International Journal of Pure and Applied Mathematics
Volume 97 No. 2 2014, 195-200
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v97i2.7
ijpam.eu

SOME APPLICATION OF PRIME IDEALS OF AG-RING

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Abstract: In this paper we define c-prime, 3-prime and weakly prime ideal of AG-ring which we will study relation of c-prime, 3-prime, weakly prime ideal and prime ideal.

Key Words: c-prime, 3-prime, weakly prime ideal

1. Introduction

M.A. Kazim and MD. Naseeruddin [2, Proposition 2.1] asserted that, in every LA-semigroups G a *medial law* hold

$$(a \cdot b) \cdot (c \cdot d) = (a \cdot c) \cdot (b \cdot d), \quad \forall\, a, b, c, d \in G.$$

Q. Mushtaq and M. Khan [4, p.322] asserted that, in every LA-semigroups G with left identity

$$(a \cdot b) \cdot (c \cdot d) = (d \cdot c) \cdot (b \cdot a), \quad \forall\, a, b, c, d \in G.$$

Further M. Khan, Faisal, and V. Amjid [3], asserted that, if a LA-semigroup G with left identity the following law holds

$$a \cdot (b \cdot c) = b \cdot (a \cdot c), \quad \forall\, a, b, c \in G.$$

Received: June 19, 2014

M. Sarwar (Kamran) [5, p.112] defined LA-group as the following; a groupoid G is called a left almost group, abbreviated as LA-group, if (i) there exists $e \in G$ such that $ea = a$ for all $a \in G$, (ii) for every $a \in G$ there exists $a' \in G$ such that, $a'a = e$, (iii) $(ab)c = (cb)a$ for every $a, b, c \in G$.

S.M. Yusuf in [6, p.211] introduces the concept of a left almost ring (LA-ring). That is, a non-empty set R with two binary operations "+" and "$\cdot$" is called a left almost ring, if $\langle R, + \rangle$ is an LA-group, $\langle R, \cdot \rangle$ is an LA-semigroup and distributive laws of "$\cdot$" over "+" holds. T. Shah and I. Rehman [6, p.211] asserted that a commutative ring $\langle R, +, \cdot \rangle$, we can always obtain an LA-ring $\langle R, \oplus, \cdot \rangle$ by defining, for $a, b, c \in R$, $a \oplus b = b - a$ and $a \cdot b$ is same as in the ring. We can not assume the addition to be commutative in an LA-ring. An LA-ring $\langle R, +, \cdot \rangle$ is said to be LA-integral domain if $a \cdot b = 0$, $a, b \in R$, then $a = 0$ or $b = 0$. Let $\langle R, +, \cdot \rangle$ be an LA-ring and S be a non-empty subset of R and S is itself and LA-ring under the binary operation induced by R, the S is called an *LA-subring* of R, then S is called an LA-subring of $\langle R, +, \cdot \rangle$. If S is an LA-subring of an LA-ring $\langle R, +, \cdot \rangle$, then S is called a left ideal of R if $RS \subseteq S$. Right and two-sided ideals are defined in the usual manner. An ideal I of R is called prime if $AB \in I$ implies $A \in I$ or $B \in I$.

In this note we prefer to called left almost rings (LA-rings) as Abel-Grassmann rings (abbreviated as an"AG-rings").

In [1] An ideal I of N is called *c-prime* if $a, b \in N$ and $ab \in I$ implies $a \in I$ or $b \in I$. R is called c-prime nearring if $\{0\}$ is a c-prime ideal of R.

An ideal I of R is called *3-prime* if $a, b \in N$ and $anb \in I$ for all $n \in N$ implies $a \in I$ or $b \in I$.

The notions of c-ideal, 3-prime ideal and prime ideal coincide in rings.

In [7] A proper ideal I of an ring R to be *weakly prime* if $0 \neq AB \subseteq I$ implies either $A \subseteq I$ or $B \subseteq I$ for any ideals A, B of R.

The following implications are well known in rings:

(1) c-prime ideal $\Rightarrow$ 3-prime ideal $\Rightarrow$ prime ideal;

(2) prime ideal $\Rightarrow$ weakly prime ideal.

2. Main Results

In this paper, we define c-prime and 3-prime of AG-ring.

Definition 2.1. An ideal I of an AG-ring R is called *c-prime* if $a, b \in R$ and $ab \in I$ implies $a \in I$ or $b \in I$.

Definition 2.2. An ideal I of an AG-ring R is called *3-prime* if $a, b \in R$ and $arb \in I$ for all $r \in R$ implies $a \in I$ or $b \in I$.

The following lemmas and theorem we will study relation of c-prime, 3-prime and prime ideals.

Lemma 2.3. *Every c-prime ideal is a 3-prime ideal*

Proof. Suppose that I is a c-prime ideal of AG-ring R, let $a, b \in R$ and $arb \in I$ for all $r \in R$. Since I is a c-prime ideal we have $a \in I$ or $b \in I$. Then I is a 3-prime ideal of R. □

Lemma 2.4. *Every 3-prime ideal is a prime ideal*

Proof. Suppose that I is a 3-prime ideal of AG-ring R, let $a, b \in I$ and $ab \in I$. Since I is a 3-prime ideal we have $a \in I$ or $b \in I$. Then I is a prime ideal of R. □

Lemma 2.5. *Every c-prime ideal is a prime ideal*

Proof. Suppose that I is a c-prime ideal of AG-ring R, let $a, b \in I$ and $ab \in I$. Since I is a c-prime ideal we have $a \in I$ or $b \in I$. Then I is a prime ideal of R. □

In [6, p.221]. studied if I is a prime ideal in AG-ring R if and only if R/I is an AG-integral domain. The following theorems are application by lemmas 2.4 and 2.5

Theorem 2.6. *Let R be an AG-ring. Then I is a 3-prime ideal in R if and only if R/I is an AG-integral domain.*

Proof. ($\Rightarrow$) Let I is a 3-prime ideal in R. By Lemma 2.4 then I is a prime ideal. Thus R/I is an AG-integral domain.

($\Leftarrow$) Assume that R/I is an AG-integral domain with $arb \in I$ for all $r \in R$. Then $I + arb = I$ so $(I + a)r(I + b) = I$. Since R/I is an AG-integral domain we have $I + a = I$ or $I + b = I$. Then $a \in I$ or $b \in I$. Thus P is a 3-prime ideal of R. □

Theorem 2.7. *Let R be an AG-ring. Then I is a c-prime ideal in R if and only if R/I is an AG-integral domain.*

Proof. ($\Rightarrow$) Let I is a c-prime ideal in R. By Lemma 2.5 then I is a prime ideal. Thus R/I is an AG-integral domain.

($\Leftarrow$) Assume that R/I is an AG-integral domain with $ab \in I$ for all $a, b \in R$. Then $I + ab = I$ so $(I + a)(I + b) = I$. Since R/I is an AG-integral domain we have $I + a = I$ or $I + b = I$. Then $a \in I$ or $b \in I$. Thus P is a c-prime ideal of R. □

The next following we define of weakly prime ideal.

Definition 2.8. A proper ideal I of an AG-ring R to be *weakly prime* if $0 \neq AB \subseteq I$ implies either $A \subseteq I$ or $B \subseteq I$ for any ideals A, B of R.

Clearly every prime ideal is weakly prime and $\{0\}$ is always weakly prime ideal of R. The following theorem we will study properties

Theorem 2.9. *If I is weakly prime but not prime, then $I^2 = 0$.*

Proof. Since I is weakly prime (but not prime), there exist ideals $A \nsubseteq I$ and $B \nsubseteq P$ but $0 = AB \subseteq I$. Since $I \subseteq A + I$ and $I \subseteq B + I$. But if $I^2 \neq 0$,. By distributive laws "$\cdot$" over "$+$" of AG-ring we have

$$\begin{aligned} 0 \neq I^2 = II &\subseteq (A + I)(B + I) \\ &= [(A + I)B] + [(A + I)P] \\ &= AB + IB + AI + II \\ &\subseteq I \end{aligned}$$

which implies $(A + I) \subseteq I$ and $(B + I) \subseteq I$, since I is a weakly prime; that is $A \subseteq I$ or $B \subseteq I$, a contradiction. Hence, $I^2 = 0$. □

If $R^2 = 0$, then it is evident that every ideal of R is weakly prime. In particular, if an ideal I of an AG-ring R is weakly prime but not a prime ideal, then every ideal of I as an AG-ring is weakly prime by Theorem 2.9.

Theorem 2.10. *Every ideal of an AG-ring R is weakly prime if and only if for any ideals A and B of R, $AB = A$, $AB = B$, or $AB = 0$.*

Proof. Suppose that every ideal of R is weakly prime. Let A, B be ideals of R. Then AB is a left ideals of R, if $AB \neq R$, then by hypothesis, AB is weakly prime. We are consider two situation, that is $AB = 0$. or $AB \neq 0$. If $0 \neq AB \subseteq AB$, then by Definition 2.8, we have $A \subseteq AB$ or $B \subseteq AB$. Since A and B are ideals of R, we have $AB \subseteq A$ and $AB \subseteq B$. Therefore $A = AB$ or $B = AB$. If $AB = R$, then we have $A = B = R$ whence $R^2 = R$.

Conversely, let K be any proper ideal of R and suppose that $0 \neq AB \subseteq K$ for ideals A and B of R. Then we have either $A = AB \subseteq K$ or $B = AB \subseteq K$. □

Corollary 2.11. *Let R be an-AG-ring and every ideal of R is weakly prime. Then for any ideal I of R, either $I^2 = I$ or $I^2 = 0$.*

The following theorem we will study relation of c-prime, 3-prime, weakly prime ideals.

Theorem 2.12. *Every c-prime ideal is a weakly prime ideal*

Proof. Suppose that I is a c-prime ideal of AG-ring R. By Lemma 2.3 we have I is a prime ideal. Since every prime ideal is weakly prime ideal we have I is a weakly prime ideal of R. □

Theorem 2.13. *Every 3-prime ideal is a weakly prime ideal*

Proof. Suppose that I is a 3-prime ideal of AG-ring R. By Lemma 2.4 we have I is a prime ideal. Since every prime ideal is weakly prime ideal we have I is a weakly prime ideal of R. □

Acknowledgments

The author is very grateful to the anonymous referee for stimulating comments and improving presentation of the paper.

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