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NEW EXACT SOLUTIONS OF SOME NONLINEAR SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS USING MODIFIED EXTENDED DIRECT ALGEBRAIC METHOD

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Abstract: The modified extended direct algebraic method (MEDA) is a powerful solution method for obtaining new exact complex solutions of some nonlinear system of partial differential equations such as classical Drinfel'd-Sokolov-Wilson system (DSWE),(2+1)-dimensional Davey-Stewartson system and generalized Hirota-Satsuma coupled KdV system.

Key Words: The MEDA method, classical Drinfel'd-Sokolov-Wilson system (DSWE), (2+1)-dimensional Davey-Stewartson system and generalized Hirota-Satsuma coupled KdV system

1. Introduction

Recently many new approach to obtain the exact solutions of nonlinear differential equations have been proposed. Among these are variational iteration method [1]-[7], tanh function method [8], [9], modified extended tanh function method [10]-[16], sine-cosine method [17], [18], Exp-method [19], inverse scattering method [20], Hirota's bilinear method [21], the homogeneous balance method [22], the Riccati expansion method with constant coefficients [23], [24].

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Recently, the direct algebraic method and symbolic computation have been suggested to obtain the exact complex solutions of nonlinear partial differential equations [25], [26].

The aim of this paper is to extend the modified extended direct algebraic (MEDA) method to solve three different types of nonlinear systems of partial differential equations such as the classical Drinfel' d-Sokolov-Wilson system, (2+1)-dimensional Davey-Stewartson system and generalized Hirota-Satsuma coupled KdV system.

2. Modified Extended Direct Algebraic Method

Consider the following nonlinear system of partial differential equations with independent variables x and t and dependent variables u and v,

$$\begin{aligned} &F_1(u, v, u_t, v_t, u_x, v_x, u_{tt}, v_{tt}, u_{xx}, v_{xx}, ...) = 0,\\ &F_2(u, v, u_t, v_t, u_x, v_x, u_{tt}, v_{tt}, u_{xx}, v_{xx}, ...) = 0. \end{aligned} \tag{1}$$

Applying the transformation $u(x,t) = u(z)$ and $v(x,t) = v(z)$, where $z = i(x - ct) + \gamma$, $i^2 = -1$ where γ is an arbitrary constant, converts (1) into a system of ordinary differential equations (ODEs)

$$\begin{aligned} &Q_1(u, v, -ciu', -civ', iu', iv', ...) = 0,\\ &Q_2(u, v, -ciu', -civ', iu', iv', ...) = 0, \end{aligned} \tag{2}$$

where the prime denote the derivative with respect to the same variable z. Using some mathematical operations, the system (2) is converted into a second -order ordinary differential equation (ODE)

$$G(u, -ciu', iu', c^2u'', -u'', ...) = 0. \tag{3}$$

In order to seek the solutions of equation (1), we introduce the following ansatze

$$u(z) = a_0 + \sum_{j=1}^{M}(a_j\phi^j + b_j\phi^{-j}), \tag{4}$$

$$\phi' = b + \phi^2, \tag{5}$$

where b is a parameter to be determined, $\phi = \phi(z)$, $\phi' = d\phi/dz$. The parameter M can be found by balancing the highest-order derivative term with the nonlinear terms, see [27]. Substituting (4) into (3) with (5) with yield a system of algebraic equations with respect to a_j, b_j, b and c (where j=1,...,M) because all the coefficients of ϕ^j have to vanish. We can then determine a_0, a_j, b_j, b and c equation (4) has the general solutions:

(I) If $b < 0$: $\phi = -\sqrt{-b}\ tanh(\sqrt{-b}z)$ or $\phi = -\sqrt{-b}\ coth(\sqrt{-b}z)$, It depends on initial conditions.

(II) If $b > 0$: $\phi = \sqrt{b}\ tan(\sqrt{b}z)$, or $\phi = -\sqrt{b}\ cot(\sqrt{b}z)$, It depends on initial conditions.

(III) If $b = 0$: $\phi = (-1)/z$.

Substituting the results into (3), then we obtain the exact travelling wave solutions of equation (1). To illustrate the procedure, three examples related to classical Drinfel'd-Sokolov-Wilson system,(2+1)-dimensional Davey-Stewartson system, generalized Hirota -Satsuma coupled KdV system are given in the following.

3. Applications

3.1. Classical Drinfel'd-Sokolov-Wilson System

Consider

$$\begin{aligned} &u_t + pvv_x = 0,\\ &v_t + qv_{xxx} + ruv_x + su_xv = 0, \end{aligned} \tag{6}$$

where p, q, r and s are arbitrary constants. Recently, DSWE and the coupled DSWE, a special case of the classical DSWE, have been studied by several authors [25] and the references therein. Using a complex variation z defined as $z = i(x - ct) + \gamma$, we can convert (6) into ODEs, which read

$$-ciu' + ipvv' = 0 \ \Rightarrow \ u' = (p/c)vv', \tag{7}$$

$$-civ' - qiv''' + ru(iv') + siu'v = 0, \tag{8}$$

where the prime denotes the derivative with respect to z. Integrating (7), we obtain

$$u = (p/2c)v^2 + c_1, \tag{9}$$

where c_1 is an arbitrary integration constant. Substituting u into (8) yields

$$-cv' - qv''' + [(p(r + 2s))/2c]v^2v' + rc_1v' = 0.$$

Let $(p(r+2s)/2c) = k$, $rc_1 = h$,

$$\Rightarrow (h-c)v' - qv''' + kv^2v' = 0. \tag{10}$$

Integrating (11), we obtain

$$(h-c)v - qv'' + (k/3)v^3 = c_2, \tag{11}$$

where c_2 is an arbitrary integration constant.

Balancing the order of v^3 with the order of v'' in equation (11), we find $M = 1$.

So the solution of equation (11) takes the form:

$$v(z) = a_0 + a_1\phi + b_1\phi^{-1}. \tag{12}$$

Inserting equation (12) into equation (11) and making use of equation (5),

$$\begin{aligned} &-c_2 + (h-c)[a_0 + a_1\phi + b_1\phi^{-1}] \\ &- q[2a_1b\phi + 2b_1b\phi^{-1} + 2a_1\phi^3 + 2b_1b^2\phi^{-3}] \\ &+ k/3[(a_0^3 + 6a_0a_1b_1) + (3a_0^2a_1 + 3a_1^2b_1)\phi \\ &+ (3a_0^2b_1 + 3a_1b_1^2)\phi^{-1} + (3a_0a_1^2)\phi^2 \\ &+ (3a_0b_1^2)\phi^{-2} + (a_1^3)\phi^3 + (b_1^3)\phi^{-3}] = 0. \end{aligned} \tag{13}$$

We get a system of algebraic equations, for a_0, a_1, b_1 and b in the form:

$$\begin{aligned} -c_2 + ha_0 - ca_0 + (k/3)a_0^3 + 2ka_0a_1b_1 &= 0, \qquad (1') \\ ha_1 - ca_1 - 2qa_1b + ka_0^2a_1 + ka_1^2b_1 &= 0, \qquad (2') \\ hb_1 - cb_1 - 2qb_1b + ka_0^2b_1 + ka_1b_1^2 &= 0, \qquad (3') \\ ka_0a_1^2 &= 0, \qquad (4') \\ ka_0b_1^2 &= 0, \qquad (5') \\ -2qa_1 + (k/3)a_1^3 &= 0, \qquad (6') \\ -2qb_1b^2 + (k/3)b_1^3 &= 0. \qquad (7') \end{aligned} \tag{14}$$

These equations give the following three case:

Case (I): $a_1 = 0$. From equation (7'): $b = \sqrt{k/6q}b_1$. From equation (3'): $c = h + ka_0^2 - \sqrt{2qk/3}b_1$ (8'). From equation (1'): $c = h + (k/3)a_0^2 - (c_2/a_0)$ $\Rightarrow$ substituting c into(8') we get $b_1 = (2ka_0^3 + 3c_2)/(\sqrt{6qk}a_0) \Rightarrow b = (2ka_0^3 + 3c_2)/(6qa_0)$. Let $A = \sqrt{((2ka_0^3 + 3c_2)/(ka_0))}$, with a_0 being arbitrary constant,

$$v(x,t) = a_0 + Acot(\sqrt{b}z). \tag{15}$$

So, the travelling wave solution is given by:

$$u(x,t) = (p/2c)[a_0 + A(cot(\sqrt{b}z))]^2 + c_1. \tag{16}$$

Case (II): $b_1 = 0$, from equation ($6'$): $a_1 = \sqrt{6q/k}$,
from equation ($2'$) : $c = h - 2qb + ka_0^2 \qquad (*)$,
from equation ($1'$): $c = h + (k/3)a_0^2 - (c_2/a_0) \Rightarrow$ substituting c into $(*)$ we get $b = (2ka_0^3 + 3c_2)/(6qa_0)$, with a_0 being arbitrary constant,

$$v(x,t) = a_0 + Atan(\sqrt{b}z) \tag{17}$$

so, the travelling wave solution is given by

$$u(x,t) = (p/2c)[a_0 + A(tan(\sqrt{b}z))]^2 + c_1. \tag{18}$$

Case (III): From equation ($7'$): $b = \sqrt{k/6q}b_1$, and from equation ($6'$): $a_1 = \sqrt{6q/k}$. From equation ($3'$): $h - c - 2qb + ka_0^2 + ka_1b_1 = 0$, therefore equation ($1'$): $h - c + (k/3)a_0^2 + 2ka_1b_1 - (c_2/a_0)$, Hence $b_1 = (2ka_0^3 + 3c_2)/(4\sqrt{6kq}a_0) \Rightarrow b = (2ka_0^3 + 3c_2)/(24qa_0)$, where a_0 is arbitrary constant,

$$v(x,t) = a_0 + (A/2)[tan(\sqrt{b}z) + cot(\sqrt{b}z)], \tag{19}$$

so, the travelling wave solution is given by

$$u(x,t) = (p/2c)[a_0 + (A/2)(tan(\sqrt{b}z) + cot(\sqrt{b}z))]^2 + c_1. \tag{20}$$

Thus, we have been obtained three solutions for the system (6).

3.2. (2+1)-Dimensional Davey-Stewartson System

The (2+1)-dimensional Davey-Stewartson System [29] reads :

$$\begin{gathered} iu_t + u_{xx} - u_{yy} - 2|u|^2u - 2uv = 0, \\ v_{xx} + v_{yy} + 2(|u|^{(}2))_{xx} = 0. \end{gathered} \tag{21}$$

Using the wave variables,

$$u(x,y,t) = u(z), v(x,y,t) = v(z), z = i(\alpha x + \beta y - ct) + \gamma, \tag{22}$$

where α, β, c,and γ are constants,by similar manner as above convert (21) into the ODE :

$$cu' - \alpha^2u'' + \beta^2u'' - 2u^3 - 2uv = 0, \tag{23}$$

$$-\alpha^2 v'' - \beta^2 v'' + (u^2)'' = 0. \tag{24}$$

Integrating (24) in the system and neglecting constants of integration, We have found :

$$v = u^2/(\alpha^2 + \beta^2). \tag{25}$$

Substituting (25) into (23) of the system we find

$$cu' - (\alpha^2 - \beta^2)u'' - 2((\alpha^2 + \beta^2 + 1)/((\alpha^2 + \beta^2)))u^3 = 0.$$

Let $(\alpha^2 - \beta^2) = c_1$,

$$((\alpha^2 + \beta^2 + 1)/(\alpha^2 + \beta^2)) = c_2 \Rightarrow cu' - c_1 u'' - 2c_2 u^3 = 0. \tag{26}$$

Balancing the order of u^3 with the order of u'' in equation (26), we find: M=1. So, the solution takes the form :

$$u(z) = a_0 + a_1\phi + b_1\phi^{-1}. \tag{27}$$

Inserting equation (27) into equation (26) and making use of equation (5),

$$\begin{aligned} &c[(a_1 b - b_1) + a_1\phi^2 - b_1 b\phi^{-2}] - c_1[2a_1 b\phi + 2b_1 b\phi^{-1} + 2a_1\phi^3 + 2b_1 b^2\phi^{-3}] \\ &- 2c_2[(a_0^3 + 6\alpha_0 a_1 b_1) + (3a_0^2 a_1 + 3a_1^2 b_1)\phi + (3a_0^2 b_1 + 3a_1 b_1^2)\phi^{-1} \\ &+ (3a_0 a_1^2)\phi^2 + (3a_0 b_1^2)\phi^{-2} + (a_1^3)\phi^3 + (b_1^3)\phi^{-3}] = 0. \end{aligned} \tag{28}$$

We get a system of algebraic equations, for a_0, a_1, b_1 and b.

$$\begin{aligned} ca_1 b - cb_1 - 2c_2 a_0^3 - 12c_2 a_0 a_1 b_1 =&0, \quad (1'') \\ -2c_1 a_1 b - 6c_2 a_0^2 a_1 - 6c_2 a_1^2 b_1 =&0, \quad (2'') \\ -2c_1 b_1 b - 6c_2 a_0^2 b_1 - 6c_2 a_1 b_1^2 =&0, \quad (3'') \\ ca_1 - 6c_2 a_0 a_1^2 =&0, \quad (4'') \\ -cb_1 b - 6c_2 a_0 b_1^2 =&0, \quad (5'') \\ -2c_1 a_1 - 2c_2 a_1^3 =&0, \quad (6'') \\ -2c_1 b_1 b^2 - 2c_2 b_(1)^(3) =&0. \quad (7'') \end{aligned} \tag{29}$$

We solve the obtained system of algebraic equations give the following three cases:

Case (I): $a_1 = 0$, then from equation $(7'')$: $b_1 = \sqrt{c_1/c_2}bi$, from equation $(3'')$: $b = (-3c_2 a_0^2)/c_1$, $\Rightarrow$ $b_1 = -3a_0^2 i\sqrt{c_2/c_1}$, with a_0 being arbitrary constant, the travelling wave is given by :

$$u(x, y, t) = a_0(1 - \sqrt{3}(cot(Biz))), \tag{30}$$

so, the travelling wave solution is given by :

$$v(x,y,t) = (1/A)[a_0(1-\sqrt{3}(cot(Biz)))]^2, \tag{31}$$

where $A = \alpha^2 + \beta^2$; $B = a_0(\sqrt{(3c_2)/c_1})$.

Case (II): $b_1 = 0$, then from equation $(6'')$: $a_1 = \sqrt{c_1/c_2 1}$, and from equation $(2'')$: $b = (-3c_2a_0^2)/c_1$. Here a_0 is arbitrary constant. The travelling wave solution is given by

$$u(x,y,t) = a_0(1-\sqrt{3}(tan(Biz))). \tag{32}$$

Therefore, the travelling wave solution is given by:

$$v(x,y,t) = (1/A)[a_0(1-\sqrt{3}(tan(Biz)))]^2. \tag{33}$$

Case (III): From equation $(6'')$: $a_1 = \sqrt{c_1/c_2}\ i$, and from equation $(7'')$: $b_1 = \sqrt{c_1/c_2}\ bi$, moreover from equation $(3'')$: $b = (-3c_2a_0^2)/(2c_1)$. Hence

$$b_1 = (3/2)a_0^2 i(\sqrt{c_2/c_1}).$$

Here a_0 is arbitrary constant. The travelling wave solution in this case is given by :

$$u(x,y,t) = a_0[1+\sqrt{3/2}\ i(tan(B/\sqrt{2}z)+cot(B/\sqrt{2}z))]. \tag{34}$$

So, the travelling wave solution is given by

$$v(x,y,t) = (1/A)[a_0(1+\sqrt{3/2}\ i(tan(B/\sqrt{2}z)+cot(B/\sqrt{2}z)))]^2. \tag{35}$$

3.3. Generalized Hirota-Satsuma Coupled KdV System

Consider the generalized Hirota-Satsuma coupled KdV system [30],

$$u_t = (1/4)u_{xxx} + 3uu_x + 3(w-v^2)_x, \tag{36}$$

$$v_t = -(1/2)v_{xxx} - 3uv_x, \tag{37}$$

$$w_t = -(1/2)w_{xxx} - 3uw_x. \tag{38}$$

When $w = 0$, (36)-(38) reduce to be the well-known Hirota-Satsuma coupled KdV system. We seek traveling wave solutions for (36)-(38)in the form

$$u(x,t) = u(z), v(x,t) = v(z), w(x,t) = w(z), z = ik(x-ct)+h, \tag{39}$$

where h is an arbitrary constant.

Substituting (39) into (36)-(38) yields an

$$-ckiu' = -(1/4)k^3 iu''' + 3kiuu' + 3ki(w - v^2)', \tag{40}$$

$$-ckiv' = (1/2)k^3 iv''' - 3kiuv', \tag{41}$$

$$-ckiw' = (1/2)k^3 iw''' - 3kiuw'. \tag{42}$$

Integrating (40), and divided the above three equations by ki,we get:

$$-cu = -(1/4)k^2 u'' + (3/2)u^2 + 3(w - v^2), \tag{43}$$

$$-cv' = (1/2)k^2 v''' - 3uv', \tag{44}$$

$$-cw' = (1/2)k^2 w''' - 3uw'. \tag{45}$$

From equation (44) and (45)we get:

$$(1/2)k^2 v''' = (3u - c)v', \tag{46}$$

$$(1/2)k^2 w''' = (3u - c)w'. \tag{47}$$

By dividing equation (46) on (47) we get:

$$v'''/w''' = v'/w' \Rightarrow w'/w''' = v'/v''' \Rightarrow w' = A_0 v'. \tag{48}$$

Integrating (48), we get:

$$w = A_0 v + B_0. \tag{49}$$

Let

$$u = \alpha v^2 + \beta v + \gamma, \tag{50}$$

where A_0, B_0, α, β and γ are constants, see [31].

From equation (50), we have:

$$u' = 2\alpha v v' + \beta v' = (2\alpha v + \beta)v', \quad u'' = (2\alpha v + \beta)v'' + 2\alpha(v')^2.$$

Multiply u'' by k^2 we obtain

$$k^2 u'' = (2(\alpha)v + (\beta))k^2 v'' + 2(\alpha)k^2(v')^2. \tag{51}$$

From equation (43):

$$k^2 u'' = 6u^2 + 4cu + 12(w - v^2). \tag{52}$$

Substituting (49) and (50) into (52):

$$\begin{aligned} k^2u'' &= 6[\alpha v^2+\beta v+\gamma]^2+4c[\alpha v^2+\beta v+\gamma]+12[A_0v+B_0-v^2] \Rightarrow \\ k^2u'' &= 6[\alpha^2v^4+2\alpha\beta v^3+\beta^2+2\alpha\gamma v^2+2\beta\gamma v+\gamma^2] \\ &\quad +4c\alpha v^2+4c\beta v+4c\gamma-12v^2+12A_0v+12B_0, \\ k^2u'' &= 6\alpha^2v^4+12\alpha\beta v^3+[6\beta^2+12\alpha\gamma+4c\alpha-12]v^2 \\ &\quad +[12\beta\gamma+4c\beta+12A_0]v+[6\gamma^2+4c\gamma+12B_0]. \end{aligned} \tag{53}$$

Substituting $u=\alpha v^2+\beta v+\gamma$ into (46) we receive

$$1/2k^2v''' = 3\alpha v^2v'+3\beta vv'+(3\gamma-c)v'.$$

Multipling by 2:

$$k^2v''' = 6\alpha v^2v'+6\beta vv'+2(3\gamma-c)v'. \tag{54}$$

Integrating (54)m yields

$$k^2v'' = 2\alpha v^3+3\beta v^2+2(3\gamma-c)v+c_1, \tag{55}$$

where c_1 is an integration constant. Integrating (55) once again and multipling by 2 we have:

$$k^2(v')^2 = \alpha v^4+2\beta v^3+2(3\gamma-c)v^2+2c_1v+c_2, \tag{56}$$

where c_2 is an integration constant.

Substituting (53),(56)onto (51) we get:

$$\begin{aligned} (2\alpha v+\beta)k^2v'' &= 4\alpha^2v^4+8\alpha\beta v^3+(6\beta^2+8c\gamma-12)v^2 \\ &\quad +(12\beta\gamma+4c\beta+12A_0-4\alpha c_1)v+(6\gamma^2+4c\gamma+12B_0-2\alpha c_2). \end{aligned} \tag{57}$$

Let $4\alpha^2=k_0$, $8\alpha\beta=k_1$, $6\beta^2+8c\alpha-12=k_2$, $12\beta\gamma+4c\beta+12A_0-4\alpha c_1=k_3$, $6\gamma^2+4c\gamma+12B_0-2\alpha c_2=k_4$, $2\alpha k^2=k_5$, $\beta k^2=k_6$. Then equation (57) becomes:

$$(k_5v+k_6)v'' = k_0v^4+k_1v^3+k_2v^2+k_3v+k_4. \tag{58}$$

Balancing the order of v^4 with the order of vv'' in equation (58), gives $M=1$.

So,the solution takes the form:

$$v = a_0+a_1\phi+b_1\phi^{-1}. \tag{59}$$

Inserting equation (59) into equation (58) and making use of equation (5), we get:

$$\begin{aligned}
&[k_0(a_0^4 + 12a_0^2a_1b_1 + 6a_1^2b_1^2) + k_1(a_0^3 + 6a_0a_1b_1)\\
&+ k_2(a_0^2 + 2a_1b_1) + k_3(a_0) + k_4 - (4k_5a_1b_1b)]\\
&+ [k_0(4a_0^3a_1 + 12a_0a_1^2b_1) + k_1(3a_0^2a_1 + 3a_1^2b_1)\\
&+ k_2(2a_0a_1) + k_3(a_1) - (2k_5a_0a_1b + 2k_6a_1b)]\phi\\
&+ [k_0(4a_0^3b_1 + 12a_0a_1b_1^2) + k_1(3a_0^2b_1 + 3a_1b_1^2)\\
&+ k_2(2a_0b_1) + k_3(b_1) - (2k_5a_0b_1b + 2k_6b_1b)]\phi^{-1}\\
&+ [k_0(6a_0^2a_1^2 + 4a_1^3b_1) + k_1(3a_0a_1^2) + k_2(a_1^2) - (2k_5a_1^2b + 2k_5a_1b_1)]\phi^2\\
&+ [k_0(6a_0^2b_1^2 + 4a_1b_1^3) + k_1(3a_0b_1^2) + k_2(b_1^2) - (2k_5b_1^2b + 2k_5a_1b_1b^2)]\phi^{-2}\\
&+ [k_0(4a_0a_1^3) + k_1(a_1^3) - (2k_5a_0a_1 + 2k_6a_1)]\phi^3\\
&+ [k_0(4a_0b_1^3) + k_1(b_1^3) - (2k_5a_0b_1b^2 + 2k_6b_1b^2)]\phi^{-3}\\
&+ [k_0(a_1^4) - (2k_5a_1^2)]\phi^4 + [k_0(b_1^4) - (2k_5b_1^2b^2)]\phi^{-4} = 0.
\end{aligned} \tag{60}$$

We get a system of algebraic equations, for a_0, a_1, b_1 and b

$$\begin{aligned}
&k_0a_0^4 + 12k_0a_0^2a_1b_1 + 6k_0a_1^2b_1^2 + k_1a_0^3 + 6k_1a_0a_1b_1\\
&+ k_2a_0^2 + 2k_2a_1b_1 + k_3a_0 - 4k_5a_1b_1b_+k_4 = 0, (1''')\\
&4k_0a_0^3a_1 + 12k_0a_0a_1^2b_1 + 3k_1a_0^2a_1 + 3k_1a_1^2b_1 + 2k_2a_0a_1\\
&+ k_3a_1 - 2k_5a_0a_1b - 2k_6a_1b = 0, (2''')\\
&4k_0a_0^3b_1 + 12k_0a_0a_1b_1^2 + 3k_1a_0^2b_1 + 3k_1a_1b_1^2 + 2k_2a_0b_1\\
&+ k_3b_1 - 2k_5a_0b_1b - 2k_6b_1b = 0, (3''')\\
&6k_0a_0^2a_1^2 + 4k_0a_1^3b_1 + 3k_1a_0a_1^2 + k_2a_1^2 - 2k_5a_1^2b{-}2k_5a_1b_1 = 0, (4''')\\
&6k_0a_0^2b_1^2 + 4k_0a_1b_1^3 +_3 k_1a_0b_1^2 + k_2b_1^2 - 2k_5b_1^2b - 2k_5a_1b_1b^2 = 0, (5''')\\
&4k_0a_0a_1^3 + k_1a_1^3 - 2k_5a_0a_1 - 2k_6a_1 = 0, (6''')\\
&4k_0a_0b_1^3 + k_1b_1^3 - 2k_5a_0b_1b^2 - 2k_6b_1b^2 = 0, (7''')\\
&k_0a_1^4 - 2k_5a_1^2 = 0, (8'''); k_0b_1^4 - 2k_5b_1^2b^2 = 0.(9''')
\end{aligned} \tag{61}$$

We solve the obtained system of algebraic equations give the following three cases:

Case (I): $a_1 = 0$, from equation $(5''') : b = (6k_0a_0^2 + 3k_1a_0 + k_2)/(2k_5)$; from equation $(9''') : b_1 = (6k_0a_0^2 + 3k_1a_0 + k_2)/\sqrt{2k_0k_5}$,

$$v(x,t) = a_0 + A(cot(A_1z)), \tag{62}$$

so, the travelling wave solution is given by:

$$w(x,t) = A_0[a_0 + A(cot(A_1 z))] + B_0, \tag{63}$$

so, the travelling wave solution is given by:

$$u(x,t) = \alpha[a_0 + A(cot(A_1 z))]^2 + \beta[a_0 + A(cot(A_1 z))] + \gamma, \tag{64}$$

where

$$A = \sqrt{(6k_0 a_0^2 + 3k_1 a_0 + k_2)/k_0};$$

$$A_1 = \sqrt{b} = \sqrt{(6k_0 a_0^2 + 3k_1 a_0 + k_2)/(2k_5)}.$$

Case (II): $b_1 = 0$, from equation $(8^{\prime\prime\prime})$: $a_1 = \sqrt{(2k_5)/k_0}$. From equation $(4^{\prime\prime\prime})$:$b = (6k_0 a_0^2 + 3k_1 a_0 + k_2)/(2k_5)$,

$$v(x,t) = a_0 + A(tan(A_1 z)), \tag{65}$$

so, the travelling wave solution is given by:

$$w(x,t) = A_0[a_0 + A(tan(A_1 z))] + B_0, \tag{66}$$

so, the travelling wave solution is given by:

$$u(x,t) = \alpha[a_0 + A(tan(A_1 z))]^2 + \beta[a_0 + A(tan(A_1 z))] + \gamma. \tag{67}$$

Case (III): From equation $(8^{\prime\prime\prime})$: $a_1 = \sqrt{(2k_5)/k_0}$.

From equation $(9^{\prime\prime\prime})$: $b_1 = \sqrt{(2k_5)/k_0} b$ and from equation $(4^{\prime\prime\prime})$: $b = (-(6k_0 a_0^2 + 3k_1 a_0 + k_2))/(4k_5)$. Hence

$$b_1 = -(6k_0 a_0^2 + 3k_1 a_0 + k_2)/2\sqrt{2k_0 k_5},$$

$$v(x,t) = a_0 + (A_i/\sqrt{2})(tan((A_1 i/\sqrt{2})z) + cot((A_1 i/\sqrt{2})z)). \tag{68}$$

So, the travelling wave solution is given by:

$$w(x,t) = A_0[a_0 + (A_i/\sqrt{2}(tan((A_1 i/\sqrt{2})z) + cot((A_1 i/\sqrt{2})z)) + B_0. \tag{69}$$

Therefore, the travelling wave solution is given by:

$$u(x,t) = \alpha[a_0 + (A_i/\sqrt{2}(tan((A_1 i/\sqrt{2})z) + cot((A_1 i/\sqrt{2})z))]^2 + \beta[a_0 + (Ai/\sqrt{2}(tan((A_1 i/\sqrt{2})z) + cot((A_1 i/\sqrt{2})z))] + \gamma. \tag{70}$$

4. Conclusion

In this paper, the MEDA method has been successfully applied to obtain solutions of some important nonlinear systems of partial differential equations namely, the classical Drinfel'd-Sokolov-Wilson system (DSWW), the (2+1)-dimensional Davey-Stewartson system, and the generalized Hirota-Satsuma coupled KdV system, it is also a promising method to solve other nonlinear systems.

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