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THE SOLUTION OF FERMAT EQUATION IN THE RATIONAL POINTS OF THE UNITARY CIRCUMFERENCE

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Abstract: In this work we resolve the Fermat equation over rational points in the unitary circumference. For this we take a point (p_0, q_0) in the unitary circumference $x^2 + y^2 = 1$, where $p_0, q_0 \in \mathbb{Q}$ or $q_0 \in \mathbb{I}$ and $p_0 \in \mathbb{I}$. Then the straight line $y = q_0$ intersects the curve $x^d + y^d = 1$, in the point (p_1, q_0) we demonstrated that p_1 is an irrational number.

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1. Introduction

We know that Fermat's equation is $a^n + b^n = c^n$, this doesn't have different integer solution of the trivial for $a, b, c, n \in \mathbb{Z}^+$ and $n \geq 3$, it was solved for the English Mathematician Andrew Wile in 1996; I listened that this only was possible using a technique of extremely difficult for its comprehension. Motivated by these comments, I took the decision investigate, so I discovered a simple technique that can help solve the general case.

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The used technique, consist in the following, We took a point in the Cartesian plane whose coordinates are (p_0, q_0) in the unitary circumference, where $p_0, q_0 \in \mathbb{Q}$ or $q_0 \in \mathbb{Q}$ and $p_0 \in \mathbb{I}$, then we get $p_0^2 + q_0^2 = 1$, these points are known as

$$p_0 = \frac{2t_0}{1+t_0^2}, \quad q_0 = \frac{1-t_0^2}{1+t_0^2},$$

here note that $p_0, q_0 \in \mathbb{Q}^+$ if and only if $t_0 \in \mathbb{Q}^+, t_0 \in [0,1]$, if $q_0 \in \mathbb{Q}^+$ and $p_0 \in \mathbb{I}^+$ then $t_0^2 \in \mathbb{Q}^+$.

The straight line $y = q_0$, in the first quadrant intersects the curve $x^d + y^d = 1$ in the point (p_1, q_0), then we have the following equation $p_1^d + q_0^d = 1$.

From the last equation we demonstrate that p_1 never is a rational number. So it is important to note that (p_0, q_0) also can be written as:

$$q_0 = \frac{2t_1}{1+t_1^2}, \quad p_0 = \frac{1-t_1^2}{1+t_1^2}$$

where the relation between t_0, t_1 is $t_0 = \frac{1-t_1}{1+t_1}$.

Let us introduce the notations used in this work: $\mathbb{Z}^+$ represents the set of positive integer numbers, $\mathbb{Q}$ represents the set rational numbers, $\mathbb{I}$ represents the set of irrational numbers.

2. Fermat Had Reason

Theorem 1. *Let $a, b, c, d \in \mathbb{Z}^+$ the equation $a^d + b^d = c^d$ does not have integer solution different of the trivial.*

Proof. This theorem is equivalent to prove the following $p^d + q^d = 1$ it does not have rational solution para $p, q \in \mathbb{Q} \cap [0,1]$. So we take (p_0, q_0) a point in the unitary circumference. Then following cases happens:

I) $p_0, q_0 \in \mathbb{Q}$ and satisfies the equation $p_0^2 + q_0^2 = 1$, where

$$p_0 = \frac{2t_0}{1+t_0^2} = \frac{1-t_1^2}{1+t_1^2} \text{ and } q_0 = \frac{1-t_0^2}{1+t_0^2} = \frac{2t_1}{1+t_1^2} \tag{1}$$

Clearly $t_0, t_1 \in \mathbb{Q}$ and t_0, t_1 are related by the following equation

$$t_0 = \frac{1-t_1}{1+t_1} \tag{2}$$

Let $t_0 = \frac{m}{n}, t_1 = \frac{m_1}{n_1}$, where $m, n, m_1, n_1 \in \mathbb{Z}^+$ and m, n are relative primes.

The same way m_1, n_1 are relative primes and satisfies the relation $m < n$, $m_1 < n_1$.

From (2) we have

$$t_0 = \frac{m}{n} = \frac{n_1 - m_1}{n_1 + m_1} \tag{3}$$

The line $y = q_0$ intersects the curve $x^d + y^d = 1$ in the point (p_1, q_0), therefore this point satisfies the following :

$$p_1^d + q_0^d = 1 \tag{4}$$

From the relation (1) ,(3) and (4) we have:

$$p_1 = \frac{\sqrt[d]{(m^2+n^2)^d - (n^2-m^2)^d}}{n^2+m^2} = \frac{\sqrt[d]{(n_1^2+m_1^2)^d - (2n_1m_1)^d}}{n_1^2+m_1^2} \tag{5}$$

Supposing that $p_1 \in \mathbb{Q}$ from the relation (4) and (5) we have the equation

$$A^d + (2n_1m_1)^d = (n_1^2+m_1^2)^d \tag{6}$$

If $A = n_1^2 + m_1^2 - j, j = 1, 2, \ldots, n_1^2 + m_1^2 - 1$, we have

$$(n_1^2+m_1^2-j)^d + (2n_1m_1)^d = (n_1^2+m_1^2)^d$$

don't have a solution, for all $n_1, m_1, d \in \mathbb{N}$ and $\forall j$ odd number.

Therefore

$$(n_1^2+m_1^2-j)^d + (2n_1m_1)^d > (n_1^2+m_1^2)^d \tag{7}$$

or

$$(n_1^2+m_1^2-j)^d + (2n_1m_1)^d < (n_1^2+m_1^2)^d \tag{8}$$

Then We supposing that exists j_0 even number such that (6) have a solution

$$(n_1^2+m_1^2-j_0)^d + (2n_1m_1)^d = (n_1^2+m_1^2)^d \tag{9}$$

From the relation (9) and (7) we have

$$(n_1^2+m_1^2-j)^d > (n_1^2+m_1^2-j_0)^d,$$

if d is an odd number we have

$$-j > -j_0 \Rightarrow j_0 > j, \forall j \text{ odd number.}$$

Then $j_0 = n_1^2 + m_1^2 - 1$, if n_1 is an odd number and m_1 is an even number or viceversa. This relation and (9) is

$$1^d + (2n_1m_1)^d = (n_1^2 + m_1^2)^d$$

which don't have an integer solution.

From the relation (8) and (9), we have

$$(n_1^2 + m_1^2 - j)^d < (n_1^2 + m_1^2 - j_0)^d,$$

therefore, if d is an odd number

$$j_0 < j, \forall j \text{ odd number},$$

for $j = 1$, we have that j_0 doesn't exist.

When d is a even number the analysis is the same

II) If $q_0 \in \mathbb{Q}$ and $p_0 \in \mathbb{I}$. We have

$$p_0^d + (n - m)^d = (n + m)^d,$$

for $B = n + m - j, j = 1, 2, \ldots, n + m - 1$, we have

$$(n + m - j)^d + (n - m)^d = (n + m)^d, \tag{10}$$

for $j = n + m - k$ on (10), we have

$$k^d + (n - m)^d = (n + m)^d \tag{11}$$

If k is an odd number, then (11) don't have a solution for every $n, m \in \mathbb{N}$ and $\forall k$ odd number. by analogy with the item (I), We have that (11) only has a solution if

$$(n + m - 1)^d + (n - m)^d = (n + m)^d \tag{12}$$

where n is odd and m is even number.

□

Theorem 2. *For $d = 4$ the equation (12) doesn't solution*

Proof. Suppose that (12) have a solution

$$(n+m-1)^4 = (n+m)^4 - (n-m)^4$$

$$(n+m-1)^4 = 8nm(n^2+m^2); n \text{ is odd and } m \text{ is even} \tag{13}$$

We have $n^2+m^2 = c^2, c \in \mathbb{N}$ then

$$n = u^2 - v^2, m = 2uv, c = u^2 + v^2 \tag{14}$$

Therefore u and v have a square root exact. Furthermore n has a square root exact and $n = (u-v)(u+v)$ then $u = A^2, v = B^2; A, B \in \mathbb{N}$. Hence

$$\begin{cases} A^2 - B^2 = \tau^2, \tau \in \mathbb{N} \\ A^2 + B^2 = \lambda^2, \lambda \in \mathbb{N} \end{cases} \tag{15}$$

Then (15) don't have an integer solution □

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