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A FRUIT DRYING MATHEMATICAL MODEL

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Abstract: In this paper we assess the time of drying for any fruits under natural heat radiation. In order to assess the time period of drying process we use the method described in [1] being supplemented by an experimental technique. We show that the drying speed is a polynomial of third degree w.r.t. the quantity of the solid substance. This help us to estimate the time period of drying.

AMS Subject Classification: drying process, evolution, differential equation, time period assessment
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1. Introduction

Drying technologies play a basic role in the food industry. Nowadays using of the solar energy in drying process gets more and more popular. The food drying is a physical process producing a very simple and at the same time useful effect. The drying is characterized by removing the water to a level that minimizes the deterioration phenomena due to microorganisms, enzymes or ferments [1], [3]-[6]. The drying process is very complicated of physical point of view. However,

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we propose a method of assessment of the time period of drying which is a modification of that used in [1].

There exist different models of the drying process to different type materials which are known in the literature, [4], [7]. So we refer the reader to [1], [4]. The diffusion phenomena is described in [3], [5], and the technology of the drying as well as the mathematical description of practical point of view is represented in [4]. There are many open questions about the influence of different water fazes on the quality of the drying organic material during the drying process. The chemical and biochemical reaction rates depending on water activity as well as qualitative equilibrium isotherms for various types of food materials can be seen in [4] (Graph 1.3 - 1.5).

In this paper we use the data announced by Cooperative Extension Service at University of Kentucky, [8]. In accordance with this source we have for the most popular raw apples the following water content: on 138 [g] mean apple weight the water weight is 116 [g], then the percent water is equal approximately to 84%.

In the next section we remind some methods of assessment of the drying period for apples and other fruits, [1], [4].

In the third section we propose a modification of the time assessment method described in [1]. Introducing an experimental technique we determine analytically the drying speed W. It turns out that W is a polynomial of third degree w.r.t. the quantity of the solid substance x, [7]. Thus using the so obtained polynomial $W(x)$ one may estimate the time period of complete drying for any fruits.

2. Preliminaries

It is known that if we use natural drying process by hot air in order to dry up any fruits then this air has to be transfered to the pores of the organic material. Then one encounters inevitably with phenomena as mass transfer, capillary effect, diffusion, convection or a combined mode observed in the pore channels inside the drying organic material.

Of biological point of view the bacterial cells, fungal substances and microorganisms are susceptible to the drying process running under certain temperature environment. We emphasize that the preservation of the activities of the useful beneficial bacteria, for instance such as probiotic bacteria, depends on the drying process, that is the drying technology.

Analyzing drying process mechanism it turns out that for explenation of this process we have to involve the following surface vapour flux equation, [4]:

$$N_\nu'' = h_m(\rho_{\nu,s} - \rho_{\nu,\infty}), \tag{1}$$

where:

N_ν'' is the vapour (or drying) flux ($kg\ m^{-2}s^{-1}$);

h_m is the mass transfer coefficient ($m\ s^{-1}$);

$\rho_{\nu,s}$ is the interfacial vapour concentration at the moist material surface ($kg\ m^{-3}$);

$\rho_{\nu,\infty}$ is the vapour concentration in the surrounding space that the vapour travels into ($kg\ m^{-3}$);

h_m is determined by the flow field around the material being dried (ms^{-1}), which may be viewed as a velocity of mass movement.

For the above mentioned dynamics of the drying process, and for the literature on the present consideration we refer the reader to [4]. Taking into account the equality (1) we observe that if the surrounding vapour concentration $\rho_{\nu,\infty}$ decreases then the drying rate increases. We note here that if the vapour concentration difference is positive, then vapour leaves the material, and thus drying exists.

Remark. The reverse is the wetting process or so called humidification.

Consider a gadget which has a horizontal basin in the shape of a simple table with a rectangular holder of fruits. We are interested only in the rectangular holder. Suppose it has length and width $l_t \times b_t$ $(0 < b_t < l_t)$, respectively. Also suppose there are some quantity of fresh fruits placed on the rectangular holder. Assume also that the dry air goes through the fruits in the vertical direction. It is known that the speed W of drying can be computed by the following formula:

$$W = \frac{-L}{A}\frac{dx}{d\tau}, \tag{2}$$

where W is the speed of drying

$$W\left[\frac{kg\ \text{(evaporating moisture)}}{m^2\ \text{(surface of solid substance)}\ h}\right],$$

L is the quantity of the solid substance

$$x\quad\left[\frac{kg\ \text{(moisture)}}{kg\ \text{(dry remainder)}}\right]$$

is the quantity of moisture containing in the solid substance,
A $[m^2]$ is the measure of the drying surface,
τ $[h]$ is the period of time.

Our next step is to integrate the differential equation (2),

$$\tau = \int_0^\tau ds = \frac{L}{A} \int_{s_2}^{s_1} \frac{ds}{W}. \tag{3}$$

Consider two basic cases (see, e.g., [1]).

1. *Drying process with constant speed.* Here suppose that $s_1, s_2 > s_c$, where s_c is a critical value, $W = W_c \equiv \text{const}$, thus obtain

$$\tau = \frac{L(s_1 - s_2)}{AW_c}. \tag{4}$$

2. *Drying process with decreasing speed.* Here suppose that $s_1, s_2 < s_c$, then two cases of interest hold:

(i) *General case.* There exist two methods of computation of τ. The first method can be applied in the case when there is experimentally obtained graph of the speed W as a function of s, that is, in the formula (3) $W = W(s)$ is an integrable (in the sense of Riemann) function.

(ii) *Linear case.* In this case the quantity $W = ks + m$ is a linear function with $k = \tan\alpha \equiv \text{const} \in \mathbb{R}$ is a known slope measured by the angle α, and $m \in \mathbb{R}$ is the W intercept of point of view of a Cartesian coordinate system. In this case we may approximate the function $W(s)$ with finite number of linear functions defined on finite number of subintervals of $[s_1, s_2]$ which are mutually disjoint. Substituting the above stated linear function W in (3), thus obtain

$$\tau = \frac{L}{A} \int_{s_2}^{s_1} \frac{ds}{ks + m} = \frac{L}{kA} \ln\left(\frac{ks_1 + m}{ks_2 + m}\right). \tag{5}$$

Next set $V_1 = ks_1 + m$, $V_2 = ks_2 + m$, thus $k = \dfrac{V_1 - V_2}{s_1 - s_2}$, therefore (5) can be represented by the formula:

$$\tau = \frac{L(s_1 - s_2)}{AV_m}, \tag{6}$$

where $V_m = \dfrac{V_1 - V_2}{\ln V_1 - \ln V_2}$ is the mean logarithmic difference of V_1 and V_2 w.r.t. s_1 and s_2, respectively.

Following the theory in [1] there exists a case, when the speed of drying can be taken linear. Then the formula for calculation of the time takes the form:

$$\tau = \frac{L(s_c - s^*)}{AV_{const}} \ln\left(\frac{s - s^*}{s_2 - s^*}\right). \tag{7}$$

3. Main Result

Most of authors consider the simplest linear case (ii) and subsequent formulas (5)-(7). We note that the result in this section is based on the case of drying process concerning fruits, for instance apples, pears, peaches, apricots, etc., as for this purpose we chose the apple as a test model. To the water content of the widespread fruits and vegetables we refer the reader to the list of Cooperative Extension Service - University of Kentucky (UK), [8]. One may see there that the water content for the apple is about 84%. For the other above mentioned fruits the water content defers in no way than one in the apple.

Next we consider the nonlinear (in general case) ordinary differential equation (2), which leads to the Cauchy problem

$$\frac{dx}{d\tau} = -\frac{A}{L}W(x) \tag{8}$$

with initial condition

$$x(\tau_0) = s_2, \tag{9}$$

where s_2 is some real number determining the initial point of the dynamical process at the initial time τ_0. Thus Cauchy problem (8), (9) must satisfies the standard existence and uniqueness requirement. In the general case may the vector field W depends on both τ and x, i.e. $W = W(\tau, x)$.

Theorem (Picard-Lindelöf). *Suppose $W(\tau, x)$ is continuous in the set*

$$P \equiv \{(\tau, x):\ \tau_0 \le \tau \le \tau_0 + a,\ |x - s_2| \le b\}, \quad a, b > 0,$$

and Lipschitzian in x. Let $M > 0$ is the upper limit for $|(-A/L)W(\tau, x)|$ on P, and $\alpha = \min(a, b/M)$. Then, there exists a unique solution $x = x(\tau)$ to the initial value problem (8), (9) on the interval $[\tau_0, \tau_0 + \alpha]$.

Next, we intend to integrate the above stated initial problem (8), (9) by a method based on the experimental determining of drying speed W. Assume that W depends only on x. In order to determine the functional dependence of

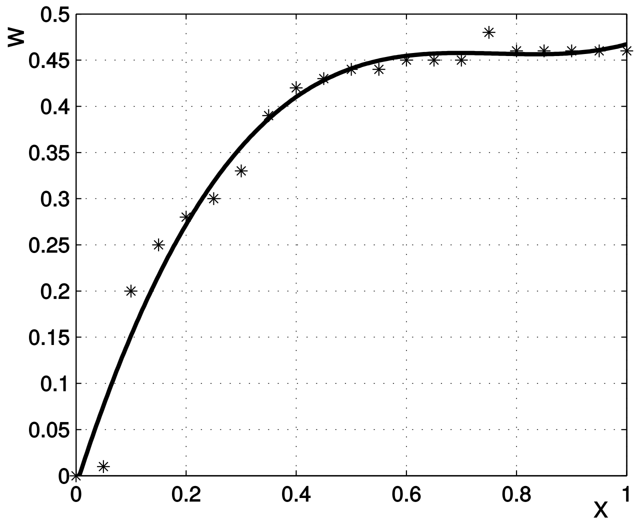

Figure 1: Graph of $W = W(x)$

W w.r.t. the quantity of moisture containing in the solid substance x we use an test technique, [2]. Thus establish the experimentally obtained ratio between quantities x and W given below, i.e. it turns out that the function $W = W(x)$ is a polynomial of degree three which has the form:

$$W \ = \ 1.0665 \ x^3 \ - \ 2.4515 \ x^2 \ + \ 1.8635 \ x \ - \ 0.011239. \tag{10}$$

Its graph is shown below: From the graph of the polynomial (1) we conclude that the function W increases monotonically until $x \approx 0.6$, whereafter it is almost constant $W \approx 0.45$ for all points x in the approximate interval $[0.6;\ 1]$. Obviously, its graph crosses the axes Ox at a point staying on the right hand of O. In other words the only real root of $W(x)$ would be of physical interest; it is positive and nearby O.

In order to calculate the time period τ to the complete drying of the fruits we make use of the equality (3)

$$\tau = \int_0^\tau ds = \frac{L}{A} \int_{s_2}^{s_1} \frac{dx}{W(x)}$$

$$= \frac{L}{A} \int_{s_2}^{s_1} \frac{dx}{1.0665x^3 - 2.4515x^2 + 1.8635x - 0.011239}$$

$$= \frac{L}{A} \big(-0.960083 \arctan (1.77 - 1.54412x) \tag{11}$$

$$+ 0.545313 \ln (-0.00607962 + x)$$

$$- 0.272656 \ln (1.73337 - 2.29256x + x^2) \big) \Big|_{s_2}^{s_1}$$

$$= \frac{L}{A} \big(\Phi(s_1) - \Phi(s_2) \big),$$

where $\Phi(\cdot)$ is the primitive to $1/W(\cdot)$. So we find the time period τ more precisely in comparison with the linear case considered above. Also one may assess τ analytically for different fruits. For instance if the functional dependence of $\tilde{W}$ w.r.t. x is

$$\tilde{W} = 1.0665x^3 - 2.4515x^2 + 1.8635x,$$

and the corresponding time period is $\tilde{\tau}$ then one has that

$$\frac{L}{A} \int_{s_2}^{s_1} \frac{dx}{1.0665x^3 - 2.4515x^2 + 1.8635x - 0.011239}$$

$$\geq \frac{L}{A} \int_{s_2}^{s_1} \frac{dx}{1.0665x^3 - 2.4515x^2 + 1.8635x},$$

hence $\tau \geq \tilde{\tau}$.

Therefore, we conclude that using the method in [1] being supplemented by the experimental technique we get the polynomial $W(x)$ and are able to calculate the time period τ of drying process for fruits as apples, pears, peaches, apricots, etc.

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