Proof. Let E and F be disjoint closed subsets of X. Since f is closed and injective $f(E)$ and $f(F)$ are disjoint closed sets in Y . Since Y is ultra normal there exists disjoint clopen sets U and V in Y such that $f(E) \subset U$ and $f(F) \subset V$. This implies $E \subset f^{-1}(U)$ and $F \subset f^{-1}(V)$. Since f is an almost contra rgb-continuous injection, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint rgb-open sets in X. This shows X is rgb-normal. □

Theorem 43. *If $f : X \to Y$ is an almost contra rgb-continuous and Y is semi-regular, then f is rgb-continuous.*

Proof. Let $x \in X$ and V be an open set of Y containing $f(x)$. By definition of semi-regularity of Y, there exists a regular open set G of Y such that $f(x) \in G \subset V$. Since f is almost contra rgb-continuous, there exists $U \in rgb-O(X, x)$ such that $f(U) \subset G$. Hence we have $f(U) \subset G \subset V$. This shows that f is rgb-continuous function. □

7. Compactness

Definition 44. A space X is said to be:

(1) rgb-compact if every rgb-open cover of X has a finite subcover.

(2) rgb-closed compact if every rgb-closed cover of X has a finite subcover.

(3) Nearly compact if every regular open cover of X has a finite subcover.

(4) Countably rgb-compact if every countable cover of X by rgb-open sets has a finite subcover.

(5) Countably rgb-closed compact if every countable cover of X by rgb-closed sets has a finite sub cover.

(6) Nearly countably compact if every countable cover of X by regular open sets has a finite sub cover.

(7) rgb-Lindelof if every rgb-open cover of X has a countable sub cover.

(8) rgb-Lindelof if every rgb-closed cover of X has a countable sub cover.

(9) Nearly Lindelof if every regular open cover of X has a countable sub cover.

(10) S-Lindelof if every cover of X by regular closed sets has a countable sub cover.

(11) Countably S-closed if every countable cover of X by regular closed sets has a finite sub-cover.

(12) S-closed if every regular closed cover of x has a finite sub cover.

Theorem 45. *Let $f : X \to Y$ be an almost contra rgb-continuous surjection. Then, the following properties hold:*

(1) If X is rgb-closed compact, then Y is nearly compact.

(2) If X is countably rgb-closed compact, then Y is nearly countably compact.

(3) If X is rgb-Lindelof, then Y is nearly Lindelof.

Proof. (1) Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is almost contra rgb-continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is rgb-closed cover of X. Since X is rgb-closed compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I\}$. Since f is surjective, $Y = \cup\{(V_\alpha) : \alpha \in I\}$ which is finite sub cover of Y, therefore Y is nearly compact.

(2) Let $\{V_\alpha : \alpha \in I\}$ be any countable regular open cover of Y . Since f is almost contra rgb-continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is countable rgb-closed cover of X. Since X is countably rgb-closed compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I\}$. Since f is surjective, $Y = \cup\{(V_\alpha) : \alpha \in I\}$ is finite subcover for Y . Hence Y is nearly countably compact.

(3) Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is almost contra rgb-continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is rgb-closed cover of X. Since X is rgb-Lindelof, there exists a countable subset I_0 of I such that $X = \{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{(V_\alpha) : \alpha \in I_0\}$ is finite sub cover for Y . Therefore, Y is nearly Lindelof.

□

Theorem 46. *Let $f : X \to Y$ be an almost contra rgb-continuous surjection. Then, the following properties hold:*

(1) If X is rgb-compact, then Y is S-closed.

(2) If X is countably rgb-closed, then Y is is countably S-closed.

(3) If X is rgb-Lindelof, then Y is S-Lindelof.

Proof. (1) Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Since f is almost contra rgb-continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is rgb-open cover of

X. Since X is rgb-compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite sub cover for Y. Therefore, Y is S-closed.

(2) Let $\{V_\alpha : \alpha \in I\}$ be any countable regular closed cover of Y. Since f is almost contra rgb-continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is countable rgb-open cover of X. Since X is countably rgb-compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite sub cover for Y. Hence, Y is countably S-closed.

(3) Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Since f is almost contra rgb-continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is rgb-open cover of X. Since X is rgb-Lindelof, there exists a countable sub-set I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite sub cover for Y. Hence, Y is S-Lindelof.

□

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