

**EXISTENCE OF POSITIVE ALMOST PERIODIC
SOLUTIONS FOR A CLASS OF IMPULSIVE
ELAY HARVESTING NICHOLSON'S
BLOWFLIES MODEL**

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Abstract: In this paper, a class of nonlinear impulsive delay harvesting Nicholson's blowflies model is considered. By employing the fixed point theorem of contraction mapping principle, some sufficient conditions for the existence of almost periodic solutions are obtained.

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1. Introduction

Recently, many works have appeared about impulsive equations which have been used extensively in modeling many practical problems arising in the fields of sciences and technology. A lot of interesting results on impulsive effect such as stability, oscillations, asymptotic behavior have been reported, e.g., see [1-

12], and the references cited therein. Some authors have studied the periodicity of impulsive equations [13-16]. Also there are many works devoted to the investigation of almost periodicity for impulsive delay differential equations [17-22]. It is known that the periodically and almost periodically varying environment play important roles in many ecological and biological systems. Compare with periodic effects, investigation of almost periodicity than periodicity may more possibly draw close to the reality in ecology and biological systems especially.

In [21], Li and Fan have considered the following impulsive delay Nicholson's blowflies model:

$$\begin{cases} x'(t) = -\delta(t)x(t) + p(t)x(t - m\omega)e^{-\alpha(t)x(t-m\omega)}, t > 0, t \neq \tau_k; \\ \Delta x(\tau_k) = b_k x(\tau_k), k = 1, 2, \dots \end{cases} \quad (1)$$

In the nondelay case, system (1) has been shown that there exists a unique positive periodic solution which is globally asymptotically stable. In the delay case, the authors have presented sufficient conditions for the global attractivity of the solutions. The results imply that under the appropriate linear periodic impulsive perturbations, the impulsive delay equation (1) preserves the original periodic property of the nonimpulsive delay equation. In [22], Long has discussed the positive almost periodic solution for a class of Nicholson's blowflies model with linear harvesting term as follows:

$$x'(t) = -\alpha(t)x(t) + \sum_{j=1}^m \beta_j(t)x(t - \tau_j(t))e^{-\gamma_j(t)x(t-\tau_j(t))} - H(t)x(t - \sigma(t)) \quad (2)$$

By using the fixed point theorem and the Lyapunov functional method, the author has established some criteria to guarantee that the solutions of this model converge locally exponentially to a positive almost periodic solution.

Motivated by the above discussions, in this paper, we consider the existence of positive almost periodic solutions for the following nonlinear impulsive delay Nicholson's blowflies model with linear harvesting term:

$$\begin{cases} x'(t) = -a(t)x(t) + \sum_{i=1}^m b_i(t)x(t - \delta_i(t))e^{-\gamma_i(t)x(t-\delta_i(t))} \\ \hspace{15em} -h(t)x(t - \sigma(t)), \quad t \neq \tau_k; \\ \Delta x(\tau_k) = I_k(x(\tau_k)), t = \tau_k, k \in Z. \end{cases} \quad (3)$$

By employing the fixed point theorem of contraction mapping principle which is different from [21] and [22], some sufficient conditions for the existence of almost periodic solutions are provided.

2. Preliminaries

Definition 2.1. (see [23]) The set of sequences $\{\tau_k^l\} = \{\tau_{k+l} - \tau_k\}, k, l \in Z$ is said to be uniformly almost periodic if for any $\varepsilon > 0$, there exists a relatively dense set in R , i.e., for any $\varepsilon > 0, k \in Z$, it is possible to find a $q \in Z$ such that $|\tau_{k+q} - \tau_k| < \varepsilon$.

By $B = \{\tau_k | \tau_k \in R, \tau_k < \tau_{k+1}, k \in Z, \lim_{k \rightarrow \pm\infty} \tau_k = \pm\infty\}$, we denote the set of all sequences unbounded and strictly increasing with distance $\rho(\tau_k^1, \tau_k^2)$, the set $PC(R, R) = \{\varphi | R \rightarrow R \text{ is continuously differentiable everywhere except at the points } \tau_k, \tau_k \in B \text{ at which } \varphi(\tau_k^-) \text{ and } \varphi(\tau_k^+) \text{ exist, and } \varphi(\tau_k^-) = \varphi(\tau_k^+)\}$.

Definition 2.2. (see [23]) The function $\varphi \in PC(R, R)$ is said to be almost periodic, if the following conditions hold:

(C1) The set of sequences $\{\tau_k^l\}, k, l \in Z$ is uniformly almost periodic;

(C2) For any $\varepsilon > 0$, there exists a real number $\delta > 0$, such that if the points τ_1 and τ_2 belong to same one interval of continuity of $\varphi(t)$ and satisfy the inequality $|\varphi(\tau_1) - \varphi(\tau_2)| < \varepsilon$ if $|\tau_1 - \tau_2| < \delta$.

(C3) For and $\varepsilon > 0$, there exists a relatively dense set M such that if $\tau \in M$ then $|\varphi(t+\tau) - \varphi(t)| < \varepsilon$ for all $t \in R$ satisfying the condition $|t - \tau_k| > \varepsilon, k \in Z$. The elements of M are called ε -almost periods.

For system (3), we introduce the following conditions:

(H1) The functions $a(t), h(t), \sigma(t), b_i(t), \gamma_i(t), \delta_i(t) (i = 1, 2, \dots, n)$ are positive almost periodic in the sense of Bohr;

(H2) The set of sequence $\{\tau_k^l\} = \{\tau_{k+l} - \tau_k\}, k, l \in Z$ is uniformly almost periodic and there exists $l > 0$ such that $\inf_{k \in Z} \tau_k^l = \theta > 0$.

(H3) $I_k \in PC(R, R)$ and I_k is uniformly almost periodic satisfying $|I_k(u) - I_k(v)| < L(t)|u - v|$.

For convenience, let $\phi^+ = \sup_{t \in R} \phi(t), \phi^- = \inf_{t \in R} \phi(t)$ for a given bounded continuous function $\phi(t)$ defined on R . According to the condition (H1), it will be assumed that $a^-, h^-, \sigma^-, b_i^-, \gamma^-, \delta^- > 0 (i = 1, 2, \dots, n)$.

Together with the system (3) we consider the following linear system

$$\begin{cases} x'(t) = -a(t)x(t), t \neq \tau_k, \tau_{k-1} < t < \tau_k; \\ \Delta x(\tau_k) = 0, t = \tau_k, k \in Z. \end{cases} \tag{4}$$

The solutions of (4) are written in the form

$$x(t; t_0, x_0) = W(t, t_0)x_0, \quad t_0, x_0 \in R. \tag{5}$$

where $W(t, s) = \exp(-\int_s^t a(u)du)$, $t \geq s, t, s \in R$.

Lemma 2.1. (see [23]) *If $a(t)$ is positive almost periodic function, then:*

1. *For the Cauchy matrix $W(t, s)$ of system (4) there exists a $\lambda > 0$, such that*

$$|W(t, s)| \leq \exp(-\lambda(t - s)), \quad t \geq s, t, s \in R. \tag{6}$$

2. *For any $\varepsilon > 0, t \geq s, t, s \in R, |t - \tau_k| > \varepsilon, |s - \tau_k| > \varepsilon, k \in Z$ there exists a $\Gamma > 0$ such that*

$$|W(t + \tau, s + \tau) - W(t, s)| \leq \varepsilon\Gamma \exp(-\frac{\lambda}{2}(t - s)). \tag{7}$$

3. Main Result

Theorem 3.1. *If conditions (H1)-(H3) hold. And furthermore assume that:*

(H4) *For any $k \in Z, I_k(0) = 0, 0 < \lim_{t \rightarrow +\infty} \sum_{\tau_k < t} L(\tau_k) \leq g$.*

(H5) $0 < N = \frac{1}{\lambda}(\sum_{i=1}^n b_i^+ + \sum_{i=1}^n \gamma_i^+ \rho + h^+) + \frac{g}{1 - \exp(-\lambda\theta)} < 1$, where $0 < \theta = \inf_{k \in Z} \{\tau_{K+1} - \tau_k\}$. *Then there exists a unique almost periodic solution of system (3).*

Proof. We denote $X = \{x(t)|x(t) \in PC(R, R), x(t)$ is an almost periodic function and the norm $\|x\| = \sup_{t \in R} |x(t)|$, so X is a Banach space. We define in X an operator Φ as follows

$$\Phi x(t) = \int_{-\infty}^t W(t, s) \{ \sum_{i=1}^n b_i(s)x(s - \delta_i(s))$$

$$\times \exp[-\gamma_i(s)x(s - \delta_i(s))] - h(s)x(s - \sigma(s))\}ds + \sum_{\tau_k < t} W(t, \tau_k)I_k(x(\tau_k)). \quad (8)$$

Let $E = \{x(t)|x(t) \in X, \|x\| \leq \rho\}$, then E is a nonempty closed subset in X . Firstly, we will prove that Φ is a self-mapping from E to E . Since we only consider the existence of positive almost periodic solutions, so for arbitrary $0 < x \in E$, from condition (H1)-(H5), it follows that

$$\begin{aligned} \|\Phi x\| &= \sup_{t \in R} \left| \int_{-\infty}^t W(t, s) \left\{ \sum_{i=1}^n b_i(s)x(s - \delta_i(s)) \exp[-\gamma_i(s)x(s - \delta_i(s))] \right. \right. \\ &\quad \left. \left. - h(s)x(s - \sigma(s)) \right\} ds + \sum_{\tau_k < t} W(t, \tau_k)I_k(x(\tau_k)) \right| \\ &\leq \sup_{t \in R} \left\{ \int_{-\infty}^t \exp(-\lambda(t - s)) \left\{ \sum_{i=1}^n b_i(s)x(s - \delta_i(s)) \exp[-\gamma_i(s)x(s - \delta_i(s))] \right. \right. \\ &\quad \left. \left. + h(s)x(s - \sigma(s)) \right\} ds + \sum_{\tau_k < t} \exp(-\lambda(t - \tau_k)) |I_k(x(\tau_k)) - I_k(0)| \right\} \\ &\leq \frac{1}{\lambda} \left(\sum_{i=1}^n b_i^+ + h^+ \right) \|x\| + \frac{g}{1 - \exp(-\lambda\theta)} \|x\| \\ &= \left[\frac{1}{\lambda} \left(\sum_{i=1}^n b_i^+ + h^+ \right) + \frac{g}{1 - \exp(-\lambda\theta)} \right] \|x\| < N \|x\| < \rho \end{aligned} \quad (9)$$

This means that $\Phi x(t)$ belongs to E . On the other hand, using the triangle inequality, we have

$$\begin{aligned} &\|\Phi x(t + \tau) - \Phi x(t)\| \\ &= \sup_{t \in R} \left| \int_{-\infty}^{t+\tau} W(t + \tau, s) \left[\sum_{i=1}^n b_i(s)x(s - \delta_i(s)) \exp(-\gamma_i(s)x(s - \delta_i(s))) \right. \right. \\ &\quad \left. \left. - h(s)x(s - \sigma(s)) \right] ds + \sum_{\tau_k < t+\tau} W(t + \tau, \tau_k)I_k(x(\tau_k)) \right. \\ &\quad \left. - \int_{-\infty}^t W(t, s) \left[\sum_{i=1}^n b_i(s)x(s - \delta_i(s)) \exp(-\gamma_i(s)x(s - \delta_i(s))) \right. \right. \\ &\quad \left. \left. - h(s)x(s - \sigma(s)) \right] ds - \sum_{\tau_k < t} W(t, \tau_k)I_k(x(\tau_k)) \right| \\ &= \sup_{t \in R} \left| \int_{-\infty}^t W(t + \tau, s + \tau) \left[\sum_{i=1}^n b_i(s + \tau)x(s + \tau - \delta_i(s + \tau)) \right. \right. \end{aligned}$$

$$\begin{aligned}
& \exp(-\gamma_i(s+\tau)x(s+\tau-\delta_i(s+\tau))) - h(s+\tau)x(s+\tau-\sigma(s+\tau))]ds \\
& + \sum_{\tau_k < t+\tau} W(t+\tau, \tau_k) I_k(x(\tau_k)) \\
& - \int_{-\infty}^t W(t, s) \left[\sum_{i=1}^n b_i(s)x(s-\delta_i(s)) \exp(-\gamma_i(s)x(s-\delta_i(s))) \right. \\
& \left. - h(s)x(s-\sigma(s)) \right] ds - \sum_{\tau_k < t} W(t, \tau_k) I_k(x(\tau_k)) | \\
\leq & \sup_{t \in R} \left\{ \int_{-\infty}^t |W(t+\tau, s+\tau) - W(t, s)| \sum_{i=1}^n b_i(s+\tau)x(s+\tau-\delta_i(s+\tau)) \cdot \right. \\
& \exp(-\gamma_i(s+\tau)x(s+\tau-\delta_i(s+\tau))) ds \\
& + \int_{-\infty}^t W(t, s) \sum_{i=1}^n |b_i(s+\tau) - b_i(s)| x(s+\tau-\delta_i(s+\tau)) \cdot \\
& \exp(-\gamma_i(s+\tau)x(s+\tau-\delta_i(s+\tau))) ds \\
& + \int_{-\infty}^t W(t, s) \sum_{i=1}^n b_i(s) |x(s+\tau-\delta_i(s+\tau)) - x(s-\delta_i(s))| \cdot \\
& \exp(-\gamma_i(s+\tau)x(s+\tau-\delta_i(s+\tau))) ds \\
& + \int_{-\infty}^t W(t, s) \sum_{i=1}^n b_i(s)x(s-\delta_i(s)) \cdot \\
& \left. | \exp(-\gamma_i(s+\tau)x(s+\tau-\delta_i(s+\tau))) - \exp(-\gamma_i(s)x(s+\tau-\delta_i(s+\tau))) | ds \right. \\
& + \int_{-\infty}^t W(t, s) \sum_{i=1}^n b_i(s)x(s-\delta_i(s)) \cdot \\
& \left. | \exp(-\gamma_i(s)x(s+\tau-\delta_i(s+\tau))) - \exp(-\gamma_i(s)x(s-\delta_i(s))) | ds \right. \\
& + \int_{-\infty}^t |W(t+\tau, s+\tau) - W(t, s)| h(s+\tau)x(s+\tau-\sigma(s+\tau)) ds \\
& + \int_{-\infty}^t W(t, s) |h(s+\tau) - h(s)| x(s+\tau-\sigma(s+\tau)) ds \\
& + \left. \int_{-\infty}^t W(t, s) h(s) |x(s+\tau-\sigma(s+\tau)) - x(s-\sigma(s))| ds \right\} \\
& + \sup_{t \in R} \sum_{\tau_k < t} |W(t+\tau, \tau_{k+q}) - W(t, \tau_k)| I_{k+q}(x(\tau_{k+q})) \\
& + \sup_{t \in R} \sum_{\tau_k < t} W(t, \tau_k) |I_{k+q}(x(\tau_{k+q})) - I_k(x(\tau_k))| \tag{10}
\end{aligned}$$

Noting that

$$\begin{aligned} & \exp(-\gamma_i(s + \tau)x(s + \tau - \delta_i(s + \tau))) - \exp(-\gamma_i(s)x(s + \tau - \delta_i(s + \tau))) \\ &= \exp(-\gamma_i(\xi)x(\xi - \delta_i(\xi))[\gamma_i(s + \tau)x(s + \tau - \delta_i(s + \tau)) - \gamma_i(s)x(s + \tau - \delta_i(s + \tau))], \\ & \hspace{15em} \xi \in (s, s + \tau)]. \end{aligned}$$

Therefore, from (10) we have

$$\begin{aligned} & \|\Phi x(t + \tau) - \Phi x(t)\| \\ & \leq \sup_{t \in R} \left\{ \int_{-\infty}^t \varepsilon \Gamma \exp\left(-\frac{\lambda}{2}(t - s)\right) \sum_{i=1}^n b_i^+ \rho ds + \int_{-\infty}^t \exp(-\lambda(t - s)) n \varepsilon \rho ds \right. \\ & \quad + \int_{-\infty}^t \exp(-\lambda(t - s)) \sum_{i=1}^n b_i^+ \varepsilon ds + \int_{-\infty}^t \exp(-\lambda(t - s)) \sum_{i=1}^n b_i^+ \rho^2 \varepsilon ds \\ & \quad + \int_{-\infty}^t \exp(-\lambda(t - s)) \sum_{i=1}^n b_i^+ \rho \gamma_i^+ \varepsilon ds + \int_{-\infty}^t \varepsilon \Gamma \exp\left(-\frac{\lambda}{2}(t - s)\right) h^+ \rho ds \\ & \quad + \int_{-\infty}^t \exp(-\lambda(t - s)) \varepsilon \rho ds + \int_{-\infty}^t \exp(-\lambda(t - s)) h^+ \varepsilon ds \left. \right\} \\ & \quad + \sup_{t \in R} \sum_{\tau_k < t} \varepsilon \Gamma \exp\left(-\frac{\lambda}{2}(t - \tau_k)\right) |I_{k+q}(x(\tau_{k+q})) - I_{k+q}(0)| \\ & \quad + \sup_{t \in R} \sum_{\tau_k < t} \exp(-\lambda(t - \tau_k)) [|I_{k+q}(x(\tau_{k+q})) - I_k(x(\tau_{k+q}))| \\ & \quad + |I_k(x(\tau_{k+q})) - I_k(x(\tau_k))|] \end{aligned} \tag{11}$$

Noting that

$$\sup_{t \in R} \sum_{\tau_k < t} \varepsilon \Gamma \exp\left(-\frac{\lambda}{2}(t - \tau_k)\right) |I_{k+q}(x(\tau_{k+q})) - I_{k+q}(0)| \leq \frac{\varepsilon \Gamma \rho}{1 - \exp(-\frac{\lambda \theta}{2})},$$

and

$$\begin{aligned} & \sup_{t \in R} \sum_{\tau_k < t} \exp(-\lambda(t - \tau_k)) [|I_{k+q}(x(\tau_{k+q})) - I_k(x(\tau_{k+q}))| + |I_k(x(\tau_{k+q})) - I_k(x(\tau_k))|] \\ & \hspace{15em} \leq \frac{\varepsilon(1 + gL^+)}{1 - \exp(\lambda \theta)}. \end{aligned}$$

Thus, we get

$$\|\Phi x(t + \tau) - \Phi x(t)\|$$

$$\begin{aligned}
 &\leq \frac{\varepsilon}{\lambda} [2\Gamma\rho \sum_{i=1}^n b_i^+ + n\rho + \sum_{i=1}^n b_i^+ + \rho^2 \sum_{i=1}^n b_i^+ + \rho \sum_{i=1}^n b_i^+ \gamma_i^+ + 2\Gamma h^+ \rho + \rho + h^+] \\
 &\quad + \frac{\varepsilon\Gamma\rho}{1 - \exp(-\frac{\lambda\theta}{2})} + \frac{\varepsilon(1 + gL^+)}{1 - \exp(\lambda\theta)} \\
 &= \varepsilon K_1
 \end{aligned} \tag{12}$$

where

$$\begin{aligned}
 K_1 = \frac{1}{\lambda} [2\Gamma\rho \sum_{i=1}^n b_i^+ + n\rho + \sum_{i=1}^n b_i^+ + \rho^2 \sum_{i=1}^n b_i^+ + \rho \sum_{i=1}^n b_i^+ \gamma_i^+ + 2\Gamma h^+ \rho + \rho + h^+] \\
 + \frac{\Gamma\rho}{1 - \exp(-\frac{\lambda\theta}{2})} + \frac{1 + gL^+}{1 - \exp(\lambda\theta)}
 \end{aligned}$$

is a positive bounded constant. This means that $\Phi \in E$.

Finally we prove that Φ is a contracting operator in E . Let $0 < x, y \in E$, we get

$$\begin{aligned}
 &\|\Phi x(t) - \Phi y(t)\| \\
 &= \sup_{t \in R} | \int_{-\infty}^t W(t, s) [\sum_{i=1}^n b_i(s)x(s - \delta_i(s)) \exp(-\gamma_i(s)x(s - \delta_i(s))) - h(s)x(s - \sigma(s))] ds \\
 &\quad - \int_{-\infty}^t W(t, s) [\sum_{i=1}^n b_i(s)y(s - \delta_i(s)) \exp(-\gamma_i(s)y(s - \delta_i(s))) - h(s)y(s - \sigma(s))] ds \\
 &\quad + \sum_{\tau_k < t} W(t, \tau_k) I_k(x(\tau_k)) - \sum_{\tau_k < t} W(t, \tau_k) I_k(y(\tau_k)) | \\
 &\leq \sup_{t \in R} | \int_{-\infty}^t \exp(-\lambda(t - s)) \sum_{i=1}^n b_i(s) [x(s - \delta_i(s)) \exp(-\gamma_i(s)x(s - \delta_i(s))) \\
 &\quad - y(s - \delta_i(s)) \exp(-\gamma_i(s)y(s - \delta_i(s)))] + h(s)y(s - \sigma(s)) - h(s)x(s - \sigma(s)) | ds \\
 &\quad + \sum_{\tau_k < t} \exp(-\lambda(t - \tau_k)) L(\tau_k) |x(\tau_k) - y(\tau_k)| \\
 &\leq \sup_{t \in R} | \int_{-\infty}^t \exp(-\lambda(t - s)) \sum_{i=1}^n b_i(s) [|x(s - \delta_i(s)) - y(s - \delta_i(s))| \exp(-\gamma_i(s)x(s - \delta_i(s))) \\
 &\quad + y(s - \delta_i(s)) | \exp(-\gamma_i(s)x(s - \delta_i(s))) - \exp(-\gamma_i(s)y(s - \delta_i(s))) |] \\
 &\quad + h(s)y(s - \sigma(s)) - h(s)x(s - \sigma(s)) | ds \\
 &\quad + \sum_{\tau_k < t} \exp(-\lambda(t - \tau_k)) L(\tau_k) |x(\tau_k) - y(\tau_k)| \\
 &\leq \frac{1}{\lambda} [\sum_{i=1}^n b_i^+ + \sum_{i=1}^n \gamma_i^+ \rho + h^+] \|x - y\| + \frac{g}{1 - \exp(-\lambda\theta)} \|x - y\| \\
 &= \frac{1}{\lambda} [\sum_{i=1}^n b_i^+ + \sum_{i=1}^n \gamma_i^+ \rho + h^+ + \frac{g}{1 - \exp(-\lambda\theta)}] \|x - y\| \\
 &= N \|x - y\|
 \end{aligned} \tag{13}$$

From condition (H5), since $N < 1$, it follows that Φ is a contracting operator in E . According to the contraction mapping principle, there exists a unique positive almost periodic solution of system (3).

Example. We give the following illustrate example to demonstrate the results obtained in previous section.

$$\left\{ \begin{array}{l} x'(t) = -(96 + \cos t)x(t) + (4 + \sin \sqrt{2}t)x(t - \exp(1 + \frac{1}{4} \sin t)) \\ \exp[-(2 + \frac{1}{2} \sin t) \cdot \\ x(t - \exp(1 + \frac{1}{4} \sin t))] + (2 - \sin \sqrt{3}t)x(t - \exp(1 - \frac{1}{2} \cos \sqrt{3}t)) \\ \exp[-(2 - \frac{1}{2} \sin t) \cdot \\ x(t - \exp(1 + \frac{1}{2} \cos \sqrt{3}t))] - \frac{1 - \frac{1}{4} \cos \sqrt{2}t}{200} \\ x(t - \exp(1 + \frac{1}{4} \sin t)), t \neq \tau_k; \\ \Delta x(\tau_k) = \frac{k^2 + 1}{40(k^2 + 2)} \sin x(\tau_k), \quad t = \tau_k, k \in Z. \end{array} \right. \quad (14)$$

Obviously

$$a^+ = 97, a^- = 95, b_1^+ = 5, b_1^- = 3, b_2^+ = 3, b_2^- = 1,$$

$$\gamma_1^+ = \gamma_2^+ = \frac{5}{2}, \gamma_1^- = \gamma_2^- = \frac{3}{2}, h^+ = \frac{5}{800}, h^- = \frac{3}{800}.$$

Therefore, $\lambda = 95$. Noting that

$$|\sin x(\tau_k) - \sin y(\tau_k)| \leq |x(\tau_k) - y(\tau_k)|,$$

so $g = \frac{1}{40}$. If we set sequences $\{\tau_k\}$ satisfying $\theta = 1$ and $\rho = 10$, then

$$\begin{aligned} N &= \frac{1}{\lambda} \left(\sum_{i=1}^2 b_i^+ + \sum_{i=1}^2 \gamma_i^+ \rho + h^+ \right) + \frac{g}{1 - \exp(-\lambda\theta)} \\ &= \frac{1}{95} \left(8 + 50 + \frac{5}{800} \right) + \frac{1}{40(1 - \exp(-95))} < 1 \end{aligned}$$

holds. Thus system (14) exists a unique almost periodic solution $x(t)$ satisfying that $\|x\| \leq 10$.

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