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CORDIAL LABELING FOR CYCLE OF COMPLETE BIPARTITE GRAPHS AND CYCLE OF WHEELS

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Abstract: In this paper we have obtained cordial labeling for cycle of complete bipartite graphs and cycle of wheels.

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1. Introduction

The concept of cordial labeling was introduced by Cahit [2] in 1987 as a weaker version of graceful and harmonious labelings. Many researchers have studied cordiality of graphs. Ho et al. [3] proved that unicyclic graph is cordial unless it is C_{4k+2}. Kaneria et al. [6] introduced a graph known as cycle of graphs. In [7] Kaneria et al. proved that cycle of a cycle is cordial.

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The recent survey on graph labeling can be found in Gallian [4], which provide vast amount of literature on graph lableling. Labelled graph have variety of applications in coding theory. A detailed study about applications of graph labeling is carried out in Bloom and Golomb [1]. For all terminology and notations we follws Harary [5]. First of all we shall recall some definitions, which are used in this paper.

Definition 1.1. A function $f : V(G) \longrightarrow \{0,1\}$ is called *binary vertex labeling* of a graph G and $f(v)$ is called *label of the vertex* v of G under f.

For an edge $e = (u, v)$, the induced function $f^{*} : E(G) \longrightarrow \{0,1\}$ defined as $f^{*}(e) = |f(u) - f(v)|$. Let $v_f(0)$, $v_f(1)$ be number of vertices of G having labels 0 and 1 respectively under f and let $e_f(0)$, $e_f(1)$ be number of edges of G having labels 0 and 1 respectively under f^{*}.

A binary vertex labeling f of a graph G is called *cordial labeling* if

$$|v_f(0) - v_f(1)| \leq 1 \text{ and } |e_f(0) - e_f(1)| \leq 1.$$

A graph which admits cordial labeling is called *cordial graph.*

Definition 1.2. For a cycle C_n, each vertices of C_n is replace by connected graphs $G_1, G_2, \ldots, G_n$ is known as *cycle of graphs* and we shall denote it by $C(G_1, G_2, \ldots, G_n)$. If we replace each vertices by a graph G i.e. $G_1 = G$, $G_2 = G$, ..., $G_n = G$, such cycle of a graph G, we shall denote it by $C(n \cdot G)$.

2. Main Results

Theorem 2.1. *$C(t \cdot K_{m,n})$ is cordial, $\forall\ m, n, t \in N - \{1\}$.*

Proof. Let G be a cycle of t copies of the complete bipartite graph $K_{m,n}$. Let $u_{i,j}$ $(1 \leq j \leq m)$ and $v_{i,k}$ $(1 \leq k \leq n)$ be vertices of i^{th} copy of $K_{m,n}$, $\forall$ $i = 1, 2, \ldots, t$. We shall join $u_{i,m}$ vertex of i^{th} copy of $K_{m,n}$ with $v_{i+1,1}$ vertex of $(i+1)^{th}$ copy of $K_{m,n}$ by an edge, $\forall\ i = 1, 2, \ldots, t-1$. We also join $u_{t,m}$ with $v_{1,1}$ by an edge to form the cycle graph $C(t \cdot K_{m,n})$.

To define the labeling function $f : V(C(t \cdot K_{m,n})) \longrightarrow \{0,1\}$ we shall consider following three cases.

Case I. m and n are even. Then:

$$f(u_{1,j}) = \begin{cases} 0, & \forall\ j = 1, 2, \ldots, \frac{m}{2}, \\ 1, & \forall\ j = \frac{m}{2}+1, \frac{m}{2}+2, \ldots, m; \end{cases}$$

$$f(v_{1,k}) = \begin{cases} 0, & \forall\ k = 1, 2, \ldots, \frac{n}{2}, \\ 1, & \forall\ k = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n; \end{cases}$$

$$f(u_{i,j}) = \begin{cases} f(u_{1,j}), & \text{when } i \equiv 0, 1 \pmod 4, \\ 1 - f(u_{1,j}), & \text{when } i \equiv 2, 3 \pmod 4; \end{cases}$$

$$f(v_{i,k}) = \begin{cases} f(v_{1,k}), & \text{when } i \equiv 0, 1 \pmod 4, \\ 1 - f(v_{1,k}), & \text{when } i \equiv 2, 3 \pmod 4, \end{cases}$$

where $j = 1, 2, \ldots, m$, $k = 1, 2, \ldots, n$, $i = 2, 3, \ldots, t-1$. Hence

$$f(u_{t,j}) = \begin{cases} 0, & \text{when } t \equiv 0, 1, 2 \pmod 4 \text{ and } j = 1, 2, \ldots, \frac{m}{2} \text{ or} \\ & t \equiv 3 \pmod 4 \text{ and } j = \frac{m}{2}+1, \frac{m}{2}+2, \ldots, m, \\ 1, & \text{when } t \equiv 0, 1, 2 \pmod 4 \text{ and } j = \frac{m}{2}+1, \frac{m}{2}+2, \ldots, m \text{ or} \\ & t \equiv 3 \pmod 4 \text{ and } j = 1, 2, \ldots, \frac{m}{2}; \end{cases}$$

$$f(v_{t,k}) = \begin{cases} 0, & \text{when } t \equiv 0, 1 \pmod 4 \text{ and } k = 1, 2, \ldots, \frac{n}{2} \text{ or} \\ & t \equiv 2, 3 \pmod 4 \text{ and } k = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n, \\ 1, & \text{when } t \equiv 0, 1 \pmod 4 \text{ and } k = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n \text{ or} \\ & t \equiv 2, 3 \pmod 4 \text{ and } k = 1, 2, \ldots, \frac{n}{2}. \end{cases}$$

Case II. W.l.o.g. we assume that m is even and n is odd. Therefore:

$$f(u_{1,j}) = \begin{cases} 0, & \forall\ j = 1, 2, \ldots, \frac{m}{2}, \\ 1, & \forall\ j = \frac{m}{2}+1, \frac{m}{2}+2, \ldots, m; \end{cases}$$

$$f(v_{1,k}) = \begin{cases} 0, & \forall\ k = 1, 2, \ldots, \frac{n-1}{2}, \\ 1, & \forall\ k = \frac{n+1}{2}, \frac{n+3}{2}, \ldots, n; \end{cases}$$

$$f(u_{i,j}) = \begin{cases} f(u_{1,j}), & \text{when } i \equiv 0, 1 \pmod 4, \\ 1 - f(u_{1,j}), & \text{when } i \equiv 2, 3 \pmod 4; \end{cases}$$

$$f(v_{i,k}) = \begin{cases} f(v_{1,k}), & \text{when } i \equiv 0, 1 \pmod 4, \\ 1 - f(v_{1,k}), & \text{when } i \equiv 2, 3 \pmod 4, \end{cases}$$

where $j = 1, 2, \ldots, m$, $k = 1, 2, \ldots, n$, $i = 2, 3, \ldots, t-1$. So, we have

$$f(u_{t,j}) = \begin{cases} 0, & \text{when } t \equiv 0, 1, 2 \pmod 4 \text{ and } j = 1, 2, \ldots, \frac{m}{2} \text{ or} \\ & t \equiv 3 \pmod 4 \text{ and } j = \frac{m}{2}+1, \frac{m}{2}+2, \ldots, m, \\ 1, & \text{when } t \equiv 0, 1, 2 \pmod 4 \text{ and } j = \frac{m}{2}+1, \frac{m}{2}+2, \ldots, m \text{ or} \\ & t \equiv 3 \pmod 4 \text{ and } j = 1, 2, \ldots, \frac{m}{2}; \end{cases}$$

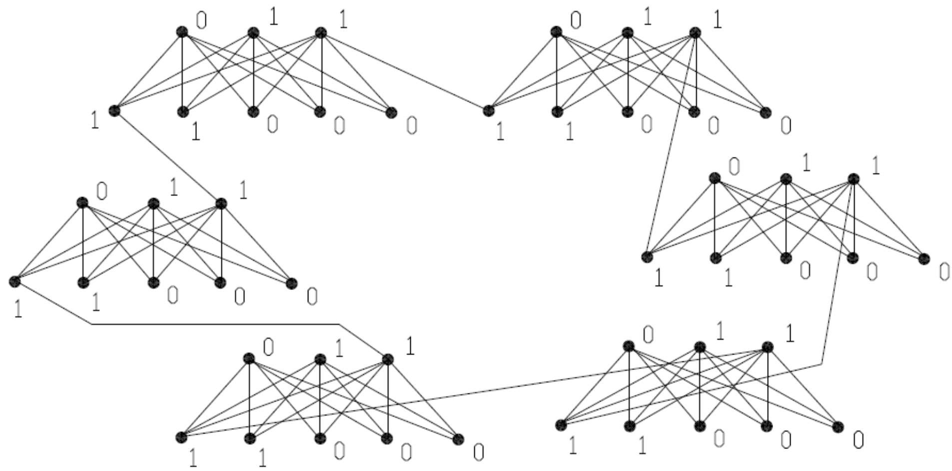

Figure 1: Cycle graph $C(6 \cdot K_{3,5})$ and its cordial labeling

$$f(v_{t,k}) = \begin{cases} 0, & \text{when } t \equiv 0,1 \pmod 4 \text{ and } k = 1,2,\ldots,\frac{n-1}{2} \text{ or} \\ & t \equiv 2,3 \pmod 4 \text{ and } k = \frac{n+1}{2},\frac{n+3}{2},\ldots,n, \\ 1, & \text{when } t \equiv 0,1 \pmod 4 \text{ and } k = \frac{n+1}{2},\frac{n+3}{2},\ldots,n \text{ or} \\ & t \equiv 2,3 \pmod 4 \text{ and } k = 1,2,\ldots,\frac{n-1}{2}. \end{cases}$$

Case III. m and n both are odd. Hence

$$f(u_{i,j}) = \begin{cases} 0, & \forall\, j = 1,2,\ldots,\frac{m-1}{2},\ \forall i = 1,2,\ldots,t, \\ 1, & \forall\, j = \frac{m+1}{2},\frac{m+3}{2},\ldots,m,\ \forall i = 1,2,\ldots,t; \end{cases}$$

$$f(v_{i,k}) = \begin{cases} 1, & \forall\, k = 1,2,\ldots,\frac{n-1}{2},\ \forall i = 1,2,\ldots,t, \\ 0, & \forall\, k = \frac{n+1}{2},\frac{n+3}{2},\ldots,n,\ \forall i = 1,2,\ldots,t. \end{cases}$$

The above labeling pattern give rise cordial labeling to the given graph G, as it satisfies $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ in above three cases. Thus $G = C(t \cdot K_{m,n})$ is a cordial graph, $\forall\, t,m,n \in N - \{1\}$. □

Example 2.2. $C(6 \cdot K_{3,5})$ (it is related with case-III) and its cordial labeling shown in Figure 1.

Theorem 2.3. *$C(t \cdot W_n)$ is cordial, $t,n \in N - \{1,2\}$.*

Proof. Let G be cycle of t copies of wheel W_n. Let $v_{i,j}$ $(0 \leq j \leq n)$ be vertices of i^{th} copy of W_n, where $v_{i,0}$ is vertex of apex of the wheel W_n, $\forall$

$i = 1, 2, \ldots, t$. We shall join $v_{i,0}$ with $v_{i+1,0}$, $\forall\ i = 1, 2, \ldots, t-1$ and $v_{t,0}$ with $v_{1,0}$ unless $t \equiv 2 \pmod 4$, otherwise join $v_{1,0}$ with $v_{t,1}$ to form the cycle graph $C(t \cdot W_n)$.

To define the labeling function $f : V(t \cdot W_n) \longrightarrow \{0, 1, \}$, we have following four cases.

Case I. Let $t \equiv 0, 2 \pmod 4$. Then

$$f(v_{i,0}) = \begin{cases} 0, & \text{when } i \equiv 0, 1 \pmod 4, \\ 1, & \text{when } i \equiv 2, 3 \pmod 4, \\ & \forall\ i = 1, 2, \ldots, t; \end{cases}$$

$$f(v_{i,j}) = \begin{cases} 1, & \text{when } i \equiv 0, 1 \pmod 4, \\ 0, & \text{when } i \equiv 2, 3 \pmod 4, \\ & \forall\ i = 1, 2, \ldots, t,\ \forall\ j = 1, 2, \ldots, n. \end{cases}$$

Case II. Let $t \equiv 1, 3 \pmod 4$ and $n \equiv 0, 1, 2 \pmod 4$. Then

$$f(v_{i,0}) = \begin{cases} 0, & \text{when } i \equiv 0, 1 \pmod 4, \\ 1, & \text{when } i \equiv 2, 3 \pmod 4, \\ & \forall\ i = 1, 2, \ldots, t-1; \end{cases}$$

$$f(v_{i,j}) = \begin{cases} 1, & \text{when } i \equiv 0, 1 \pmod 4, \\ 0, & \text{when } i \equiv 2, 3 \pmod 4, \\ & \forall\ i = 1, 2, \ldots, t-1,\ \forall\ j = 1, 2, \ldots, n; \end{cases}$$

$$f(v_{t,j}) = \begin{cases} 1, & \text{when } j \equiv 1, 2 \pmod 4, \\ 0, & \text{when } j \equiv 0, 3 \pmod 4, \\ & \forall\ j = 0, 1, 2, \ldots, n. \end{cases}$$

Case III. Let $t \equiv 1 \pmod 4$ and $n \equiv 3 \pmod 4$. Then

$$f(v_{i,0}) = \begin{cases} 0, & \text{when } i \equiv 0, 1 \pmod 4, \\ 1, & \text{when } i \equiv 2, 3 \pmod 4, \\ & \forall\ i = 1, 2, \ldots, t-1; \end{cases}$$

$$f(v_{i,j}) = \begin{cases} 1, & \text{when } i \equiv 0, 1 \pmod 4, \\ 0, & \text{when } i \equiv 2, 3 \pmod 4, \\ & \forall\ i = 1, 2, \ldots, t-1,\ \forall\ j = 1, 2, \ldots, n; \end{cases}$$

$$f(v_{t,j}) = \begin{cases} 0, & \text{when } j = 1,2 \text{ or } j \equiv 2,3 \pmod 4, \\ 1, & \text{when } j = 0,3,4,5 \text{ or } j \equiv 0,1 \pmod 4, \\ & \forall\ j = 6,7,\dots,n. \end{cases}$$

Case IV. Let $t \equiv 3 \pmod 4$ and $n \equiv 3 \pmod 4$. Then

$$f(v_{i,0}) = \begin{cases} 0, & \text{when } i \equiv 0,1 \pmod 4, \\ 1, & \text{when } i \equiv 2,3 \pmod 4, \\ & \forall\ i = 1,2,\dots,t-1; \end{cases}$$

$$f(v_{i,j}) = \begin{cases} 1, & \text{when } i \equiv 0,1 \pmod 4, \\ 0, & \text{when } i \equiv 2,3 \pmod 4, \\ & \forall\ i = 1,2,\dots,t-1,\ \forall\ j = 1,2,\dots,n; \end{cases}$$

$$f(v_{t,j}) = \begin{cases} 1, & \text{when } j = 1,2 \text{ or } j \equiv 2,3 \pmod 4, \\ 0, & \text{when } j = 0,3,4,5 \text{ or } j \equiv 0,1 \pmod 4, \\ & \forall\ j = 6,7,\dots,n. \end{cases}$$

The above labeling pattern give rise cordial labeling to the given graph G, as it satisfies $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$ in above four cases. Thus $G = C(t \cdot W_n)$ is a cordial graph, $\forall\ t, n \in N - \{1, 2\}$.

Example 2.4. $C(5W_7)$ and its cordial labeling (it is related with Case III) shown in Figure 2.

Example 2.5. $C(5W_4)$ and its cordial labeling (it is related with Case II) shown in Figure 3.

3. Concluding Remarks

Cordial labeling of some cycle of graphs discussed. Here we provide cordial labeling to $C(t \cdot K_{m,n})$ and $C(t \cdot W_n)$.

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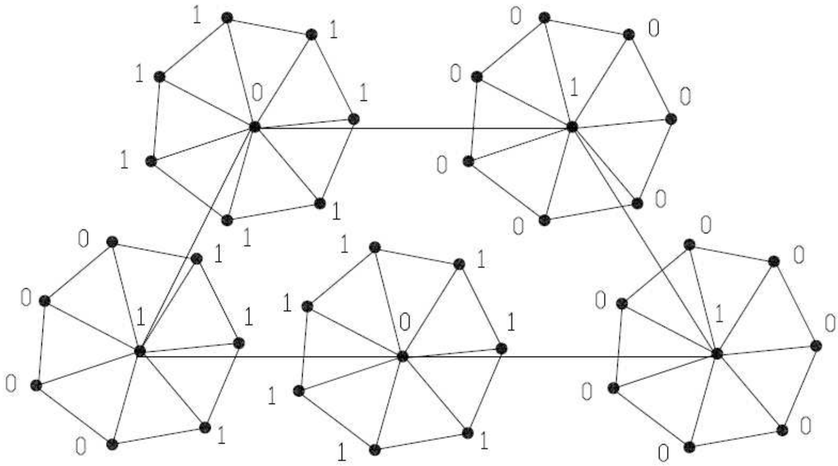

Figure 2: Cycle graph $C(5 \cdot W_7)$ and its cordial labeling

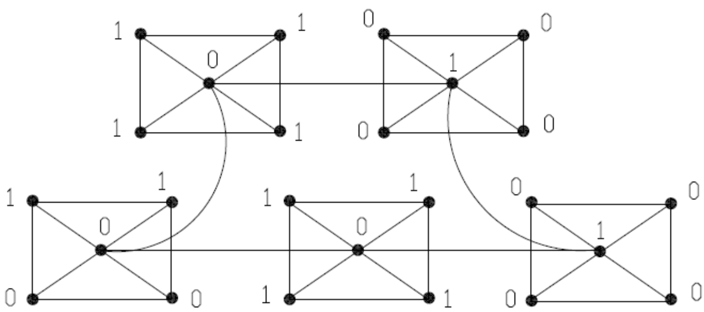

Figure 3: Cycle graph $C(5 \cdot W_4)$ and its cordial labeling

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